

# Physics 2101 Section 3 April 30<sup>th</sup>: Chap. 19

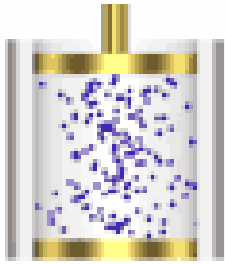
## Announcements:

- **Final Exam: May 11<sup>th</sup>**  
(Tuesday), 7:30 AM at Howe-Russell 130
- **Make up Final: May 15<sup>th</sup>**  
(Saturday) 7:30 AM at Nicholson 119

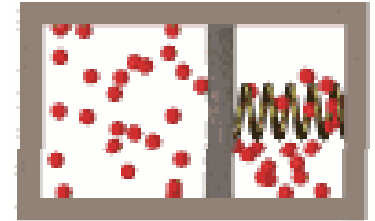
## Class Website:

<http://www.phys.lsu.edu/classes/spring2010/phys2101-3/>

<http://www.phys.lsu.edu/~jzhang/teaching.html>



# Chap. 19: Kinetic Theory of Gases



Thermodynamics = macroscopic picture

Gases = micro -> macro picture

## IDEAL GAS LAW

$n$  = number of moles

$N$  = number of particles

$$pV = nRT \quad pV = NkT$$

$$k = 1.38 \times 10^{-23} \text{ J/K}$$

$$R = kN_A$$

$$\begin{aligned} R &= 8.315 \text{ J}/(\text{mol}\cdot\text{K}) \\ &= 0.0821 \text{ (L}\cdot\text{atm)} / (\text{mol}\cdot\text{K}) \\ &= 1.99 \text{ calories}/(\text{mol}\cdot\text{K}) \end{aligned}$$

Monoatomic ideal gas : He, Ar, Ne, Kr... (no potential energies)

**Atoms**

$$E_{\text{int,monotonic}} = N \left( \frac{3}{2} kT \right) = \frac{3}{2} nRT$$

$$\Delta E_{\text{int,monotonic}} = \frac{3}{2} nR(\Delta T)$$

**The internal energy of an ideal gas depends only on the temperature**

## Ideal Gases, Ideal Gas Law

$$pV = nRT$$

$$pV = NkT$$

It was found experimentally that if 1 mole of any gas is placed in containers that have the same volume  $V$  and are kept at the same temperature  $T$ , approximately all have the same pressure  $p$ . The small differences in pressure disappear if lower gas densities are used.

Further experiments showed that all low-density gases obey the equation  $pV = nRT$ . Here  $R = 8.31 \text{ J/mol}\cdot\text{K}$  and is known as the "**gas constant.**" The equation itself is known as the "**ideal gas law.**" The constant  $R$  can be expressed as  $R = kN_A$ .

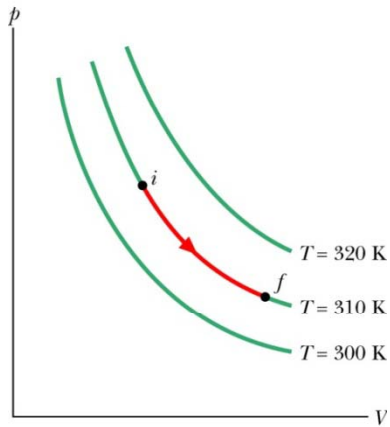
Here  $k$  is called the Boltzmann constant and is equal to  $1.38 \times 10^{-23} \text{ J/K}$ .

If we substitute  $R$  as well as  $n = \frac{N}{N_A}$  in the ideal gas law we get the equivalent form:

$pV = NkT$ . Here  $N$  is the number of molecules in the gas.

The behavior of all real gases approaches that of an ideal gas at low enough densities. Low densities means that the gas molecules are far enough apart that they do not interact with one another, but only with the walls of the gas container.

# Work done by **isothermal** ( $\Delta T = 0$ ) expansion of ideal gas



On p-V graph, the green lines are isotherms...

... each green line corresponds to a system at a constant temperature.

