Physics 2101
Section 3
March 26^{th}

Announcements:

- Next Quiz on April 1^{st}
- Don't forget supl. HW #9
- Cha. 14 today

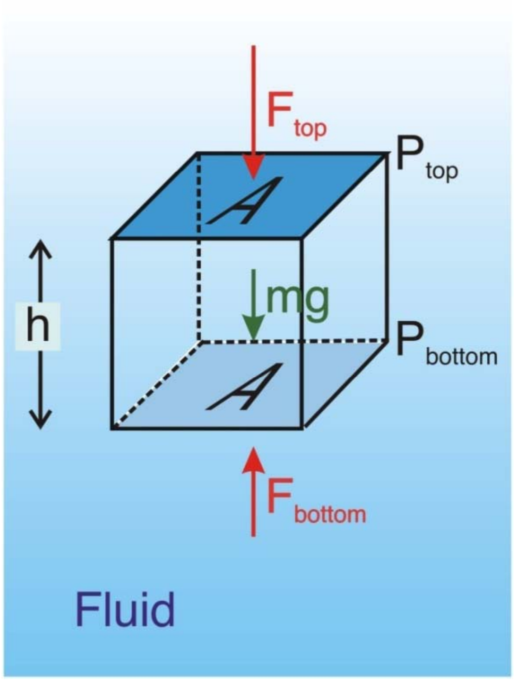
<u>Class Website</u>:

