Physics 2101
Section 3
Apr 12th

Announcements:

- Finish Ch. 15 today
- Start Ch. 16 today too

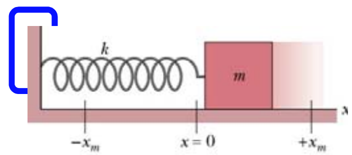
Class Website:

http://www.phys.lsu.edu/classes/spring2010/phys2101-3/

http://www.phys.lsu.edu/~jzhang/teaching.html

Torsion Pendulum

Spring

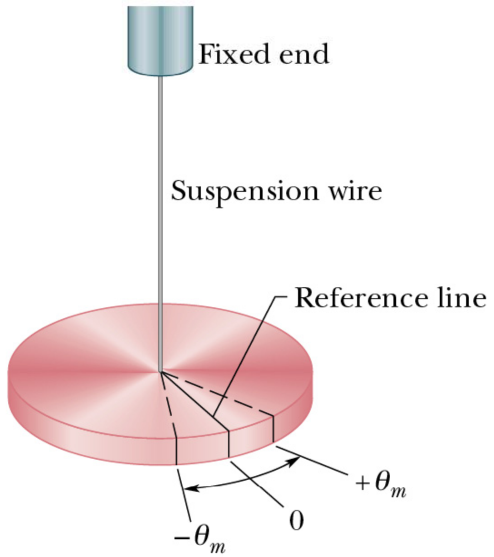


$$ma = -kx$$

$$a = -\frac{k}{m}x$$

$$\omega^2 = \frac{k}{m}$$

What is period of torsion pendulum?



Torque

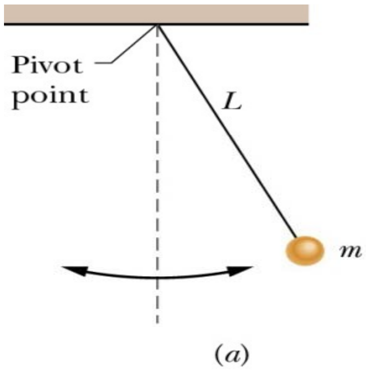
$$\tau = I\alpha = -\kappa\theta$$

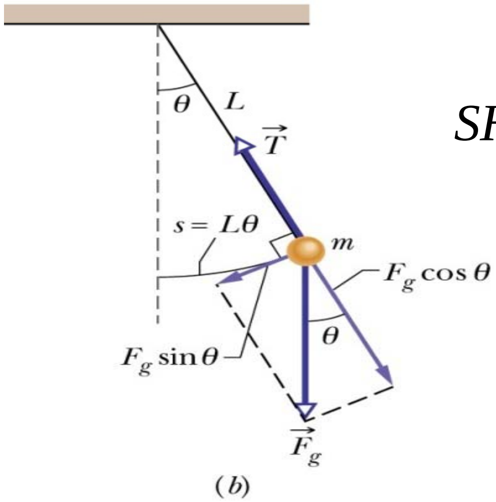
"kappa" is torsion spring constant

$$\alpha = -\frac{\kappa}{I}\theta$$

$$\omega^2 = \frac{\kappa}{I} \quad \Rightarrow \omega = \sqrt{\frac{\kappa}{I}} \quad \Rightarrow T = 2\pi\sqrt{\frac{I}{\kappa}}$$

Simple Pendulum





2) <u>Simple Pendulum</u> : gravity is "restoring force"

$$\tau = I\alpha = -L(mg\sin\theta) \Rightarrow \alpha \cong -\frac{mgL}{I}\theta$$

$$SHM: \ \theta(t) = \theta_{max}\cos(\omega t + \varphi) \Rightarrow \omega = \sqrt{\frac{mgL}{I}} = \sqrt{\frac{mgL}{(mL^2)}} = \sqrt{\frac{g}{L}}$$

- **Independent of amplitude and mass (in small angle approximation) !**
- **Dependent only on L and g**

Clicker Question Question 10-4

A bowling ball and a ping-pong ball are each tied to a string and hung from the ceiling. The distance from the ceiling to the CM of each object is the same. Which object would have a longer period of motion if they were set swinging? Neglect air resistance and frictional effects.