From ideal gas law, this means that for a given isotherm:

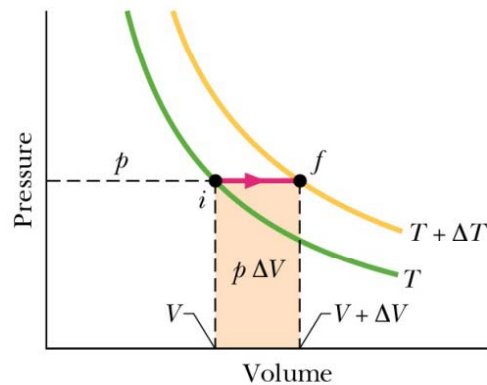
$$pV = \text{constant} \quad \Rightarrow \quad p = (nRT) \frac{1}{V} \quad \text{Relates } p \text{ and } V$$

The work done by the gas is then:

$$W_{by} = \int_{V_i}^{V_f} p dV = \int_{V_i}^{V_f} \left( \frac{nRT}{V} \right) dV = nRT \int_{V_i}^{V_f} \frac{dV}{V}$$

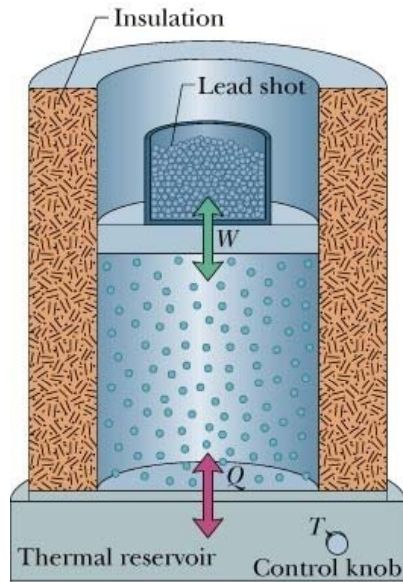
$$\Rightarrow W_{by, \text{isothermal}}^{\Delta T=0} = nRT \ln \left( \frac{V_f}{V_i} \right) = nRT \ln \left( \frac{p_i}{p_f} \right)$$

# Work done by **isobaric** ( $\Delta P = 0$ ) expansion of an ideal gas



(b)

$$\Rightarrow W_{by, \text{isobaric}}^{\Delta P=0} = p\Delta V = nR\Delta T$$



## Work Done by an Ideal Gas at Constant Temperature

Consider the gas shown in the figure. It is held at a constant temperature  $T$  and undergoes an isothermal expansion from volume  $V_i$  to volume  $V_f$ . The process follows the red line on the lower figure. The work  $W$  done by the ideal gas is given by the equation

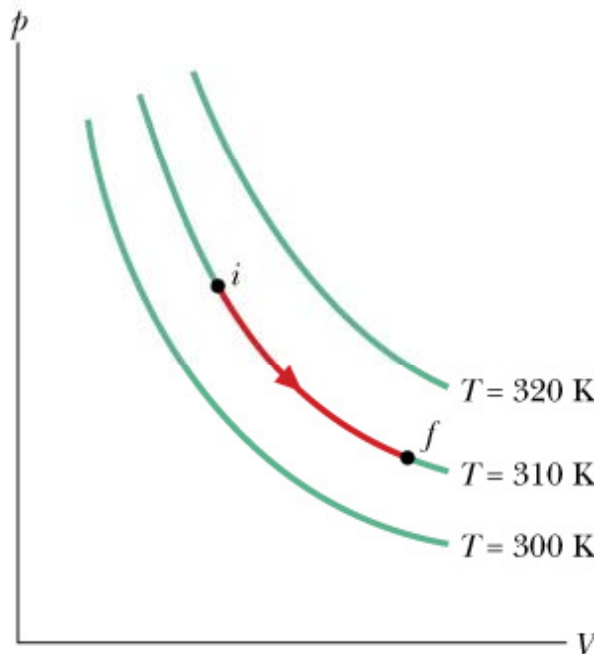
$$W = \int_{V_i}^{V_f} p dV. \quad \text{From the ideal gas law we have that}$$

$$p = \frac{nRT}{V} \rightarrow W = \int_{V_i}^{V_f} \frac{nRT}{V} dV = nRT \int_{V_i}^{V_f} \frac{dV}{V} = nRT \left[ \ln V \right]_{V_i}^{V_f};$$

$$W = nRT \ln \frac{V_f}{V_i}.$$

For **expansion** we have :  $V_f > V_i \rightarrow \ln \frac{V_f}{V_i} > 0 \rightarrow W > 0$ .

