Announcement

HELP:

- See me (office hours).
- There will be a HW help session on Monday night from 7-8 in Nicholson 109.
- Tutoring at #102 of Nicholson Hall.

Application of kinematic equation:

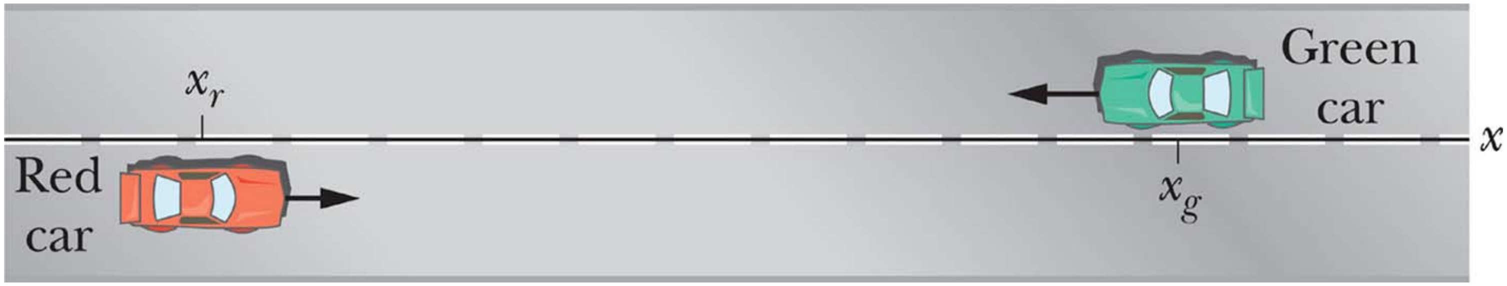
$$a = const.$$

$$v = v_0 + at$$

$$x = x_0 + v_0 t + \frac{at^2}{2}$$

$$v^2 = v_0^2 + 2a(x - x_0)$$

Example: A red car and a green car, identical except for the color, move toward each other in adjacent lanes and parallel to the x axis. At time t=0, the red car is at x_r=0 and the green car is at x_g= 220 m. If the red car has a constant velocity of 20 km/h, the cars pass each other at x =44.5 m, and if it has a constant velocity of 40 km/h, they pass each other at x =76.6m. What are (a) the initial velocity and (b) the acceleration of the green car?



Red car: a=0 so

$$x_f(1) = v_1 t_1$$
$$x_f(2) = v_1 t_2$$

$$v_1 = \frac{20km}{h} = \frac{50}{9} m/s$$
$$v_2 = \frac{40km}{h} = \frac{100}{9} m/s$$

$$t_1 = 8.0s$$
$$t_2 = 6.9s$$

Now we must simultaneously solve two equations for the green car.

$$44.5 - 220 = v_0 t_1 + \frac{a t_1^2}{2} = -175.5$$

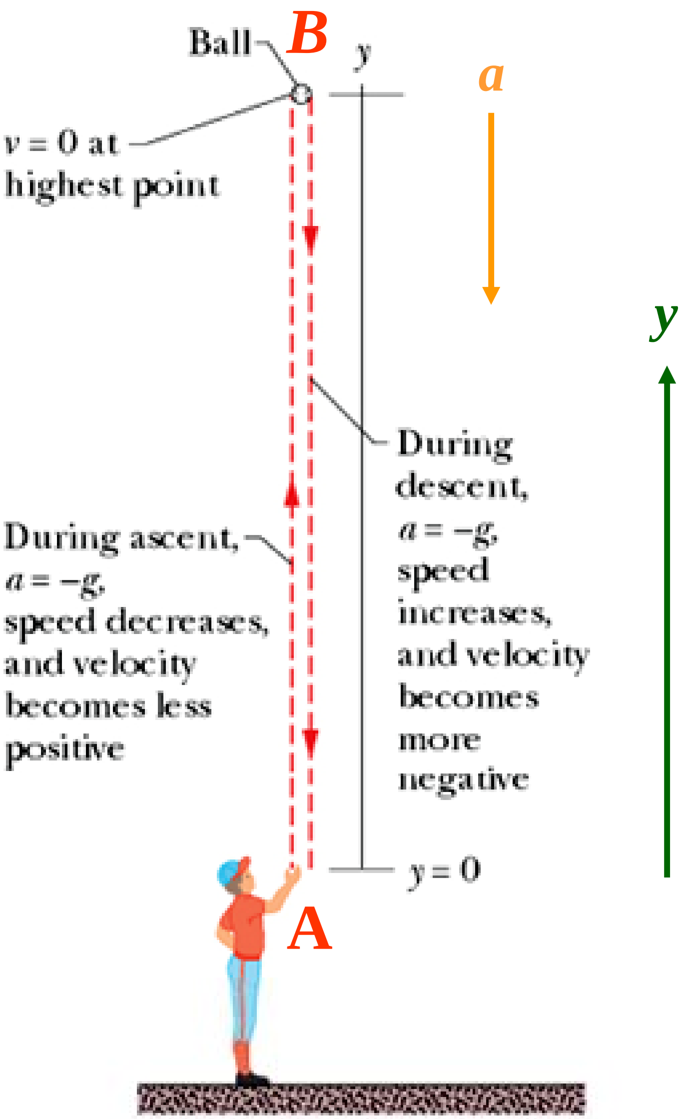
$$76.6 - 220 = v_0 t_2 + \frac{a t_2^2}{2} = -143.4$$