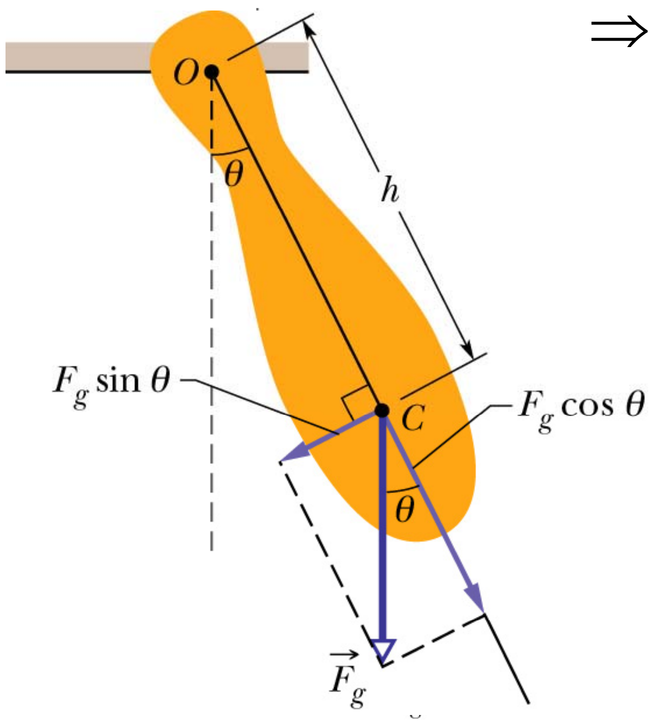
1. *Bowling ball*
2. *Ping-pong ball*
3. *Both have the same period*

$$\omega = \sqrt{(g/L)}$$

Physical Pendulum

Swinging mass under gravity

"spring" restoring force is gravity



(b)

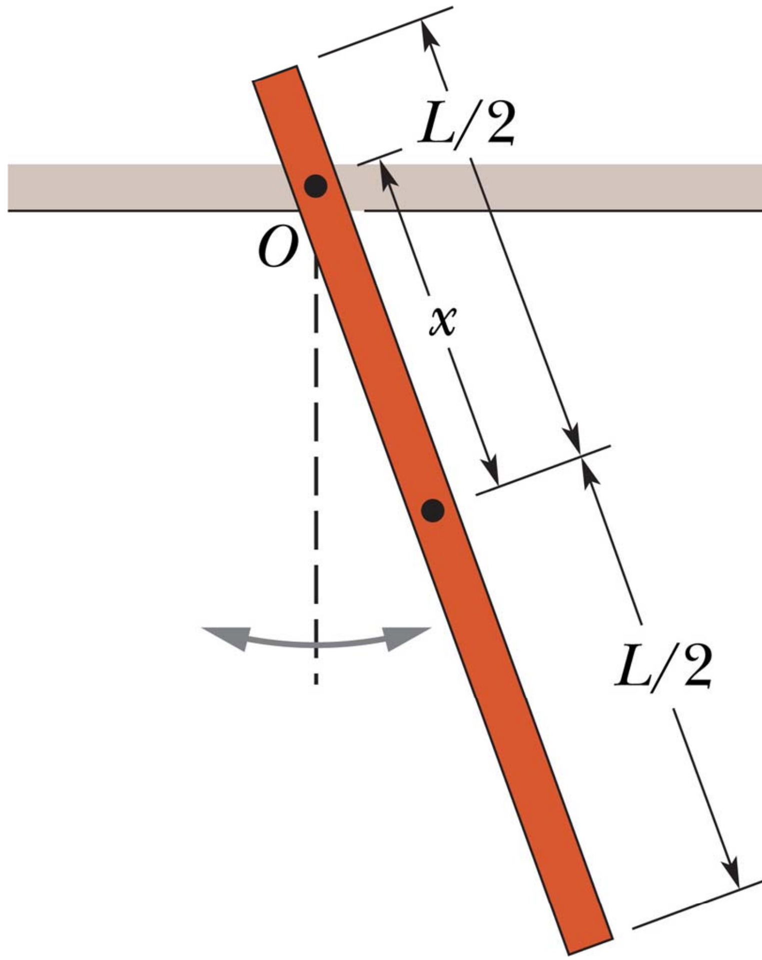
$$\tau_{pivot\,pt.} = rF_{\perp} = -h(mg\sin\theta)$$

$$\Rightarrow \tau = I_{rot}\alpha \qquad \Rightarrow \alpha \cong -\frac{mgh_{rot-com}}{I_{rot}}\theta$$

$$SHM:\ \theta(t) = \theta_{\max}\cos(\omega t + \varphi)$$

$$\Rightarrow \omega = \sqrt{\frac{mgh_{rot-com}}{I_{rot}}} \qquad \Rightarrow T = 2\pi\sqrt{\frac{I_{rot}}{mgh_{rot-com}}}$$

15-49: A stick of length L oscillates as a physical pendulum. (a) What value of distance x between the stick's COM ant is pivot point O gives the least period? (b) What is the least period?



