Physics 2101
Section 3
March 31st

Announcements:

- Quiz today about Ch. 14

Class Website:

http://www.phys.lsu.edu/classes/spring2010/phys2101-3/

http://www.phys.lsu.edu/~jzhang/teaching.html

"*Simple Harmonic Motion (SHM)*"---Kinematics

$$x(t) = x_{\max} \cos(\omega t + \varphi)$$ **Position**

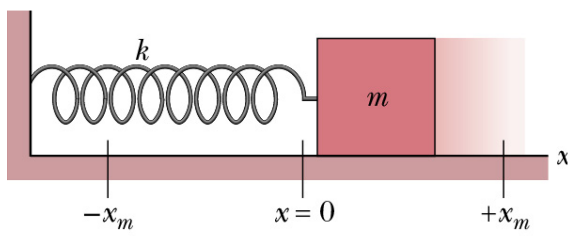
$$v(t) = \frac{dx}{dt} = -x_{\max}\omega \sin(\omega t + \varphi)$$ **Velocity** $\left[|v_{\max}| = |x_{\max}\omega| \right]$

$$a(t) = \frac{dv}{dt} = -x_{\max}\omega^2 \cos(\omega t + \varphi)$$ **Acceleration** $\left[|a_{\max}| = |x_{\max}\omega^2| \right]$

$$a(t) = -\omega^2 x(t)$$

In SHM, the acceleration is proportional to the displacement but opposite sign. The proportionality is the square of the angular frequency.

SHO and Hooke's Law (again…)----Dynamics



$$F = -kx$$

$$ma = -kx$$

$$a = -\frac{k}{m}x$$

"In SHM, the acceleration is proportional to the displacement but opposite sign. The proportionality is the square of the angular frequency".

$$a(t) = -\omega^2 x(t)$$

Block-spring is linear SHO:

$$\omega^2 = \frac{k}{m} \rightarrow \omega = \sqrt{\frac{k}{m}}$$

$$f = \frac{\omega}{2\pi} \rightarrow f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