This Gives!

$$a = -2.0 m/s^2$$
$$v_0 = -13.9 m/s$$

Special Case: free-falling body motion

Close to the surface of the Earth all objects move toward the center of the Earth with an acceleration whose magnitude is constant and equal to 9.8 m/s^2. We use the symbol *g* to indicate the acceleration of an object in free fall.



$$a = -g$$

$$v = v_0 - gt \qquad \text{(eq. 1)}$$

$$y = y_o + v_0 t - \frac{gt^2}{2} \qquad \text{(eq. 2)}$$

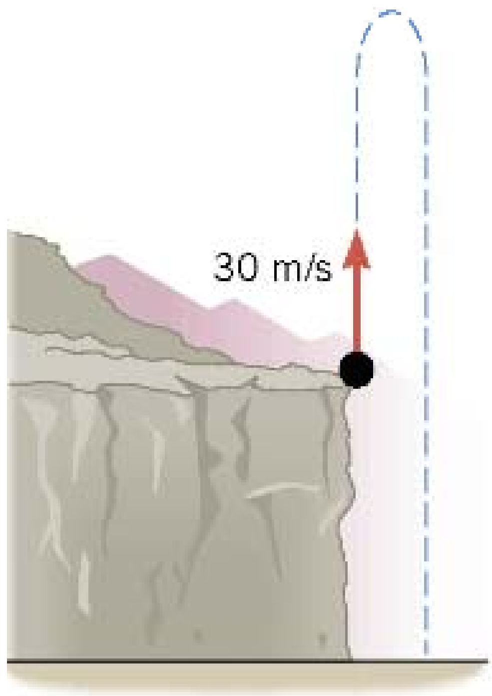
$$v^2 - v_0^2 = -2g\left(y - y_o\right) \qquad \text{(eq. 3)}$$

Question

A person standing at the edge of a cliff throws one ball straight up and another ball straight down at the same initial speed. Neglecting air resistance, which ball with the greater speed hits the ground below the cliff?

1. upward.
2. downward.
3. neither—they both hit at the same speed.

Kinematics: Taking Advantage of Symmetry



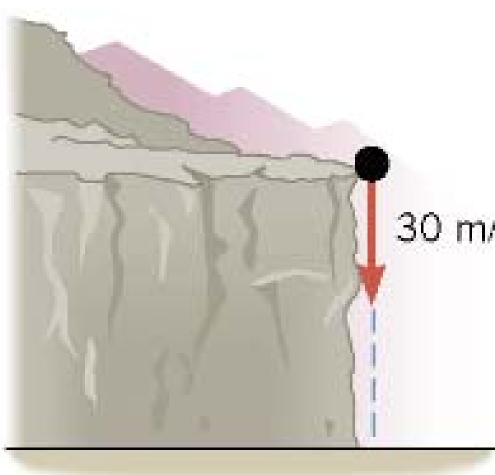
(a)