General Solution of 15-49

$$\omega = \sqrt{\frac{mgh}{I}}$$

$$I = I_{COM} + mh^2 = \frac{mL^2}{12} + mx^2$$

$$\omega = \sqrt{\frac{gx}{\frac{L^2}{12} + x^2}}$$

$$T = 2\pi\sqrt{\frac{\frac{L^2}{12} + x^2}{gx}}$$

Find the minimal T

$$\frac{12gT^2}{(2\pi)^2} = \frac{L^2}{x} + 12x$$

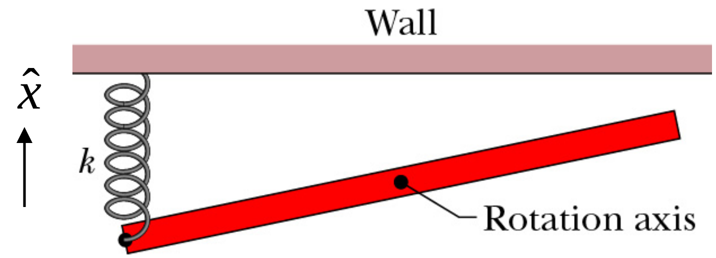
Take derivative

$$\frac{d}{dx}\left(\frac{12gT^2}{(2\pi)^2}\right) = \frac{d}{dx}\left(\frac{L^2}{x} + 12x\right) = -\frac{L^2}{x^2} + 12 = 0$$

$$x = \frac{L}{\sqrt{12}}$$

Problem

Problem 16-53: In an overhead view, a long uniform rod of length **L** and mass m is free to rotate in a horizontal axis through its center. A spring with force constant **k** is connected to the rod and the fixed wall. When the rod is in equilibrium, it is parallel to the wall. What is the period of small oscillations that results when the rod is rotated slightly and released?



TEST QUESTION LAST YEAR

$$\tau_{pivot\ pt.} = rF_{\perp} = -\left(\frac{L}{2}\right)\left(F_{spring}\right) = -\left(\frac{L}{2}\right)(kx)$$

$$= -\frac{Lk}{2}x = -\frac{Lk}{2}\left[\left(L/2\right)\theta\right] = -\frac{L^2k}{4}\theta$$

$$\tau = I_{rot}\alpha = \left(\frac{1}{12}mL^2\right)\alpha \Rightarrow \left(\frac{1}{12}mL^2\right)\alpha = -\frac{L^2k}{4}\theta$$

$$SHM: \ \theta(t) = \theta_{\max}\cos(\omega t + \varphi)$$

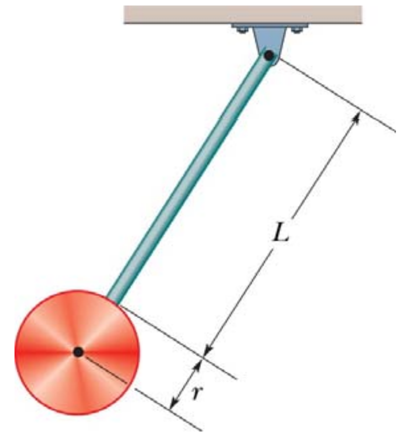
$$\Rightarrow \alpha = -\frac{L^2k}{4}\left(\frac{12}{mL^2}\right)\theta = -\left(\frac{3k}{m}\right)\theta \quad \Rightarrow \ \omega = \sqrt{\frac{3k}{m}}$$

Problem: The pendulum consists of a uniform disk with radius R and mass M attached to a uniform rod of length L and mass m.

(a) what is the rotational inertia of the pendulum?

(b) What is the distance between the pivot and the COM?

(c) What is the period?



(a) The disc has an I=MR²/2 about the COM so.

$$I(Disc) = \frac{MR^2}{2} + Mh^2 = \frac{MR^2}{2} + M\left(r + L\right)^2$$

$$I(Rod) = \frac{mL^2}{12} + mh^2 = \frac{nL^2}{12} + m\left(\frac{L}{2}\right)^2 = \frac{mL}{3}$$

(b) COM $\quad d_{COM} = \frac{Ml_d + ml_r}{M + m}$ With l_d and l_r being the the COM of the disc and rod.