http://www.phys.lsu.edu/classes/spring2010/phys2101-3/

http://www.phys.lsu.edu/~jzhang/teaching.html

Fluids at Rest ρ = constant

Static Equilibrium



$$\sum \vec{F} = 0 \quad \Rightarrow \quad \vec{a} = 0$$

$$\sum \vec{F} = 0$$

$$F_{bottom} - F_{top} - mg = 0$$

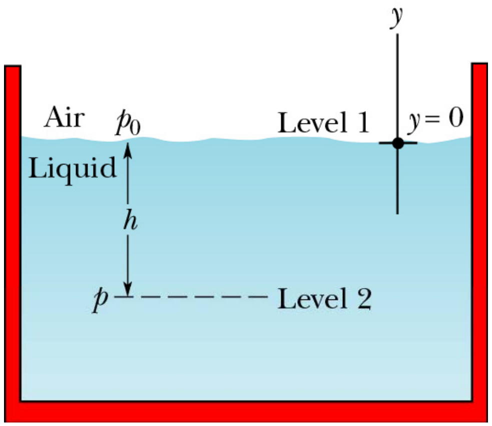
$$p_{bottom} A = p_{top} A + mg$$

$$p_{bottom} = p_{top} + \frac{mg}{A} = p_{top} + \frac{mg}{A}\frac{h}{h}$$

$$p_{bottom} = p_{top} + gh\frac{m}{V}$$

$$p_{bottom} = p_{top} + \rho gh$$

Pressure depends on depth NOT horizontal dimensions



$$p_{at\ h} = p_{atm} + \rho gh$$

NOTE

$$p_{at\ h} > p_{atm} \quad for\ h\ down$$

$$p_{at\ h} < p_{atm} \quad for\ h\ up$$

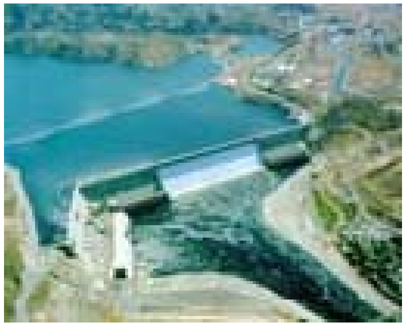
Examples

1. What is the pressure head at the bottom of a 98 ft (30 m) water tower?

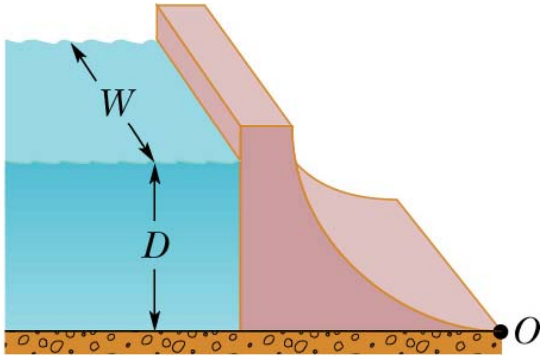
$$\Delta p = p_{at\ h} - p_{atm} = \rho g h$$
$$= \left(1000 \text{ kg/m}^3\right)\left(9.8 \text{ m/s}^2\right)\left(30 \text{ m}\right)$$
$$= 2.9\times10^5 \text{ Pa}$$
$$= 42 \text{ psig } \left(56.7 \text{ psi} - absolute\right)$$

Gauge pressure vs absolute pressure

2. What is the NET force on Grand Coulee dam (width 1200 m - height 150 m)?



$$p_{at\ h} = p_{atm} + \rho g h$$
$$dF = \left(\rho g y\right) dA$$
$$F = \int_0^D \left(\rho g y\right) W dy$$
$$= \tfrac{1}{2}\rho g W D^2$$
$$= 1.3\times10^{11} N$$



Examples

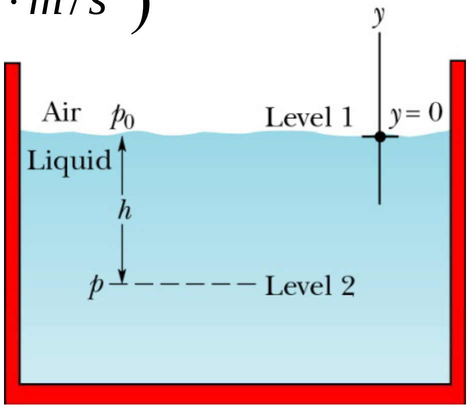
1. At what depth is the pressure two-times that of atmosphere?

$$p_{at\ h} = p_{atm} + \rho g h$$

$$2p_{atm} = p_{atm} + \rho g h$$

$$h = \frac{p_{atm}}{\rho g} = \frac{1.01\times10^5 \cdot Pa}{\left(1000\cdot kg/m^3\right)\left(9.8\cdot m/s^2\right)}$$

$$= 10.3\cdot m = 33.8\cdot ft$$



2. What is the maximum height you can suck water up a straw?

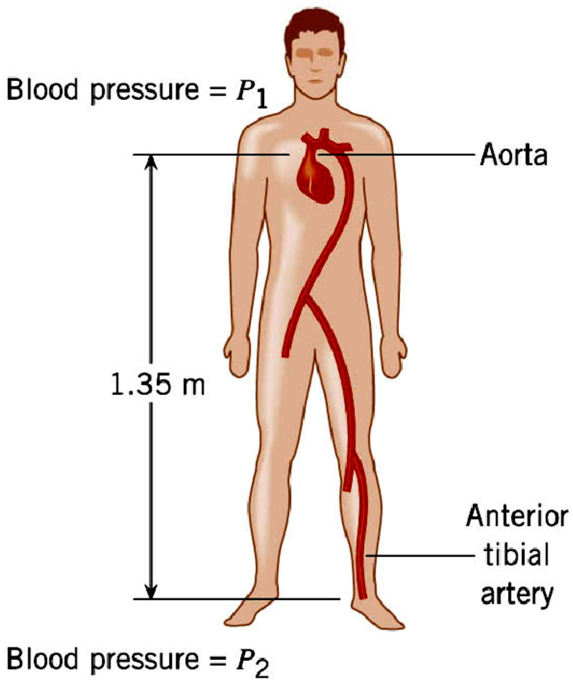
Make the pressure at the end of the straw to zero

$$p = p_{atm} + \rho g h$$

$$0 \equiv p_{atm} + \rho g h$$

$$h = -\frac{p_{atm}}{\rho g} = -10.3\cdot m = -33.8\cdot ft$$

Blood Pressure



Blood pressure of 120/80 is considered normal… what are these units? How much pressure is this? WHY?

$$\rho_{Hg} gh = (13{,}600\ {}^{kg}/_{m^3})(9.8\ {}^{m}/_{s^2})(0.12 m)$$

$$120 mmHg = 1.6 \times 10^4 Pa$$

$$80 mmHg = 1.1 \times 10^4 Pa$$

$$Difference = 0.5 \times 10^4 Pa$$

What is the pressure difference between your heart and your feet?