For **compression** we have :  $V_f < V_i \rightarrow \ln \frac{V_f}{V_i} < 0 \rightarrow W < 0$ .



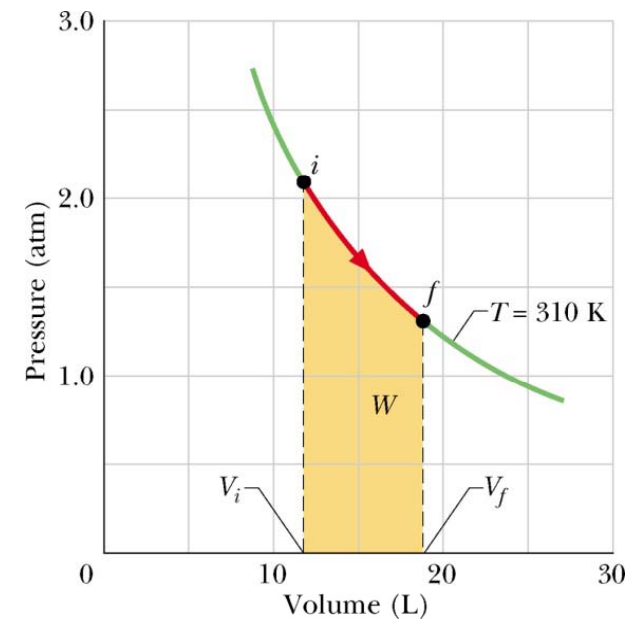
Sample problem 19-2: One mole of oxygen expands at a constant temperature  $T$  of 310 K from an initial volume  $V_i$  of 12 L to a final volume  $V_f$  of 19 L. How much work is done by the gas during the expansion.

We calculated  $W$  for isothermal expansion

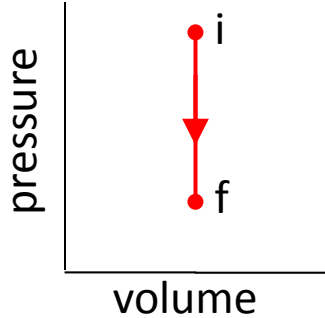
$$W = nRT \ln(V_f/V_i)$$

$$W = (1 \text{ mole})(8.31 \text{ J/mole K})(310 \text{ K}) \ln(19/12)$$

$$W = 1180 \text{ J}$$



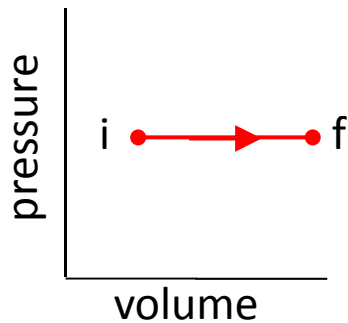
# Summary



## Work Done by an Ideal Gas at Constant Volume

Consider process  $i \rightarrow f$ . and  $V = \text{constant}$ .

The work  $W$  done by the gas is  $W = \int p dV = 0$ .

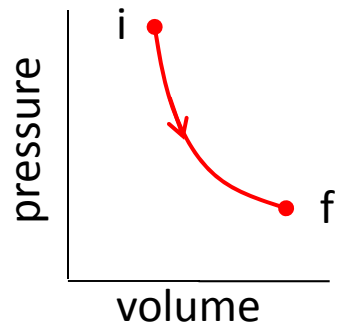


## Work Done by an Ideal Gas at Constant Pressure

Consider process  $i \rightarrow f$ . and  $P = \text{constant}$ .

