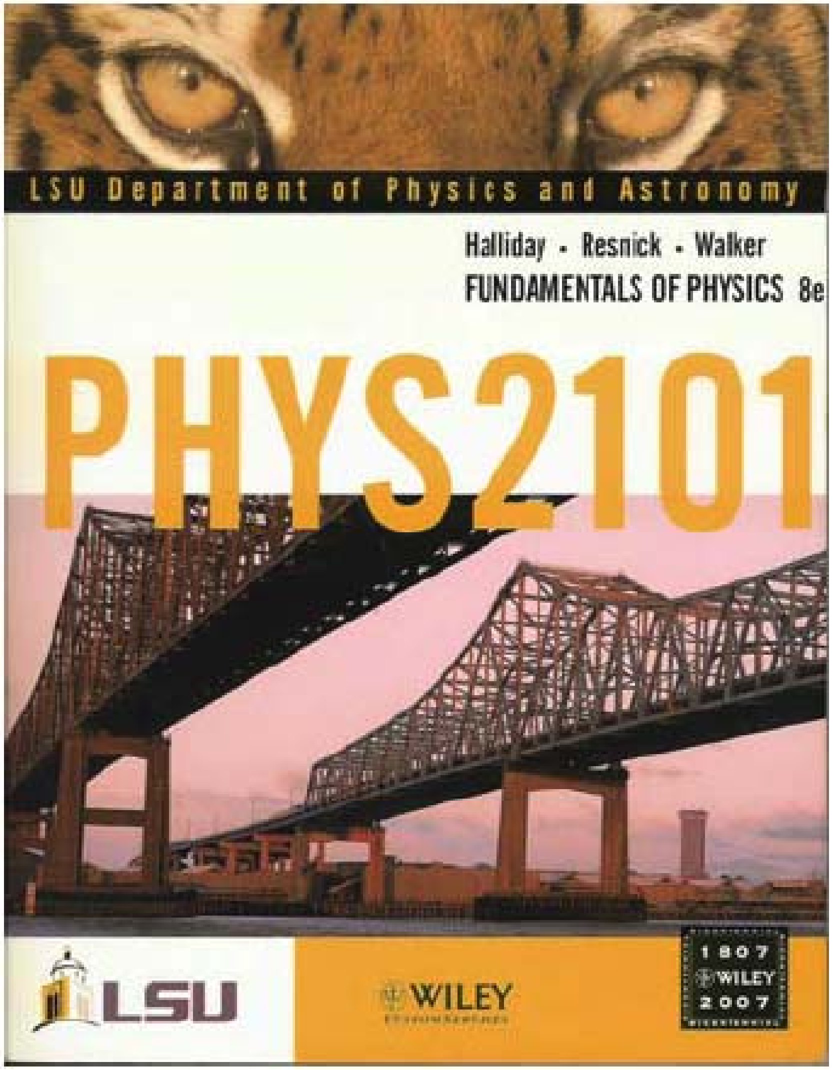


Physics 2101
Section 3
May 3rd: Chap. 19

Announcements:

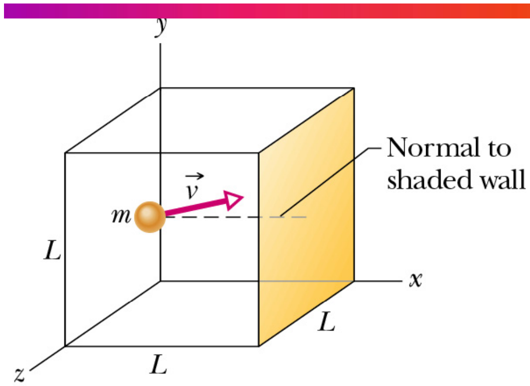
- Final Exam: May 11th (Tuesday), 7:30 AM at Howe-Russell 130
- Make up Final: May 15th (Saturday) 7:30 AM at Nicholson 119
- Final Exam for those who need extended time will be at Nicholson 109

Class Website:

http://www.phys.lsu.edu/classes/spring2010/phys2101-3/

http://www.phys.lsu.edu/~jzhang/teaching.html

Pressure, Temperature, & RMS speed



Assume the collision of the gas molecule with the wall is elastic then:

$$\Delta p_x = (-mv_x) - (mv_x) = -2mv_x$$

The molecule travels to the back wall, collides and comes back. The time it takes is $2L/v_x$.