(Density of blood is 1060 kg/m³)

$$P_2 - P_1 = \rho g h$$

$$= (1060 \cdot {}^{kg}/_{m^3})(9.8 \cdot {}^{m}/_{s^2})(1.35 m)$$

$$= 1.4 \times 10^4 Pa$$

What land animal has the highest blood pressure?

Pressure vs height: gasses

$\rho \neq$ constant

Remember: ***if h is down, pressure goes up; if h is up, pressure goes down***

$$\Delta p = \rho g h$$

What is the air pressure at 18,000 ft (5,500 m)?
(elevation affects pressure- how?)

*Assume that the density of air is proportional to the pressure (**compressible** fluid)*

$$\frac{\rho_h}{\rho_0} = \frac{p_h}{p_0} \Rightarrow \rho_h = p_h\left(\frac{\rho_0}{p_0}\right)$$

, where at 0 º C & sea level
ρ_0 = 1.29 kg/m^3 &
p_0 =1 atm = 1.01×10^5 Pa

Negative because pressure is decreasing as you go up.

$$\Delta p = \rho g h \Rightarrow dp = -\rho g\, dy$$

$$dp = -p_h\left(\frac{\rho_0}{p_0}\right) g dy$$

$$\frac{dp}{p_h} = -\left(\frac{\rho_0}{p_0}\right) g dy$$

$$\int_{p_0}^{p_H} \frac{dp}{p_h} = -\left(\frac{\rho_0}{p_0}\right) g \int_0^H dy$$

$$\ln\left(\frac{p_H}{p_0}\right) = -\left(\frac{\rho_0}{p_0}\right) g (H-0)$$

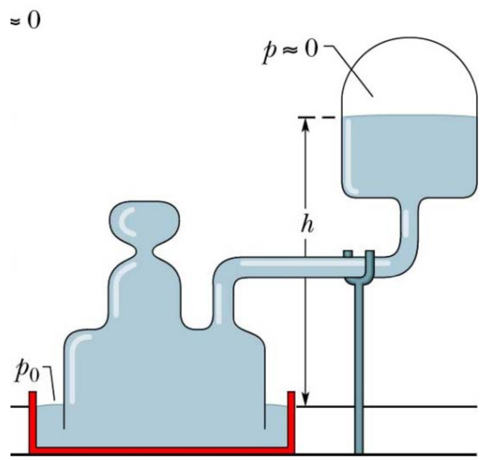
$$p_H = p_0 e^{-\left(\frac{\rho_0}{p_0}\right) gH}$$

$$p_{18,000ft} = \tfrac{1}{2} p_{atm}$$

Measuring Pressure

Torricelli (1608-1647) 1mm of Mercury = 1 torr

Closed-end Manometer (Hg Barometer)

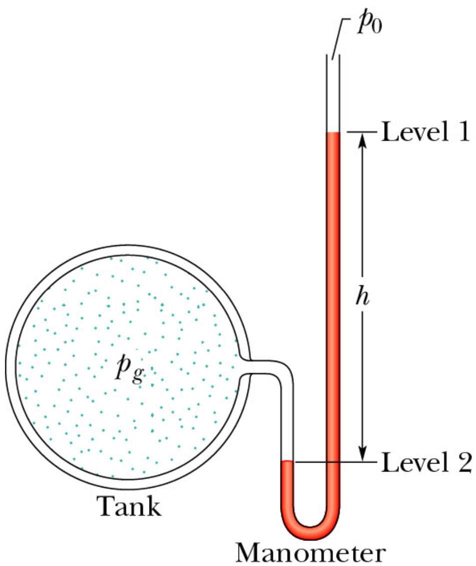


$$p_{at\ h} = 0 + \rho g h$$

$$p_{at\ h} = \rho g h$$

Absolute Pressure

Open-end Manometer



$$p_{at\ h} = p_{atm} + \rho g h$$

$$\Delta p = \rho g h$$

Gauge Pressure = $p_g = \Delta p$

$$h_{Hg} = \frac{p_{atm}}{\rho g} = \frac{1.01\times10^5 \cdot N/m^2}{\left(13{,}550 \cdot kg/m^3\right)\left(9.8 \cdot m/s^2\right)}$$