$$T = \frac{1}{f} \rightarrow T = 2\pi\sqrt{\frac{m}{k}}$$

NOTE: Independent of amplitude

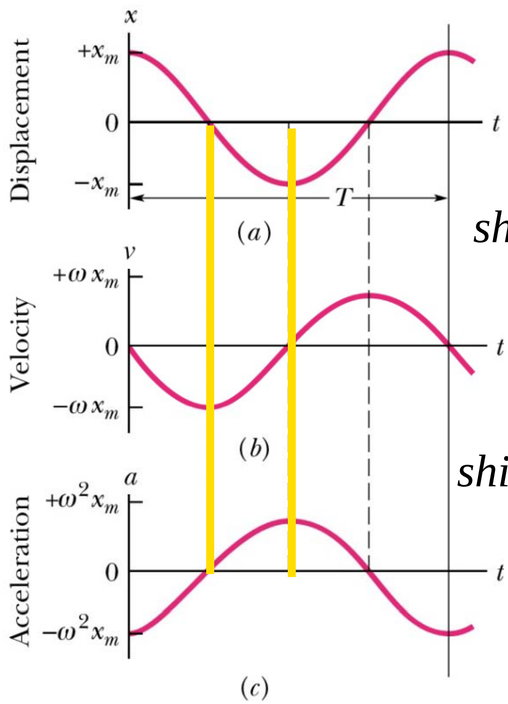
Larger mass -> longer period

SHO: relationships

$$x(t) = x_{max}\cos(\omega t + \varphi)$$

$$v(t) = -x_{max}\,\omega\sin(\omega t + \varphi)$$

$$a(t) = -x_{max}\,\omega^2\cos(\omega t + \varphi)$$



shifted by $-\dfrac{\pi}{2}$

shifted by $-\dfrac{\pi}{2}$

$$v(t) = 0 \; when \; x(t) = \pm x_{max}$$

$$v(t) = \pm v_{max} \;\; when \;\; x(t) = 0$$

<u>How to determine x_{max} and ϕ : INITIAL CONDITIONS</u>

Knowing x(0) and v(0),

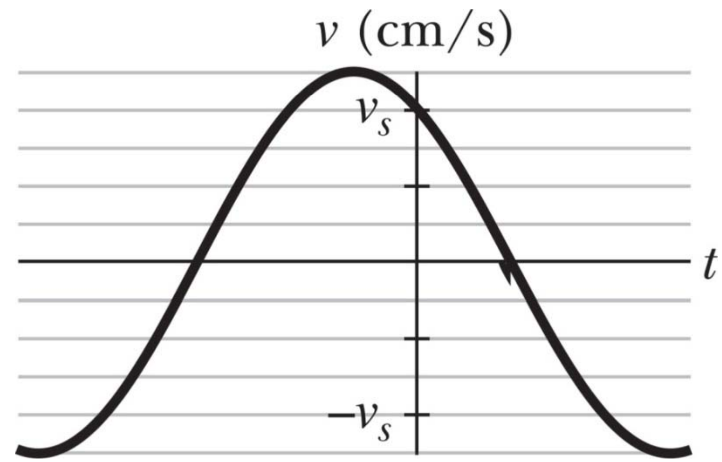
$$v(0) = -x_{max}\,\omega\sin(\omega(0) + \varphi)$$
$$x(0) = x_{max}\cos(\omega(0) + \varphi)$$

$$\longrightarrow \left(\frac{-v(0)}{\omega x(0)}\right) = \tan(\varphi)$$

$$x(0) = x_{max}\cos(\varphi) \longrightarrow x_{max} = \frac{x(0)}{\cos\left(\tan^{-1}\left(\dfrac{-v(0)}{\omega x(0)}\right)\right)}$$

Problem 15-12: What is the phase constant for the harmonic oscillator with the velocity function v(t) shown in the figure, with x(t)

$$x(t) = x_m \cos(\omega t + \phi)$$



Answer:

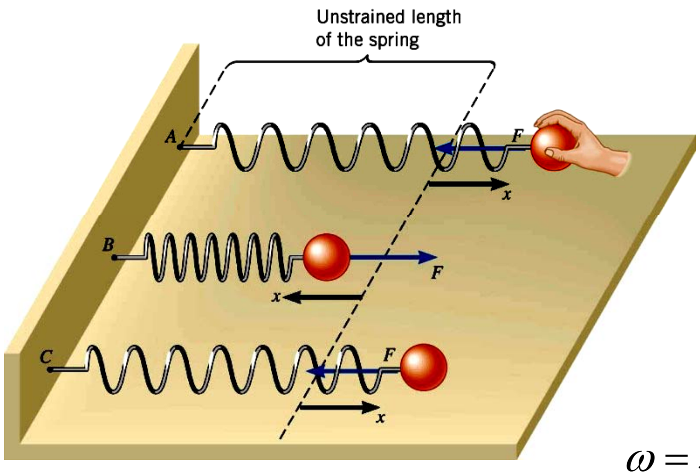
$$v(t) = -x_m \omega \sin(\omega t + \phi)$$

$$v_m = x_m \omega \qquad v_s = v(0) = -v_m \sin(\phi)$$

$$\sin(\phi) = -\frac{v_s}{v_m} = -\frac{4}{5}; \quad \phi \cong -53°$$

Example

A block whose mass m is 680 g is fastened to a spring whose constant k is 65 N/m. The block is pulled a distance x = 11 cm from its equilibrium position at x = 0 on a frictionless surface and released from rest at t = 0.



(a) What are the angular frequency, frequency, and period of the resulting motion?

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{65 \text{ N/m}}{0.68 \text{ kg}}} = 9.8 \text{ rad/s}$$

$$\rightarrow f = \frac{\omega}{2\pi} = 1.6 \text{ Hz}$$

$$\rightarrow T = \frac{1}{f} = 0.64 \text{ s}$$

(b) What is the phase angle and amplitude of the oscillation?

$$v_0 = -x_{max}\omega \sin(\omega(0) + \varphi)$$
$$x_0 = x_{max} \cos(\omega(0) + \varphi)$$

$$\longrightarrow \left(\frac{-v(0)}{\omega x(0)}\right) = \tan(\varphi) \longrightarrow \varphi = \tan^{-1}\left(\frac{-0 \text{ m/s}}{(9.8 \text{ rad/s})(0.11 \text{ m/s})}\right) = 0$$

$$x(0) = x_{max} \cos(\varphi) \longrightarrow x_{max} = \frac{x(0)}{\cos(\tan^{-1}(0))} = 11 \text{ cm}$$