(c) The period

$$T = 2\pi\sqrt{\frac{I}{(M+m)gd}}$$

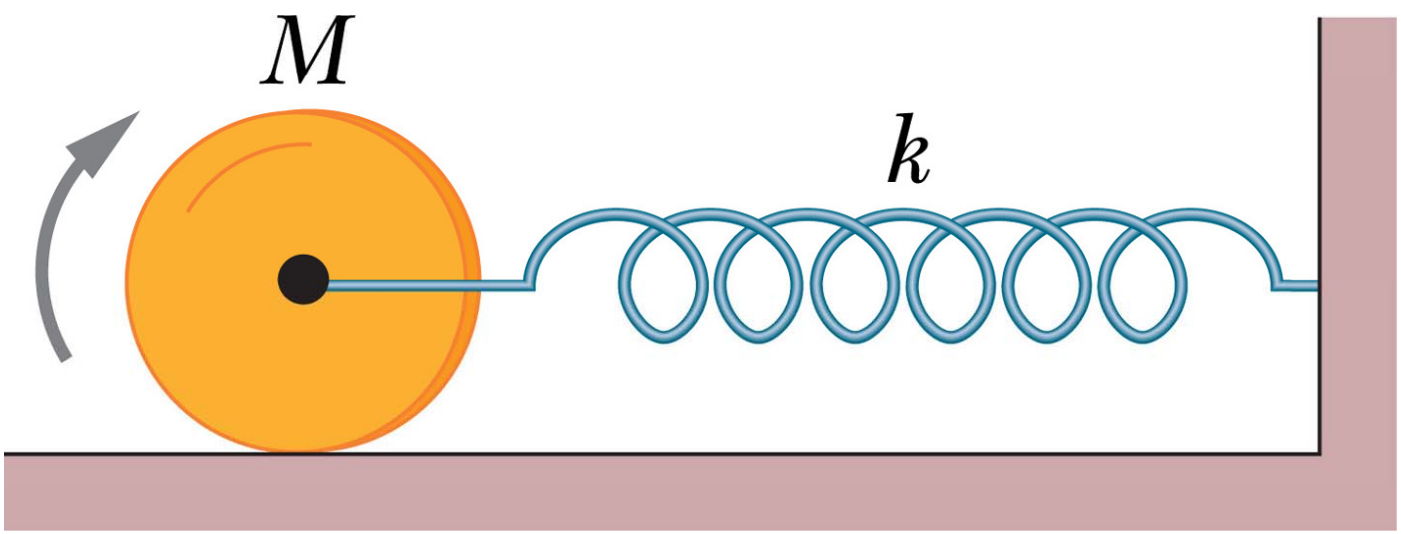
15-106: A solid Cylinder attached to a horizontal spring (k=3.0 N/m) rollos without slipping along a horizontal surface. If the system is released from rest when the spring is stretched by d=0.25 m, find

(a) The translational kinetic energy

(b) The rotational kinetic energy of the cylinder as it passes through x=0.

(c) Show that under these conditions the COM is SHM with

$$T = 2\pi\sqrt{\frac{3M}{2k}}$$



Solution:

(a) $$E_{mech} = U_s + \frac{Mv^2}{2} + \frac{I_{CM}^2\omega^2}{2}$$

$$\frac{kd^2}{2} = \frac{Mv_{cm}^2}{2} + \frac{I_{CM}^2\omega^2}{2} = \frac{Mv_{cm}^2}{2} + \frac{1}{2}\left(\frac{MR^2}{2}\right)\left(\frac{v_{cm}}{R}\right)^2$$

$$\frac{kd^2}{2} = \frac{3Mv_{cm}^2}{4}; \quad \text{so that } KE_{tran} = \frac{1}{2}Mv_{cm}^2 = \frac{1}{3}kd^2 = 6.25\times10^{-2}\,J$$