$$= 760 \cdot mm\ Hg$$

Question 14-1

Is the gauge pressure at the bottom of a 1-m high tube of water on the earth the same as is on the moon?

1. Yes
2. No
3. Sometimes

$$\left.\begin{array}{l}\Delta P_{moon} = \rho g_{moon} h \\ \Delta P_{earth} = \rho g_{earth} h\end{array}\right\} \quad g_{moon} < g_{earth}, \text{ so that } \Delta P_{moon} < \Delta P_{earth}$$

Pascal's Principle

Blaise Pascal (1623-1662)

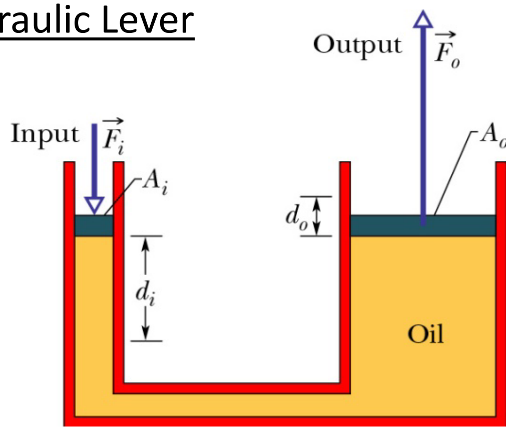
A change in the pressure applied to an enclosed incompressible fluid is transmitted throughout by the same amount.

What is the pressure 100m below sea level?

$$p_{100m} = 1 \cdot atm + 9.7 \cdot atm = 10.7 \cdot atm$$

"Transmitted" throughout the whole 100m

Hydraulic Lever



"Mechanical Advantage"

$$p_{out} = p_{in}$$

$$\frac{F_{out}}{A_{out}} = \frac{F_{in}}{A_{in}}$$

$$F_{out} = F_{in} \frac{A_{out}}{A_{in}}$$

Incompressible Fluid:

$$V = d_{in} A_{in} = d_{out} A_{out}$$

$$d_{out} = d_{in} \frac{A_{in}}{A_{out}}$$

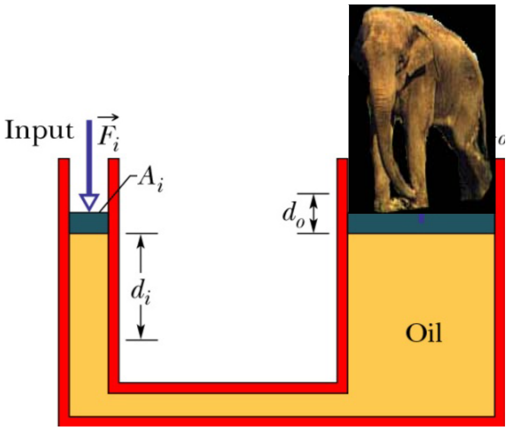
Work:

$$W_{out} = F_{out} d_{out} = \left(F_{in} \frac{A_{out}}{A_{in}} \right) \left(d_{in} \frac{A_{in}}{A_{out}} \right)$$

$$= F_{in} d_{in} = W_{in}$$

With a hydraulic lever, a given force applied over a given distance can be transformed to a greater force over a smaller distance

Examples of Pascal's Principle



A change in pressure applied to a completely enclosed fluid is transmitted undiminished to all parts of the fluid and the enclosing walls

P is the same!