$$\frac{\Delta p}{\Delta t} = \frac{2mv_x}{2L/v_x} = \frac{mv_x^2}{L}$$

But the pressure is F/A

$$p = \frac{F_x}{A} = \frac{1}{L^2}\sum_{i=1}^{n}\frac{mv_{xi}^2}{L} = \frac{m}{L^3}\sum_{i=1}^{n} v_{xi}^2$$

If we calculated the average velocity and $(v_x^2)_{avg}$ use the fact that the number in the sum is nN_A then:

$$p = \frac{nmN_A}{L^3}(v_x^2)_{avg} = \frac{nM(v_x^2)_{avg}}{V}$$

$$= \frac{nM(v^2)_{avg}}{3V} = \frac{nMv_{rms}^2}{3V}$$

$$v^2 = v_x^2 + v_y^2 + v_z^2$$

$$v_x^2 = \frac{v^2}{3}$$

$$\sqrt{(v^2)_{avg}} = v_{rms}$$

RMS = Root-Mean-Square

M ---Molar mass

RMS Speeds

We have $p = \dfrac{nM\left(v^2\right)_{ave}}{3V}$

For ideal gas $pV = nRT$

$$v_{rms} = \left(\frac{3pV}{nM}\right)^{1/2} = \left(\frac{3nRT}{nM}\right)^{1/2}$$

$$v_{rms} = \sqrt{\frac{3RT}{M}} \quad (19\text{-}22)$$

The RMS velocity depends on:

Molar mass & **Temperature**

TABLE 19-1

Some RMS Speeds at Room Temperature (T = 300 K)[a]

Gas	Molar Mass (10^{-3} kg/mol)	v_{rms} (m/s)
Hydrogen (H_2)	2.02	1920
Helium (He)	4.0	1370
Water vapor (H_2O)	18.0	645
Nitrogen (N_2)	28.0	517
Oxygen (O_2)	32.0	483
Carbon dioxide (CO_2)	44.0	412
Sulfur dioxide (SO_2)	64.1	342

[a]For convenience, we often set room temperature equal to 300 K even though (at 27°C or 81°F) that represents a fairly warm room.

Problem 19-3: Here are five numbers: 5, 11, 32, 67, and 89.
(a) What is the average value n_{avg}?
(b) What is the rms value n_{rms} of the numbers?

A close look at "RMS"

(a)

$$n_{avg} = \frac{5+11+32+67+89}{5} = 40.8$$

(b)

$$(n^2)_{avg} = \frac{1}{n}\sum_{i=1}^{n} n_i^2 = \frac{5^2+11^2+32^2+67^2+89^2}{5} = 2714.41$$

$$\sqrt{(n^2)_{avg}} = 52.1$$

Problem 19-18: Calculate the rms speed of helium atoms at 1000K. Helium has 2 protons and 2 neutrons, what is the molar mass?

Appendix F: $M = 4.00x10^{-3}\, kg/mol$

Use Eqn 19-22

$$v_{rms} = \sqrt{\frac{3RT}{M}}$$

$$v_{rms} = \sqrt{\frac{3RT}{M}} = \sqrt{\frac{3(8,31J/molK)(1000K)}{4.00\times10^{-3}\, kg/mol}} = 2.50x10^{3}\, m/s$$

Translational Kinetic Energy

$$K_{\text{avg}} = \frac{3kT}{2}$$

The kinetic energy of a gas molecule $K = \frac{mv^2}{2}$.

Its average kinetic energy $K_{\text{avg}} = \left(\frac{mv^2}{2}\right)_{\text{avg}} = \frac{mv_{\text{rms}}^2}{2}$.

Thus $K_{\text{avg}} = \frac{m}{2}\frac{3RT}{M} = \frac{3RT}{2N_{\text{A}}}$.

We finally get: $K_{\text{avg}} = \frac{3kT}{2}$

This is valid only for monoatomic system!

At a given temperature *T* all ideal gas molecules, no matter what their mass, have the same average translational kinetic energy. When we measure the temperature of a gas, we are also measuring the average translational kinetic energy of its molecules.

What is the average velocity?

Problem 19-26: What is the average translational kinetic energy of nitrogen molecules at 1600 K?