$$v = v_0 - gt$$

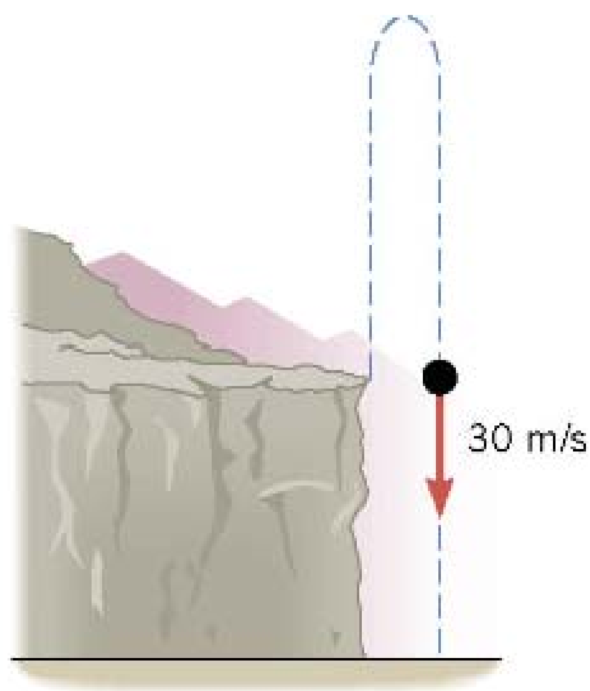
$$y = v_0 t - \tfrac{1}{2} g t^2$$

$$v^2 = v_0^2 - 2gy$$

$$y = \tfrac{1}{2}\left(v + v_0\right)t$$



(b)



(c)

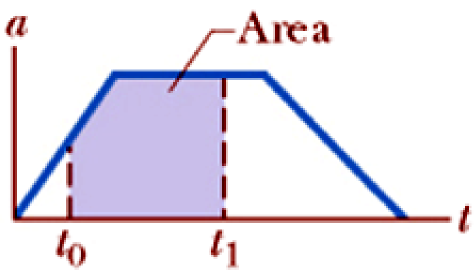
Graphical Integration in Motion Analysis (nonconstant acceleration)

When the acceleration of a moving object is not constant we must use integration to determine the velocity $v(t)$ and the position $x(t)$ of the object.

The integration can be done either using the analytic or the graphical approach:

$$a(t) = \frac{dv}{dt} \rightarrow dv = a(t)dt \rightarrow \int_{t_0}^{t_1} dv = \int_{t_0}^{t_1} a(t)dt \rightarrow v_1 - v_0 = \int_{t_0}^{t_1} a(t)dt \rightarrow v_1 = v_0 + \int_{t_0}^{t_1} a(t)dt$$

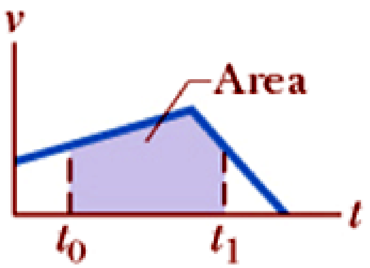
$$\int_{t_0}^{t_1} a(t)dt = \left[\text{Area under the } a \text{ versus } t \text{ curve between } t_0 \text{ and } t_1\right]$$



(a)

$$v(t) = \frac{dx}{dt} \rightarrow dx = v(t)dt \rightarrow \int_{t_0}^{t_1} dx = \int_{t_0}^{t_1} v(t)dt \rightarrow$$

$$x_1 - x_0 = \int_{t_0}^{t_1} vdt \rightarrow x_1 = x_0 + \int_{t_0}^{t_1} vdt$$

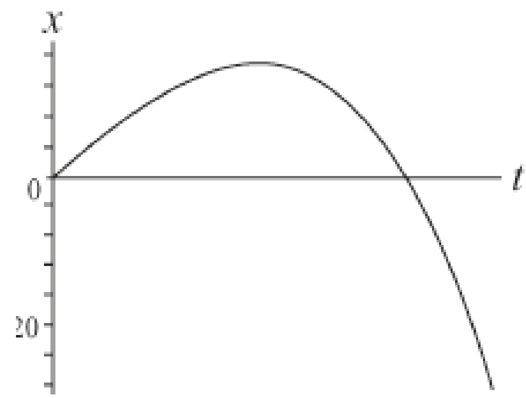


(b)