(c) What is the maximum speed and acceleration?

$$|v_{max}| = |x_{max}\omega| = (11 \text{ cm})(9.8 \text{ rad/s}) = 1.1 \text{ m/s}$$

$$|a_{max}| = |x_{max}\omega^2| = (11 \text{ cm})(9.8 \text{ rad/s})^2 = 11 \text{ m/s}^2$$

Energy swapping in SHO

The elastic property of the oscillating system (spring) stores potential energy
The inertia property (mass) stores kinetic energy: Total mechanical energy is constant

Knowing:

$$U(t) = \tfrac{1}{2}kx^2 = \tfrac{1}{2}kx_{\max}^2 \cos^2(\omega t + \varphi)$$

$$\begin{aligned} KE(t) = \tfrac{1}{2}mv^2 &= \tfrac{1}{2}m\omega^2 x_{\max}^2 \sin^2(\omega t + \varphi) \\ &= \tfrac{1}{2}m\frac{k}{m}x_{\max}^2 \sin^2(\omega t + \varphi) \\ &= \tfrac{1}{2}kx_{\max}^2 \sin^2(\omega t + \varphi) \end{aligned}$$

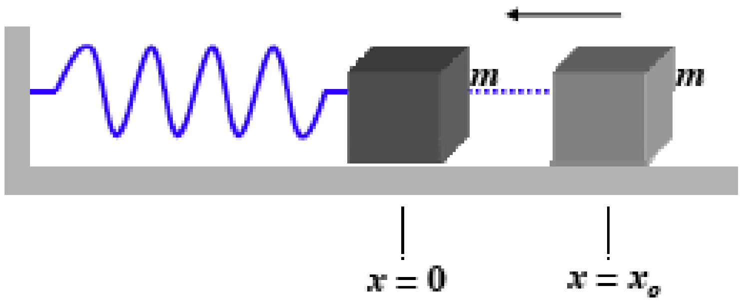
$$\begin{aligned} E_{mech} &= U(t) + KE(t) \\ &= \tfrac{1}{2}kx_{\max}^2\left(\cos^2(\omega t + \varphi) + \sin^2(\omega t + \varphi)\right) \\ &= \tfrac{1}{2}kx_{\max}^2 = \tfrac{1}{2}mv_{\max}^2 \end{aligned}$$

Exchange between *KE* and *U* periodically during oscillation

Clicker Question

Question 10-3

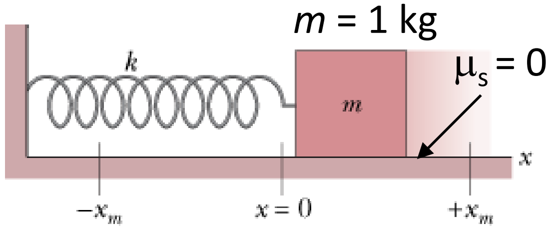
A mass of is attached to one end of a spring and is free to slide on a frictionless horizontal surface. The mass is pulled back from its equilibrium position and released. What is the speed and kinetic energy of the mass when it passes through the equilibrium position (x=0)?



1. *Maximum*
2. *Zero*
3. *Need more info*

Problem 16-17

A block of mass 1 kg (m) is on a horizontal surface (a shake table) that is moving back and forth horizontally with simple harmonic motion of frequency 2.0 Hz. (f) The coefficient of static friction between block and surface is 0.5 (μ_s).



How great can the amplitude of the SHM be if the block is not to slip along the surface?

1) Angular frequency is: $\omega = 2\pi f \; = 2\pi(2 \cdot Hz) = 12.6 \cdot rad/s$

2) Motion of shake table is: $x(t) = x_{\max}\cos(\omega t) \quad \rightarrow \quad a(t) = -x_{\max}\omega^2\cos(\omega t)$

$$|a_{\max}| = x_{\max}\omega^2$$

3) When the block just slips, it means the static friction is equal to the normal force:

$$\hat{x}: \; -f_s = ma_{SHM} \qquad f_{s,\max} = \mu_s N \qquad -\mu_s mg = -f_{s,\max} = ma_{SHM,\max}$$
$$\hat{y}: \; N - mg = 0 \qquad = \mu_s mg$$