Average Kinetic energy is

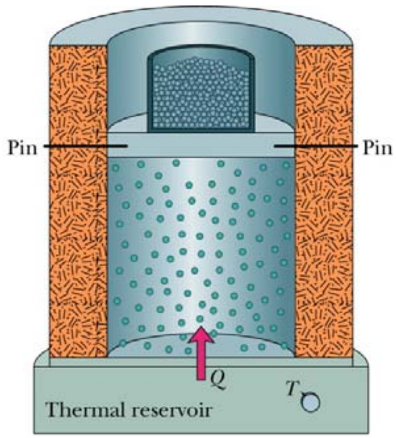
$$\mathrm{K}_{avg} = \frac{3}{2}kT$$

$$\mathrm{K}_{avg} = \frac{3}{2}kT = \frac{3}{2}\left(1.38x10^{-23}J\right)\left(1600K\right) = 3.31x10^{-20}J$$

Molar Specific Heat: Monatomic Ideal Gas

Molar Specific Heat at Constant Volume (W_{by}=0)

$$Q = nC_V\Delta T \qquad \& \qquad W_{by} = 0$$

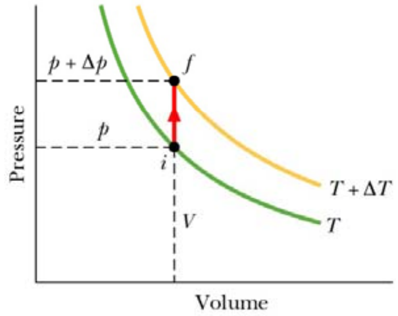


(a)

$$\Delta E_{\text{int}} = Q = nC_V\Delta T$$

Always true if *V* = const.

$$\Delta E_{\text{int}} = n\left(\tfrac{3}{2}R\right)\Delta T = n\left(C_V\right)\Delta T = Q$$

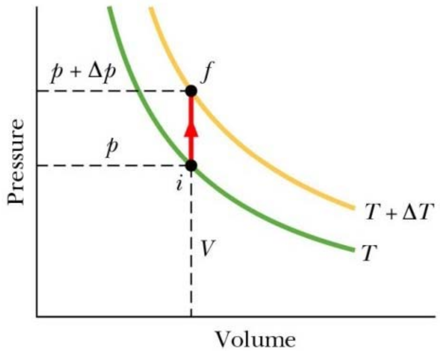


(b)

$$C_{V,monatomic} = \frac{3}{2}R = 12.5\,\text{J/mol}\cdot\text{K}$$

Molar Specific Heat: Monatomic ideal gas

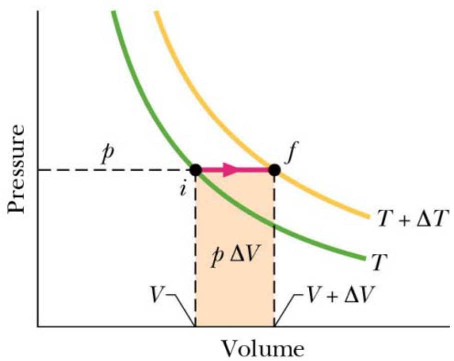
Molar Specific Heat at Constant Volume (W_{by}=0)



$$\Delta E_{\text{int}} = Q = nC_V \Delta T$$ ← Always true for const. ***V***

$$C_{V,monatomic} = \frac{3}{2} R = 12.5 \cdot \text{J/mol} \cdot \text{K}$$

Molar Specific Heat at Constant Pressure (W_{by}=p∆V)



$$\Delta E_{\text{int}} = Q - W = nC_p \Delta T - p\Delta V$$

$$nC_V \Delta T = nC_p \Delta T - nR\Delta T$$

$$C_V = C_p - R \qquad \text{or} \qquad C_p = C_V + R$$

$$C_{p,monatomic} = \frac{5}{2} R = 20.8 \cdot \text{J/mol} \cdot \text{K}$$

Molecular Specific heat Constant Volume C_V

$$C_{V,monatomic} = \frac{3}{2}R = 12.5 \cdot \text{J/mol} \cdot \text{K}$$

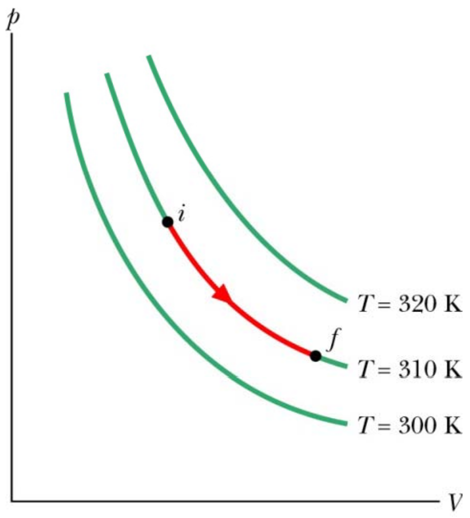
$$\Delta E_{\text{int}} = nC_V \Delta T$$