The work  $W$  done by the gas is

$$W = \int_{V_i}^{V_f} p dV = p \int_{V_i}^{V_f} dV = p(V_f - V_i).$$



## Work Done by an Ideal Gas at Constant Temperature

Consider process  $i \rightarrow f$ . and  $T = \text{constant}$ .

$W = \int_{V_i}^{V_f} p dV$ . From the ideal gas law we have that

$$p = \frac{nRT}{V} \rightarrow W = \int_{V_i}^{V_f} \frac{nRT}{V} dV = nRT \int_{V_i}^{V_f} \frac{dV}{V} = nRT \ln \frac{V_f}{V_i}$$

19-6: A quantity of ideal gas at 10.0 °C and 100 kPa occupies a volume of 2.50 m<sup>3</sup>. (a) How many moles of the gas are present? (b) If the pressure is now raised to 300 kPa and the temperature to 30.0 °C, how much volume does the gas occupy?

$$(a) \quad n = \frac{pV}{RT} = \frac{(100 \times 10^3 \text{ Pa})(2.50 \text{ m}^3)}{(8.31 \text{ J / mol} \cdot \text{K})(283 \text{ K})} = 106 \text{ moles}$$

$$(b) \quad \frac{p_f V_f}{p_i V_i} = \frac{T_f}{T_i}$$

$$V_f = V_i \left( \frac{p_i}{p_f} \right) \left( \frac{T_f}{T_i} \right) = 0.892 \text{ m}^3$$



19-9: Suppose 1.80 mole of an ideal gas is taken from a volume of 3.00 m<sup>3</sup> to a volume of 1.50 m<sup>3</sup> via an isothermal compression at 30 °C. (a) How much energy is transferred as heat during the compression, and (b) is the transfer *to* or *from* the gas?

Use Eqn. 19-45

$$\Delta E_{\text{int}} = nC_V\Delta T$$

But  $\Delta T=0$

$$\Delta E_{\text{int}} = 0$$

$$\Delta E_{\text{int}} = Q - W$$

$$Q = W$$

Eqn 19-14

$$W = nRT \cdot \ln \left[ \frac{V_f}{V_i} \right]$$

$$Q = nRT \cdot \ln \left[ \frac{V_f}{V_i} \right]$$

$$Q = (1.80)(8.31 \text{ J / mol.K})(273 + 30 \text{ K}) \cdot \ln \left[ \frac{1.5}{3} \right]$$

$$Q = -3.14 \times 10^3 \text{ J}$$

19-10: *Water bottle in a hot car.* In the American Southwest, the temperature in a closed car parked in the sun during the summer can be high enough to burn flesh. Suppose a bottle of water at a refrigerator temperature of  $5^{\circ}\text{C}$  is opened, then closed, and then left in a closed car with an internal temperature of  $75^{\circ}\text{C}$ . Neglecting the thermal expansion of the water and the bottle, find the pressure in the air pocket trapped in the bottle.

$$T_i = 5^{\circ}\text{C} = 278\text{K}$$

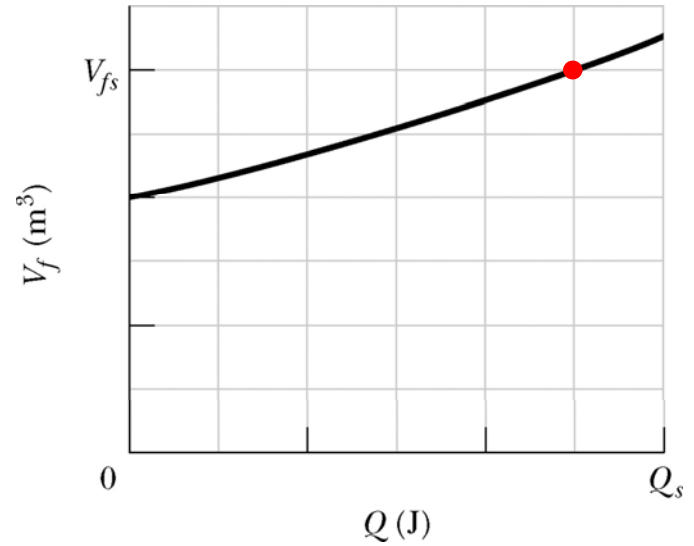
$$T_f = 75^{\circ}\text{C} = 348\text{K}$$