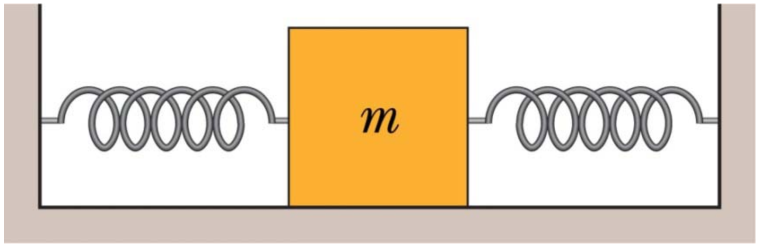
4) Putting together:

$$ma_{SHM,\max} = -\mu_s mg$$
$$|a_{SHM,\max}| = |-\mu_s g| = x_{\max}\omega^2$$

$$x_{\max} = \left|\frac{-\mu_s g}{\omega^2}\right| = \left(\frac{(0.5)(9.8 \cdot m/s^2)}{(12.6 \cdot rad/s)^2}\right) = 0.031 \cdot m$$

Problem15-13: In the figure two springs of spring constant k_l and k_r are attached to a block of mass m. Find the frequency and period.

Let the positive x be to the right then the force on the block at x is:



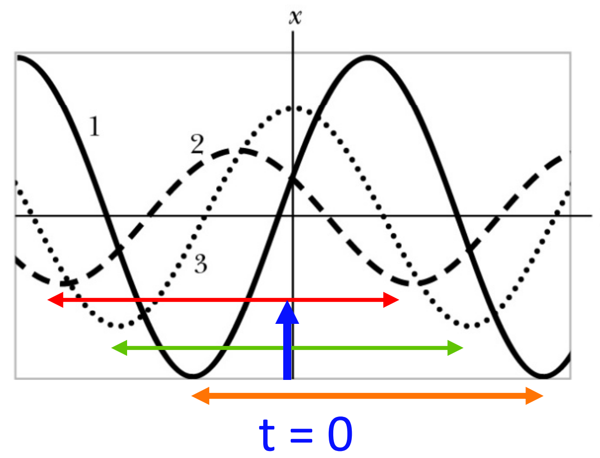
$$-\left(xk_l + xk_r\right) = ma$$

$$a = -\frac{\left(xk_l + xk_r\right)}{m} = -\omega^2 a$$

$$f = 2\pi\omega$$

$$T = \frac{1}{f}$$

Spring oscillations



x(t) curves for the experiments involving a particular spring-box systems oscillating in SHM (equal k's).

Rank (greatest-to-least) the curves according to:

a) the system's angular frequency

b) the spring's potential energy at t=0

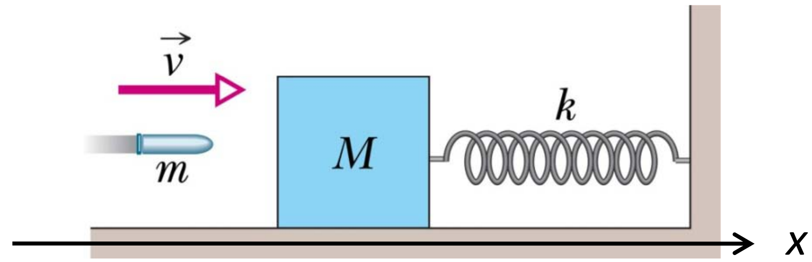
c) the box's velocity at t=0

d) the box's kinetic energy at t=0

e) the box's maximum kinetic energy

Problem 15-35: A block of Mass M at rest is attached to a spring of constant k. A bullet of mass m and velocity v strikes and is embedded in the block. (a) find the speed of the block immediately after the collision: (b) The equation for the displacement of the SHM.

(a) Use conservation of momentum.



$$(M+m)v_f = mv$$

$$v_f = \frac{mv}{M+m}$$

(b) Find the equation of SHM.

$$x(t) = x_m \cos(\omega t + \phi)$$

$$v(t) = -x_m \omega \sin(\omega t + \phi)$$

At t = 0, x = 0 and $v = v_f$

$$x(t) = x_m \cos(\omega t - \frac{\pi}{2})$$

And $x_m = \frac{v_f}{\omega}$

$$v(t) = -v_f \sin(\omega t - \frac{\pi}{2})$$