TABLE 19-2

Molar Specific Heats at Constant Volume

Molecule	Example		C_V (J/mol · K)
Monatomic	Ideal		$\frac{3}{2}R = 12.5$
	Real	He	12.5
		Ar	12.6
Diatomic	Ideal		$\frac{5}{2}R = 20.8$
	Real	N_2	20.7
		O_2	20.8
Polyatomic	Ideal		$3R = 24.9$
	Real	NH_4	29.0
		CO_2	29.7

Work Done by Isothermal ($\Delta T = 0$) Expansion/Compression of Ideal Gas



On p-V graph, the green lines are isotherms…
… each green line corresponds to a system at a constant temperature.

From ideal gas law, this means that for a given isotherm:

$$pV = \text{constant} \quad \Rightarrow \quad p = (nRT)\frac{1}{V}$$

Relates p and V

The work done by the gas is then:

$$W_{by} = \int_{V_i}^{V_f} p\,dV = \int_{V_i}^{V_f} \left(\frac{nRT}{V}\right) dV = nRT \int_{V_i}^{V_f} \frac{dV}{V}$$

$$\Rightarrow \quad W_{\substack{by,\text{isothermal} \\ \Delta T = 0}} = nRT \ln\left(\frac{V_f}{V_i}\right) = nRT \ln\left(\frac{p_i}{p_f}\right)$$

Adiabatic Expansion of an ideal gas

Because a gas is thermally insulated, or expansion/compression happens suddenly ⇒ adiabatic

Remember "Adiabatic means Q = 0"

or, by 1st Law of Thermo ⇒ $\Delta E_{int} = -W_{by}$

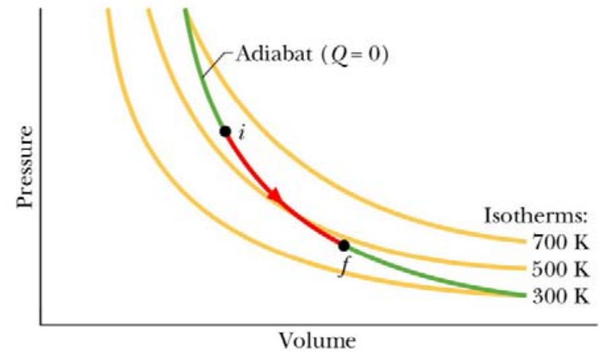
In this case **pV^{γ} = constant** where **$\gamma = C_p/C_V = (R + C_V)/C_V$**

example: <u>monatomic gas</u> **$\gamma = 5/3$**

Adiabatic Expansion

$$p_1 V_1^{\gamma} = p_2 V_2^{\gamma}$$

$$T_1 V_1^{\gamma - 1} = T_2 V_2^{\gamma - 1}$$



Compare with Isothermal Expansion ($\Delta T = 0$)

$$T_1 = T_2 \quad \Leftrightarrow \quad [p_1 V_1 = p_2 V_2]_{isothermal}$$

Free Expansion

Adiabatic Expansion of an ideal gas

Remember "Adiabatic means Q = 0"