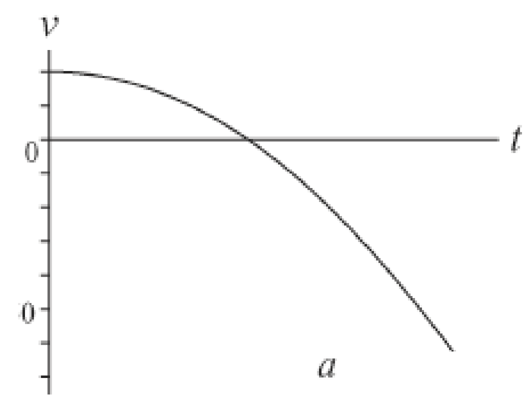
$$\int_{t_o}^{t_1} vdt = \left[\text{Area under the } v \text{ versus } t \text{ curve between } t_0 \text{ and } t_1\right]$$

Example: Acceleration: (a) If the position of a particle is given by x= 20t – 5t^3, where x is in meters and t is in seconds, when, if ever, is the particle's velocity zero?
(b) When is its acceleration a zero?
(c) For what time range (positive or negative) is **a** negative?
(d) For what time range (positive or negative) is **a** positive?
(e) Graph x(t), v(t) and a(t).

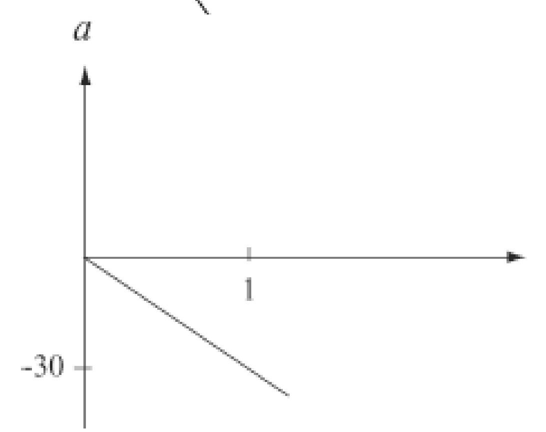
$$x(t) = 20t - 5t^3$$



$$v(t) = \frac{dx}{dt} = 20 - 15t^2$$



$$a(t) = \frac{dv}{dt} = \frac{d^2x}{dt^2} = -30t$$



Chapter 3: Vectors

In physics we have parameters that can be completely described by a number and are known as **scalars**. Temperature and mass are such parameters.

Other physical parameters require additional information about direction and are known as **vectors**. Examples of vectors are displacement, velocity, and acceleration.

This chapter covers the basic mathematical language to describe vectors. In particular we need to know the following:

- Geometric vector addition and subtraction
- Geometric vector addition and subtraction
- Resolving a vector into its components
- The notion of a unit vector
- Addition and subtraction vectors by components
- Multiplication of a vector by a scalar
- The scalar (dot) product of two vectors
- The vector (cross) product of two vectors

Vector expressed by components

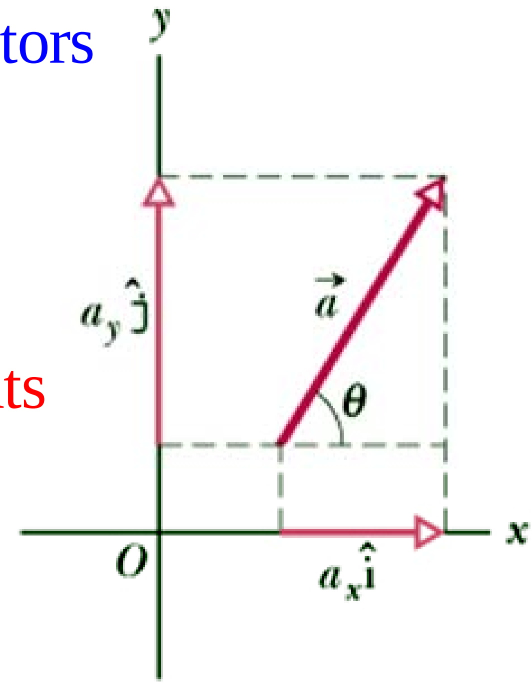
Vector $\vec{a}$ can be written with its components and unit vectors