A person m=75kg stands on circular piston A (diameter=0.40m) of a hydraulic pump. If you want to lift an elephant weighing 1500kg, what is the minimum diameter of circular piston B?

$$P = \frac{F_A}{A_A} = \frac{F_B}{A_B}$$

$$A_B = A_A \frac{F_B}{F_A}$$

$$\pi\left(\frac{d_B}{2}\right)^2 = \pi\left(\frac{d_A}{2}\right)^2 \frac{F_B}{F_A}$$

$$d_B = 2\left(\frac{d_A}{2}\right)\sqrt{\frac{F_B}{F_A}}$$

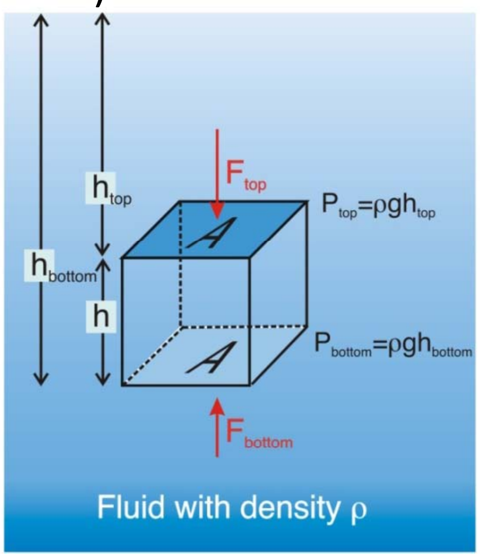
$$= 1.8m$$

Buoyancy and Archimedes' Principle

Archimedes' (287?-212 BC)

Buoyancy - lift a rock under water - its light
- take it above water - its heavy
  - many objects float - how?
  - Force of gravity points downward
  - buoyant force is exerted upward by fluid

Consider a cube in a fluid with density ρ and area A.
(forget about the pressure on top of the fluid, i.e. atmospheric pressure)



1) At top, fluid exerts a force on cube:

$$F_{top} = P_{top}A = \rho g h_{top} A \qquad \textbf{DOWNWARD}$$

2) At bottom, fluid exerts a force on cube:

$$F_{bottom} = P_{bottom}A = \rho g h_{bottom} A \qquad \textbf{UPWARD}$$

3) Net force due to "fluid" is **bouyant force**:

$$F_{buoyant} = F_{bottom} - F_{top} = \rho g(h_{bottom} - h_{top})A$$

$$= \rho g h A = \rho g V \qquad \textbf{UPWARD}$$

$$= m_{fluid} g$$ **= weight of displaced fluid**

Buoyancy and Archimedes' Principle

Buoyant Force - is equal to the weight of fluid displaced by the object

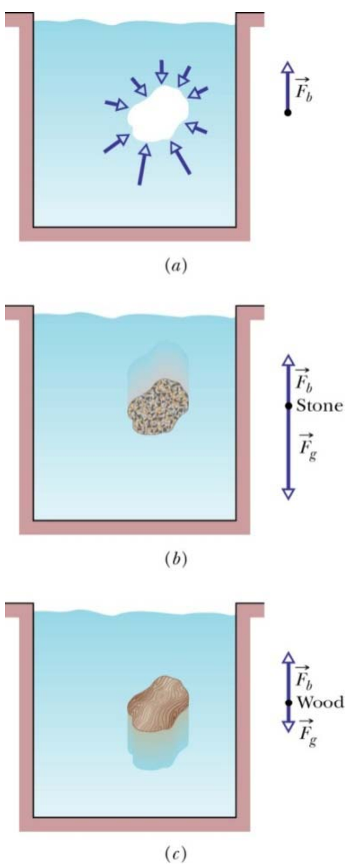
- is directed UPWARDs

$$F_B = m_F g$$

UPWARD

- Does not depend on the shape of the object ONLY volume.