or, by 1^{st} Law of Thermo $\Rightarrow$ $\Delta E_{int} = -W_{by}$

pV^{γ} = constant

Proof

$$dE_{\text{int}} = Q - pdV$$

$$dE_{\text{int}} = -pdV = nC_V dt$$

$$ndT = -\left(\frac{p}{C_V}\right) dV$$

$$pV = nRT$$

$$or \ \mathrm{pdV+Vdp=nRdT}$$

Remember $C_p - C_V = R$

giving

$$ndT = \frac{PdV + Vdp}{C_p - C_V}$$

$$-\left(\frac{p}{C_V}\right) dV = \frac{PdV + Vdp}{C_p - C_V}$$

$$\frac{dp}{p} + \left(\frac{C_p}{C_V}\right)\frac{dV}{V} = 0$$

$$\frac{dp}{p} + \gamma \frac{dV}{V} = 0$$

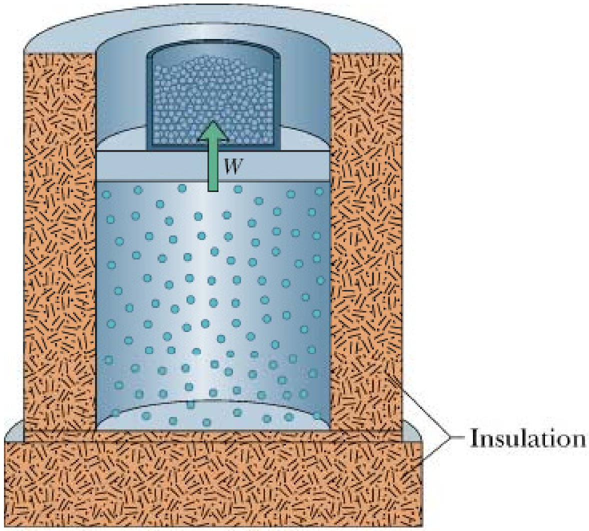
Integrating gives

$$\ln p + \gamma \ln V = \text{Constant}$$

$$or \ \mathrm{pV}^{\gamma} = \text{Constant}$$

$$p_i V_i^{\gamma} = p_f V_f^{\gamma}$$

$$T_i V_i^{\gamma-1} = T_f V_f^{\gamma-1}$$

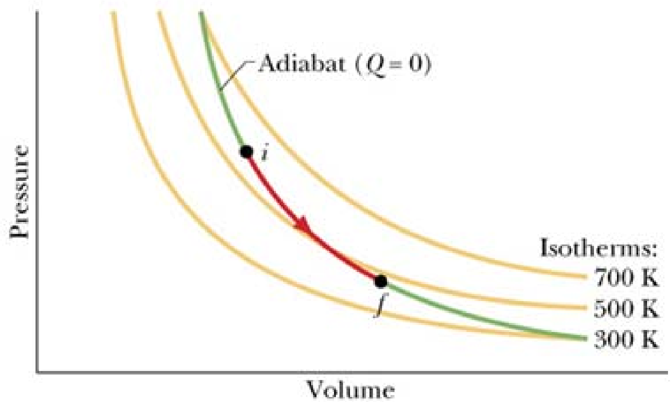


(a)

Adiabatic Expansion of an Ideal Gas

Consider the ideal gas in fig. *a*. The container is well insulated. When the gas expands, no heat is transferred to or from the gas. This process is called adiabatic. Such a process is indicated on the p-V diagram of fig. *b* by the red line. The gas starts at an initial pressure p_i and initial volume V_i. The corresponding final parameters are p_f and V_f. The process is described by the equation

$p_i V_i^{\gamma} = p_f V_f^{\gamma}$. Here the constant $\gamma = \dfrac{C_p}{C_V}$.



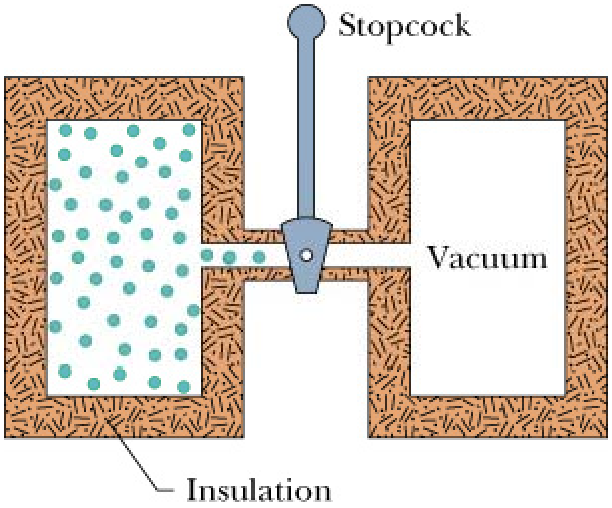
(b)

Using the ideal gas law we can get the equation

$$T_i V_i^{\gamma-1} = T_f V_f^{\gamma-1} \quad \rightarrow \quad T_f = T_i \frac{V_i^{\gamma-1}}{V_f^{\gamma-1}}$$

If $V_f > V_i$ we have adiabatic expansion and $T_f < T_i$.