$\vec{a} = a_x \hat{\mathrm{i}} + a_y \hat{\mathrm{j}}$ (two-dimensional case)

$a_x = a\cos\theta$ and $a_y = a\sin\theta.$

The quantities $a_x \hat{\mathrm{i}}$ and $a_y \hat{\mathrm{j}}$ are called the vector components

$$a = \sqrt{a_x^2 + a_y^2} \quad \text{and} \quad \tan\theta = \frac{a_y}{a_x}.$$



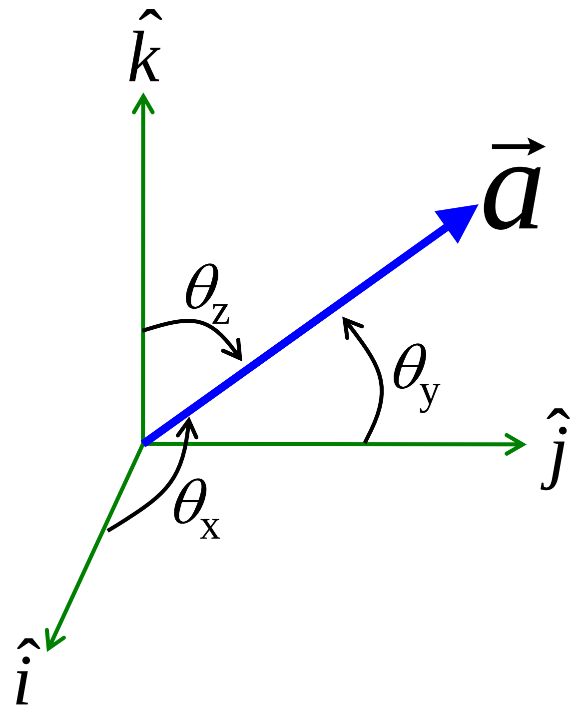
Vector $\vec{a}$ in three-dimensional case

$\vec{a} = a_x \hat{i} + a_y \hat{j} + a_z \hat{k}$

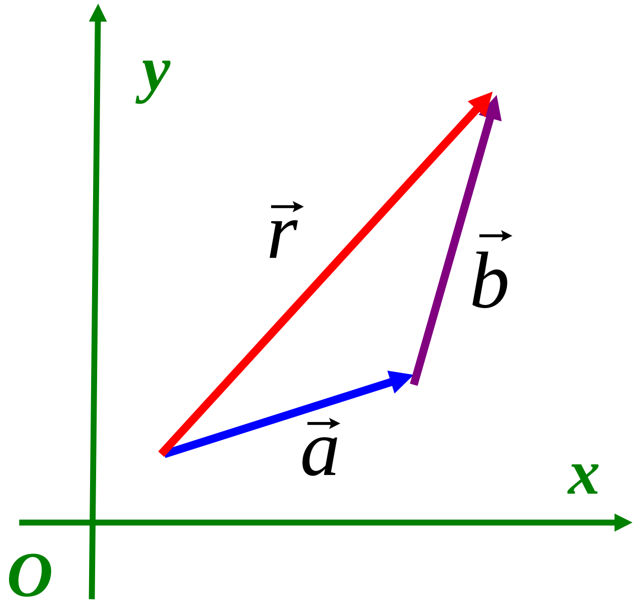
$a_x = a\cos\theta_x;\quad a_y = a\sin\theta_y;\quad a_z = a\sin\theta_z$

$$a = \sqrt{a_x^2 + a_y^2 + a_z^2}$$

$$\cos\theta_x = \frac{a_x}{a};\quad \cos\theta_y = \frac{a_y}{a};\quad \cos\theta_x = \frac{a_z}{a}$$