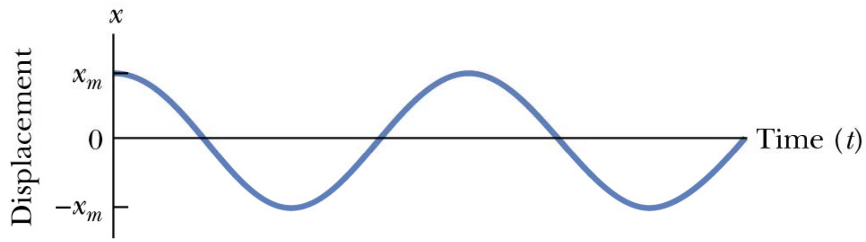
(b) $$KE_{rot} = \frac{1}{2}I\omega^2 = \frac{1}{4}Mv_{com}^2 = \frac{1}{6}kx_{\mathrm{m}}^2 = 3.13\times10^{-2}\,J$$

(c) $$\frac{dE_{mech}}{dt} = \frac{d}{dt}\left(\frac{3Mv_{cm}^2}{4} + \frac{kx^2}{2}\right) = \frac{3Mv_{cm}a_{cm}}{2} + kxv_{cm} = 0$$

$$a_{cm} = -\left(\frac{2k}{3M}\right)x \quad \Longrightarrow \quad \omega = \sqrt{\frac{2k}{3M}}$$

Chap. 15: Oscillations

$$\omega = 2\pi f \rightarrow T = \frac{2\pi}{\omega} \rightarrow \omega T = 2\pi$$



$$x(t) = x_{\max} \cos(\omega t + \varphi)$$ **Position**

$$v(t) = \frac{dx}{dt} = -x_{\max}\,\omega \sin(\omega t + \varphi)$$ **Velocity**

$$a(t) = \frac{dv}{dt} = -x_{\max}\,\omega^2 \cos(\omega t + \varphi)$$ **Acceleration**

$$a(t) = -\omega^2 x(t)$$

In SHM, the acceleration is proportional to the displacement but opposite sign. The proportionality is the square of the angular frequency.

$$\omega_{spring} = \sqrt{\frac{k}{m}} \qquad \omega_{torsion\ pendulum} = \sqrt{\frac{\kappa}{I_{rot}}}$$

$$\omega_{simple\ pendulum} = \sqrt{\frac{g}{L}} \qquad \omega_{physical\ pendulum} = \sqrt{\frac{mgh_{rot-com}}{I_{rot}}}$$

"Phase Diagram" of SHO

Knowing:

$$U(t) = \tfrac{1}{2}kx^2 = \tfrac{1}{2}kx_{\max}^2 \cos^2(\omega t + \varphi) \quad \Rightarrow \frac{x^2}{x_{\max}^2} = \cos^2(\omega t + \varphi)$$

$$KE(t) = \tfrac{1}{2}mv^2 = \tfrac{1}{2}m\omega^2 x_{\max}^2 \sin^2(\omega t + \varphi) \quad \Rightarrow \frac{v^2}{\omega^2 x_{\max}^2} = \sin^2(\omega t + \varphi)$$

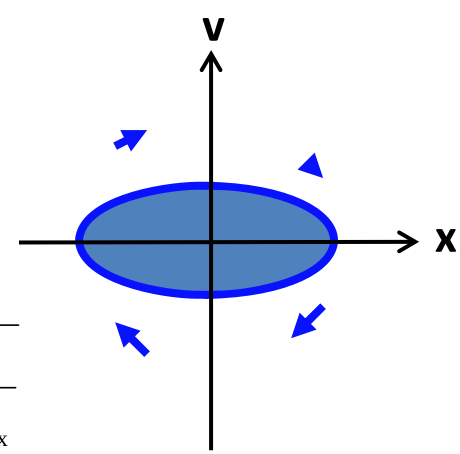
$$E_{mech} = U(t) + KE(t) = \tfrac{1}{2}kx_{\max}^2 = \tfrac{1}{2}mv_{\max}^2$$

$$\frac{x^2}{x_{\max}^2} = \frac{x^2}{2E_{mech}/k}$$

$$\frac{v^2}{\omega^2 x_{\max}^2} = \frac{v^2}{2\omega^2 E_{mech}/k^2} = \frac{v^2}{2E_{mech}/m}$$

$$\sin^2(\omega t + \varphi) + \cos^2(\omega t + \varphi) = 1$$

$$\frac{x^2}{\left(2E_{mech}/k\right)} + \frac{v^2}{\left(2E_{mech}/m\right)} = 1$$



$$v(t) = v_{\max}\sqrt{1 - \frac{x^2}{x_{\max}^2}}$$