If $V_f < V_i$ we have adiabatic compression and $T_f > T_i$.



$$T_i = T_f \qquad p_i V_i = p_f V_f$$

Free Expansion

In a free expansion, a gas of initial volume V_i and initial pressure p_i is allowed to expand in an empty container so that the final volume is V_f and the final pressure p_f.

In a free expansion $Q = 0$ because the gas container is insulated. Furthermore, since the expansion takes place in vacuum the net work $W = 0$.

The first law of thermodynamics predicts that $\Delta E_{\text{int}} = 0$.
Since the gas is assumed to be ideal there is no change in temperature: $T_i = T_f$.

Using the law of ideal gases we get the following equation, which connects the initial with the final state of the gas:
$p_i V_i = p_f V_f$.

$$E_{\text{int}} = \frac{3nRT}{2}$$

Internal Energy of an Ideal Gas

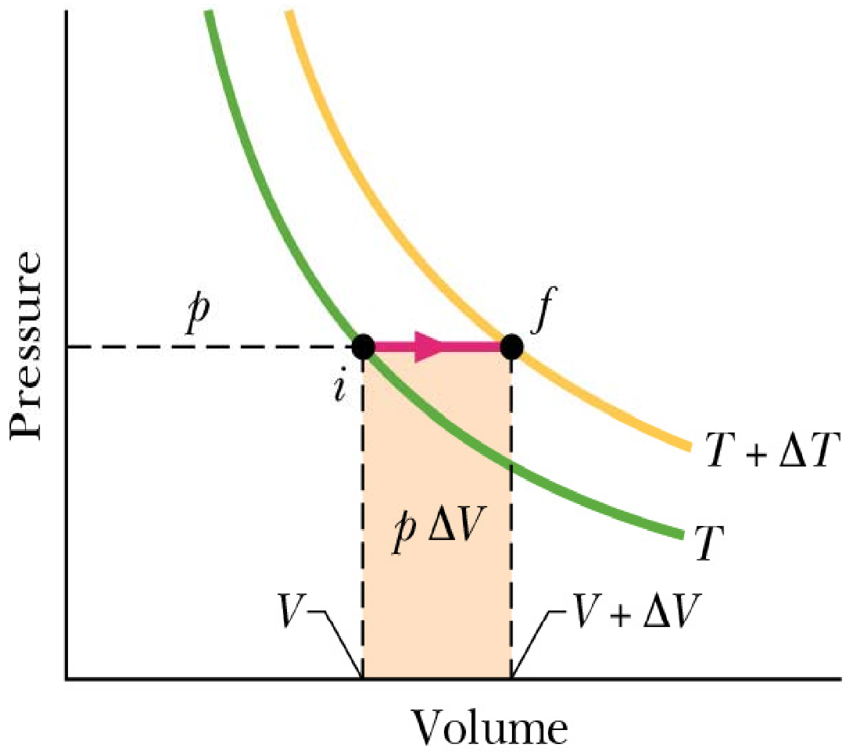
Consider a monatomic gas such as He, Ar, or Kr. In this case the internal energy E_{int} of the gas is the sum of the translational kinetic energies of the constituent atoms. The average translational kinetic energy of a single atom is given by the equation $K_{\text{avg}} = \frac{3kT}{2}$. A gas sample of n moles contains $N = nN_A$ atoms.

The internal energy of the gas $E_{\text{int}} = NK_{\text{avg}} = \frac{nN_A 3kT}{2} = \frac{3nRT}{2}$.

The equation above expresses the following important result:

The internal energy E_{int} of an ideal gas is a function of gas temperature only; it does not depend on any other parameter.