Adding vectors by components



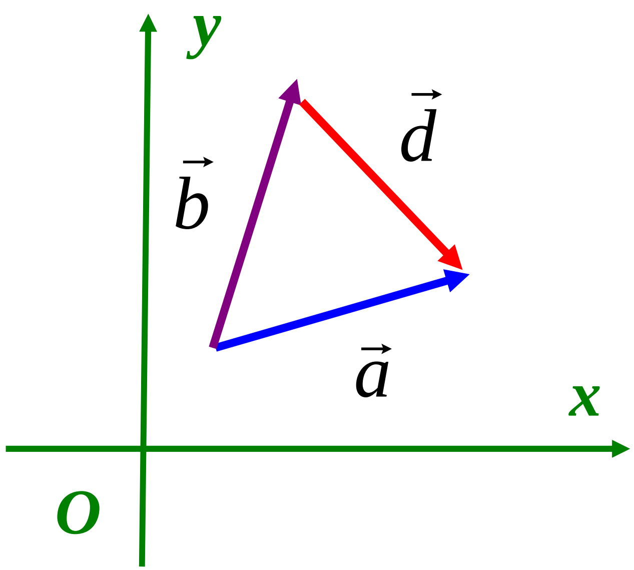
$$\vec{a} = a_x\hat{\mathrm{i}} + a_y\hat{\mathrm{j}}\ ;\ \vec{b} = b_x\hat{\mathrm{i}} + b_y\hat{\mathrm{j}}.$$

$$\vec{r} = \vec{a} + \vec{b} = r_x\hat{\mathrm{i}} + r_y\hat{\mathrm{j}}.$$

The components r_x and r_y are given by the equations

$r_x = a_x + b_x$ and $r_y = a_y + b_y$.

Subtracting vectors by components



$$\vec{a} = a_x\hat{\mathrm{i}} + a_y\hat{\mathrm{j}}\ ;\ \vec{b} = b_x\hat{\mathrm{i}} + b_y\hat{\mathrm{j}}.$$

$$\vec{d} = \vec{a} - \vec{b} = d_x\hat{\mathrm{i}} + d_y\hat{\mathrm{j}}.$$

The components d_x and d_y are given by the equations

$d_x = a_x - b_x$ and $d_y = a_y - b_y$.

Multiplying a Vector by a Scalar

Multiplication of a vector $\vec{a}$ by a scalar s results in a new vector $\vec{b} = s\vec{a}$.

The magnitude b of the new vector is given by $b = |s|a$.

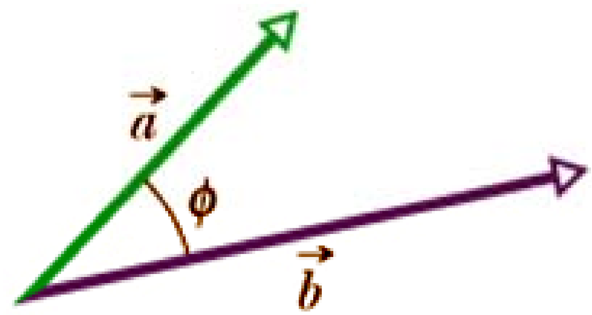
If $s > 0$, vector $\vec{b}$ has the same direction as vector $\vec{a}$.

If $s < 0$, vector $\vec{b}$ has a direction opposite to that of vector $\vec{a}$.

The Scalar Product of Two Vectors

The scalar product $\vec{a} \cdot \vec{b}$ of two vectors $\vec{a}$ and $\vec{b}$ is given by $\vec{a} \cdot \vec{b} = ab\cos\phi$. The scalar product of two vectors is also known as the "dot" product. The scalar product in terms of vector components is given by the equation

$$\vec{a} \cdot \vec{b} = a_x b_x + a_y b_y + a_z b_z.$$



Application in physics:

Work: $W = \vec{F} \cdot \vec{d} = Fd\cos\phi$

Example

What is the angle ϕ between $\vec{a} = 3.0\hat{i} - 4.0\hat{j}$ and $\vec{b} = -2.0\hat{i} + 3.0\hat{k}$?