Use Ideal Gas Law with  $V_i = V_f$

$$\frac{p_f V_f}{p_i V_i} = \frac{T_f}{T_i}$$

$$p_f = \frac{T_f}{T_i} p_i = 1.25\text{atm}$$

19-11: Suppose 0.825 mol of an ideal gas undergoes an isothermal expansion as energy is added to it as heat  $Q$ . The figure shows the final volume  $V_f$  versus  $Q$ , what is the gas temperature ? (The scale of the vertical axis is set by  $V_{fs}=0.30 \text{ m}^3$ , and the scale of the horizontal axis by  $Q_s=1200 \text{ J}$ ).



Remember  $\Delta E_{\text{int}} = 0$

$$\Delta E_{\text{int}} = Q - W = 0; \quad Q = W$$

Eqn 19-14

$$Q = nRT \cdot \ln \left[ \frac{V_f}{V_i} \right]$$

This Eqn applies anywhere

$$Q = nRT \cdot \ln \left[ \frac{V_f}{V_i} \right]$$

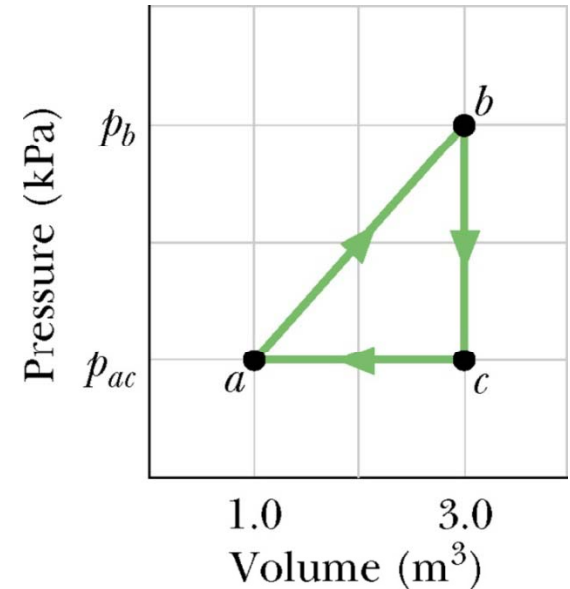
Pick a point on graph

$$Q = 1000 \text{ J}; \quad V_f = 0.30 \text{ m}^3$$

$$Q = nRT \cdot \ln \left[ \frac{V_f}{V_i} \right] \quad \text{with } V_i = 0.20 \text{ m}^3$$

$$T = 360 \text{ K}$$

19-15: A sample of an ideal gas is taken through the cyclic process  $abca$  shown in the figure. The scale is  $p_b = 7.5$  kPa and  $p_{ac} = 2.5$  kPa. At point  $a$ ,  $T = 200$  K. (a) How many moles of gas are in the sample? What are (b) the temperature of the gas at point  $b$ , (c) the temperature of the gas at point  $c$ , and (d) the net energy added to the gas as heat during the cycle?



(a) We know everything at point  $a$

$$n = \frac{pV}{RT} = \frac{(2500 \text{ Pa})(1.0 \text{ m}^3)}{(8.31 \text{ J/molK})(200 \text{ K})}$$

$$n = 1.5 \text{ mol.}$$

(b) Use Ideal Gas Law (ratios)

$$\frac{p_b V_b}{p_a V_a} = \frac{T_b}{T_a}$$

$$T_b = 1.8 \times 10^3 \text{ K}$$

(c) Use Ideal Gas Law (ratios)

$$\frac{p_c V_c}{p_a V_a} = \frac{T_c}{T_a}$$

$$T_c = 600 \text{ K}$$

(d) Energy added:  $\Delta E_{\text{int}} = 0$

$$Q = W$$

$$Q = \frac{1}{2}(\text{base})(\text{height}) = 5000 \text{ J}$$