Work Done by Isobaric ($\Delta P = 0$) Expansion of an Ideal Gas



(b)

$$\Rightarrow W_{\substack{by,isobaric \\ \Delta P=0}} = p\Delta V$$

$$= nR\Delta T$$

Problem 19-6: A quantity of ideal gas at 10.0 ^{0}C and 100 kPa occupies a volume of 2.50 m^3. (a) How many moles of the gas are present? (b) If the pressure is now raised to 300 kPa and the temperature to 30.0 ^{0}C, how much volume does the gas occupy?

$$n = \frac{pV}{RT} = \frac{(100x10^3 Pa)(2.50 m^3)}{(8.31 J/mol \bullet K)(283K)} = 106 moles$$

(b) $$\frac{p_f V_f}{p_i V_i} = \frac{T_f}{T_i}$$

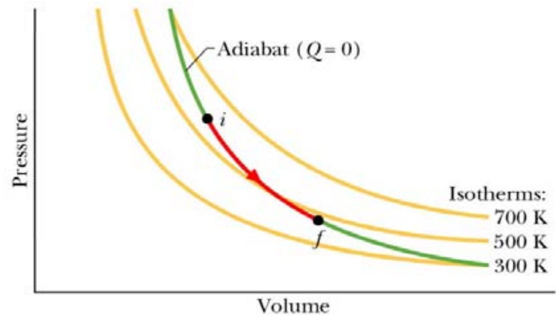
$$V_f = V_i \left(\frac{p_i}{p_f}\right)\left(\frac{T_f}{T_i}\right) = 0.892 m^3$$

Remember special cases…

Ideal Gas: $PV = nRT$ $\Delta E_{\text{int}} = \frac{3}{2} nR(\Delta T)$

Adiabatic expansion/contraction - NO TRANSFER OF ENERGY AS HEAT **Q = 0**

$$[\Delta E_{\text{int}} = -W]_{adiabatic}$$



Constant-volume processes (isochoric)-

NO WORK IS DONE **W = 0**

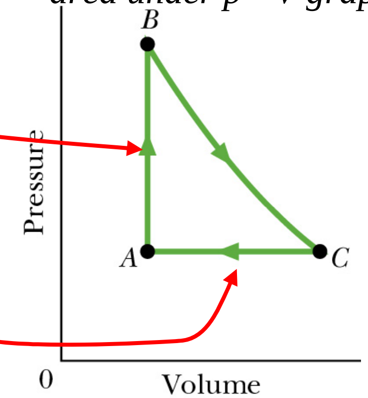
$$\Delta E_{\text{int}} = Q$$

$$W_{by} = \int_{V_i}^{V_f=V_i} pdV = 0$$

$$Q_{\Delta V=0} = nC_V \Delta T$$

$$W_{by} = \int pdV$$

$$= area\ under\ p-V\ graph$$



Constant-pressure processes

$$\Delta E_{\text{int}} = Q - p\Delta V$$

$$C_V = C_P - R$$

$$W_{by} = \int_{V_i}^{V_f=V_i} pdV = p\Delta V$$

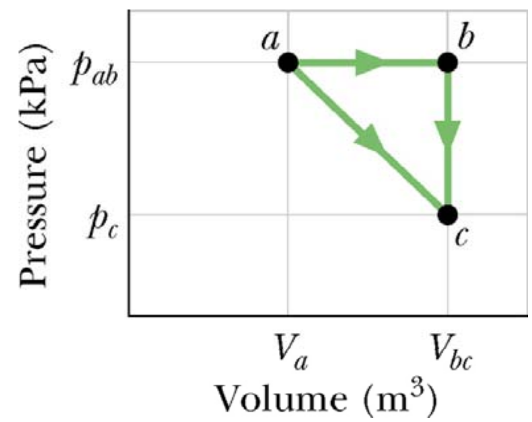
$$Q_{\Delta P=0} = nC_P \Delta T$$

3) **Cyclical process (closed cycle)** $\Delta E_{\text{int,closed cycle}} = 0$

a) net area in p-V curve is Q

$$\Delta E_{\text{int}} = 0 \Rightarrow Q = W$$

19#46: One mole of an ideal atomic gas goes from a to c along the diagonal pate. The scale of the vertical axis is set by p_{ab}=5.0 kPa and p_c=2.0 kPa, and the scale of the V axis is V_{bc}4.0 m^3 and V_a=2.0 m^3. (a) what is the change in the internal Energy? (b) How much Energy is added to the gas? (c) How much heat is required if the gas goes from a to c via abc?