Use $\vec{a}\cdot\vec{b} = ab\cos\phi$ such that $\cos\phi = \dfrac{\vec{a}\cdot\vec{b}}{ab}$

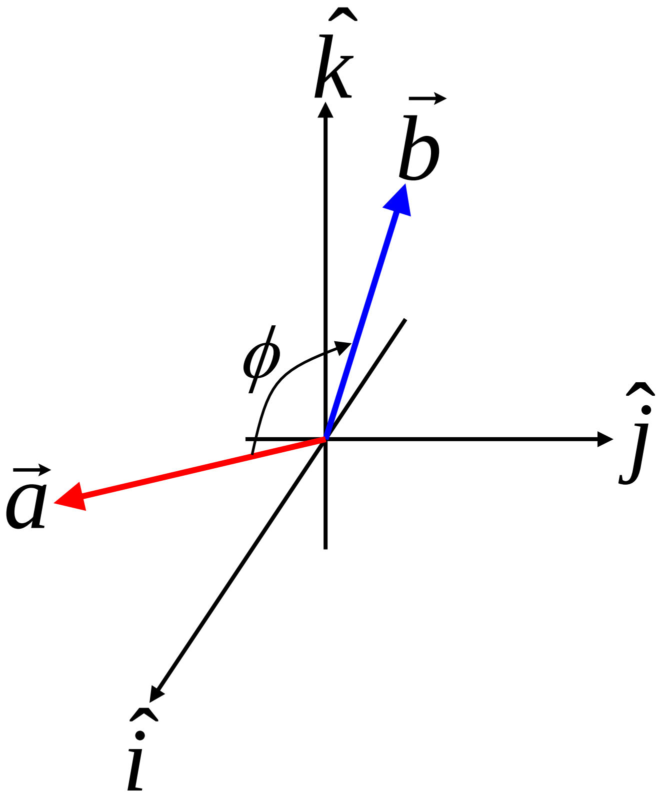
Since

$a = \sqrt{3.0^2 + (-4.0)^2} = 5.0$ and $b = \sqrt{(-2.0)^2 + 3.0^2} = 3.61$

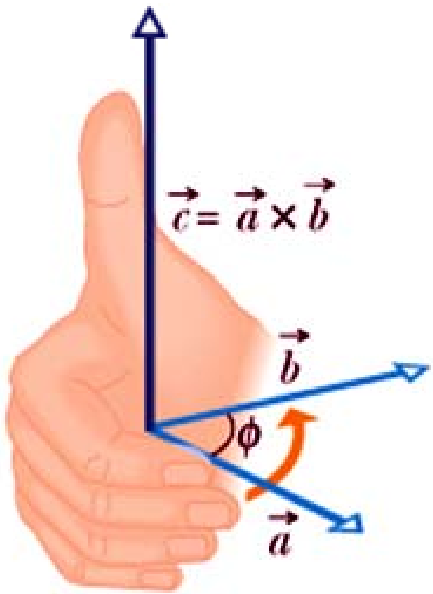
and

$$\vec{a}\cdot\vec{b} = a_x b_x + a_y b_y + a_z b_z = a_x b_x = -6.0$$

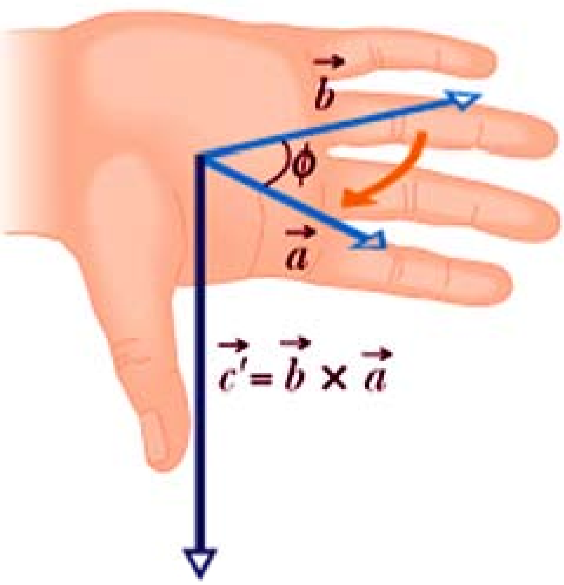
$$\cos\phi = \frac{\vec{a}\cdot\vec{b}}{ab} = \frac{-6.0}{5.0\times 3.61} \quad \text{and} \quad \phi = 109°$$



The Vector Product of Two Vectors



(a)



(b)

The vector product $\vec{c} = \vec{a} \times \vec{b}$ is a vector $\vec{c}$.

The magnitude of $\vec{c}$ is given by the equation

$c = ab \sin\phi$.

$\vec{c}$ is perpendicular to the plane P defined by $\vec{a}$ and $\vec{b}$.

The sense of the vector $\vec{c}$ is given by the right-hand rule:

a. Place the vectors $\vec{a}$ and $\vec{b}$ tail to tail.

b. Rotate $\vec{a}$ in the plane P along the shortest angle so that it coincides with $\vec{b}$.

c. Rotate the fingers of the right hand in the same direction.

d. The thumb of the right hand gives the sense of $\vec{c}$.

The vector product of two vectors is also known as the "cross" product.

The vector product $\vec{c} = \vec{a} \times \vec{b}$ in terms of vector components

$$\vec{a} = a_x\hat{\mathrm{i}} + a_y\hat{\mathrm{j}} + a_z\hat{\mathrm{k}},\ \ \vec{b} = b_x\hat{\mathrm{i}} + b_y\hat{\mathrm{j}} + b_z\hat{\mathrm{k}},\ \ \vec{c} = c_x\hat{\mathrm{i}} + c_y\hat{\mathrm{j}} + c_z\hat{\mathrm{k}}$$

The vector components of vector $\vec{c}$ are given by the equations

$$c_x = a_y b_z - a_z b_y,\quad c_y = a_z b_x - a_x b_z,\quad c_z = a_x b_y - a_y b_x.$$

or $c_x = \begin{vmatrix} a_y & a_z \\ b_y & b_z \end{vmatrix},\ c_y = \begin{vmatrix} a_z & a_x \\ b_z & b_x \end{vmatrix},\ c_z = \begin{vmatrix} a_x & a_y \\ b_x & b_y \end{vmatrix}$

Note: Those familiar with the use of determinants can use the expression

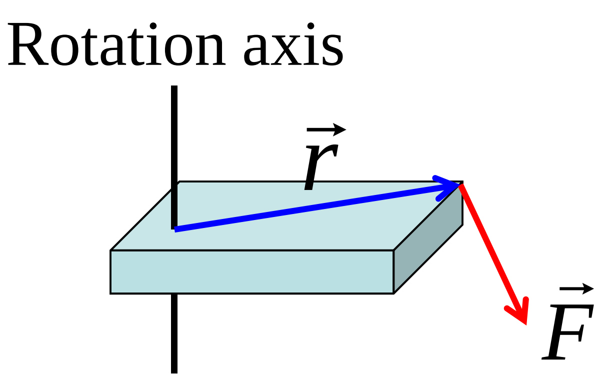
$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{\mathrm{i}} & \hat{\mathrm{j}} & \hat{\mathrm{k}} \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{vmatrix}$$

Note: The order of the two vectors in the cross product is important:

$$\vec{b} \times \vec{a} = -\left(\vec{a} \times \vec{b}\right).$$

Application in physics:

Torque: $\vec{\tau} = \vec{r} \times \vec{F}$



Example

Show $\vec{a}\cdot(\vec{b}\times\vec{c})=\vec{b}\cdot(\vec{c}\times\vec{a})=\vec{c}\cdot(\vec{a}\times\vec{b})$

$$\vec{a}\cdot(\vec{b}\times\vec{c})=(a_x\hat{i}+a_y\hat{j}+a_z\hat{k})\cdot\begin{vmatrix}\hat{\mathrm{i}} & \hat{\mathrm{j}} & \hat{\mathrm{k}}\\ b_x & b_y & b_z\\ c_x & c_y & c_z\end{vmatrix}$$

$$=a_x(b_yc_z-b_zc_y)+a_y(b_zc_x-b_xc_y)+a_z(b_xc_y-b_yc_x)=\begin{vmatrix}a_x & a_y & a_z\\ b_x & b_y & b_z\\ c_x & c_y & c_z\end{vmatrix}$$

$$=\begin{vmatrix}b_x & b_y & b_z\\ c_x & c_y & c_z\\ a_x & a_y & a_z\end{vmatrix}=\vec{b}\cdot(\vec{c}\times\vec{a})$$

$$=\begin{vmatrix}c_x & c_y & c_z\\ a_x & a_y & a_z\\ b_x & b_y & b_z\end{vmatrix}=\vec{c}\cdot(\vec{a}\times\vec{b})$$