For any straight line on pv plot it is easy to prove

$$E_{\text{int}} = n\left(\frac{3}{2}\right)RT$$

$$W_{straight} = \left(\frac{p_f + p_i}{2}\right)\Delta V$$

$$(a)\ E_{\text{int}\,c} - E_{\text{int}\,a} = \left(\frac{3}{2}\right)(T_c - T_a) = \left(\frac{3}{2}\right)(p_c V_c - p_a V_a) = -3000\,J$$

$$(b)\ W = \left(\frac{p_a + p_c}{2}\right)(V_c - V_a) = 7000\,J$$

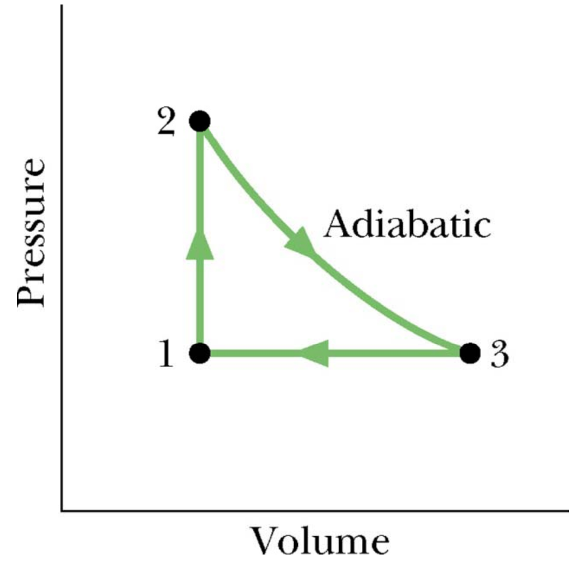
$$Q = \Delta E_{\text{int}} + W = 2000\,J$$

(c) ΔE_{int} found in (a)

$$W = (5.0x10^3\,Pa)(2m^3) = 10000\,J$$

$$Q = 5000\,J$$

19 #63: 1.00 mol of an ideal monatomic gas goes through the cycle shown in the Figure. The temperatures are T_1 =300 K, T_2=600 K, and T_3= 455K. For 1 to 2, what are (a) heat Q, (b) the change in internal energy, and (c) the work done W? For 2 to 3, what are (d) Q, (e) change in E_{int}, and (i) W? For the full cycle, what are (j) Q, (k) change in E_{int}, and (l) W. The Initial pressure at point 1 is 1.00 atm. What are the (m) volume and (n) pressure at point 2 and the (o) volume and (p) pressure at point 3.



$$\text{(a, b, c) For } 1 \Rightarrow 2,\ \Delta V = 0,\ W = 0,\ \Delta E_{\text{int}} = n\left(\tfrac{3}{2}R\right)\Delta T = n\left(C_V\right)\Delta T = Q; \quad \text{Here } \Delta T = T_2 - T_1$$

$$\text{(d, e, f) For } 2 \Rightarrow 3,\ \text{Q} = 0,\ \Delta E_{\text{int}} = n\left(\tfrac{3}{2}R\right)\Delta T = -W; \quad \text{Here } \Delta T = T_3 - T_2$$

$$\text{For } 3 \Rightarrow 1, \quad \Delta E_{\text{int}} = n\left(\tfrac{3}{2}R\right)\Delta T < 0; \quad W = p\Delta V = nR\Delta T < 0;$$

$$Q = \Delta E_{\text{int}} + W = \frac{5}{2}nR\Delta T; \quad \text{Here } \Delta T = T_1 - T_3$$

Problem 19-21: (a) Compute the rms speed of a nitrogen molecule at 20.0 ^{0}C. Each N atom has 7 protons and 7 neutrons? (a) what is the rms speed at 300K and 20.0 ^{0}C. At what temperatures will the rms speed be (b) half that value and (c) twice that value?

Table 19-1: $M = 28g/mol$

(a) Use Eqn 19-22

$$v_{rms} = \sqrt{\frac{3RT}{M}}$$

$$v_{rms} = 517m/s \quad (\text{for } T = 300K)$$

Set up ratios

$$\frac{v_2}{v_1} = \sqrt{\frac{3RT_2/M}{3RT_1/M}} = \sqrt{\frac{T_2}{T_1}}$$

$$v_1 = 517m/s \quad \text{for } T = 300\text{K}$$

$$v_2 = (517m/s)\sqrt{\frac{293K}{300K}}$$

$$v_2 = 511m/s$$

(b) Set $v_3 = \frac{v_2}{2}$ and solve

$$T_3 = 73K$$

(c) Set $v_4 = 2v_2$ and solve

$$T_4 = 1170K$$