- Applies to partially or completely immersed object

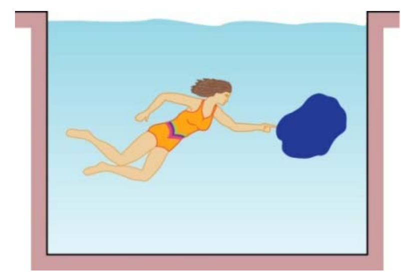


A stone drops

$(F_B - mg) < 0 \quad a \rightarrow \text{downward}$

Wood will rise

$(F_B - mg) > 0 \quad a \rightarrow \text{upward}$



A "bag" of water stays put

$(F_B - mg) = 0 \quad a = 0$

Buoyancy and Archimedes' Principle

Buoyant Force **- equal to the weight of fluid displaced by the object**

- is directed UPWARDs

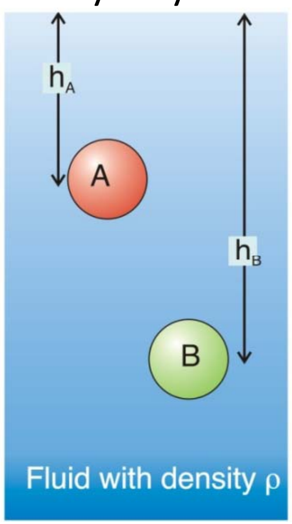
- Does not depend on shape of object - ONLY volume.

- Applies to partially or completely submerged object

$$F_B = m_F g$$

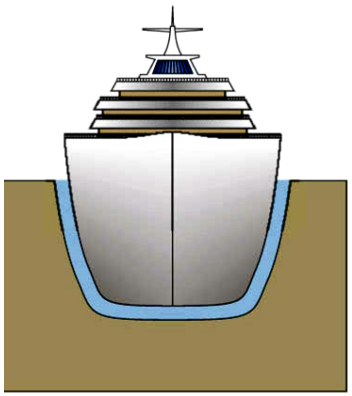
UPWARD

Which has larger buoyancy force?



If the volumes are the same, they displace the same mass of fluid so the buoyant forces are the same

Sink or float?



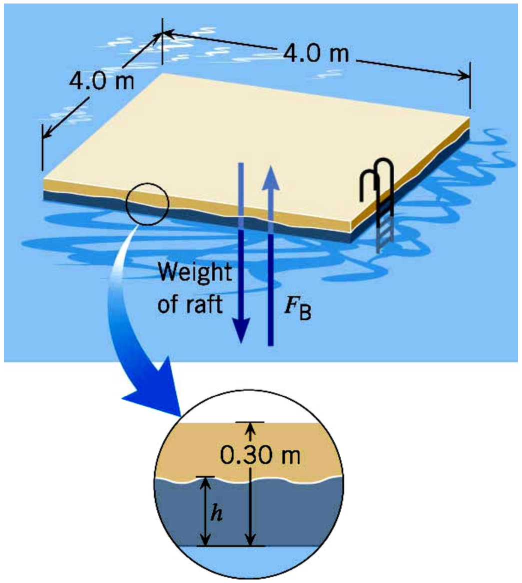
The answer is 2.
What matters is the mass of the **displaced** fluid.

A 200-ton ship is in a tight-fitting lock so that the mass of fluid left in the lock is much less than the mass of the ship. Does it float?

1. No. The ship touches the bottom since it weighs more than the water.
2. Yes, as long as the water gets up to the ship's waterline.

Buoyancy

A wooden raft is 4 m on a side and 30 cm thick. How much of the raft is below the water? (ρ_{wood} = 550 kg/m^3)



$$W_{raft} = \rho_{raft} V g$$
$$= (550 kg/m^3)(4 \cdot 4 \cdot 0.3 m^3)(9.8 m/s^2)$$
$$= 25900 N$$

$$W_{raft} = W_{waterdisplaced} = \rho_{water} V_{water} g$$
$$V_{water} = \frac{25900 N}{(1000 kg/m^3)(9.8 m/s^2)}$$
$$= 2.64 m^3$$

$$V_{water} = 2.64 m^3 = (16 m^2) h$$
$$h = 0.17 m$$
$$= 17 cm$$

What happens if the density of wood is more or less?

Buoyancy and Archimedes' Principle

Two cups are filled to the same level with water. One cup has ice cubes floating in it. Which weighs more? (yes, some of the ice is sticking up out of the water)

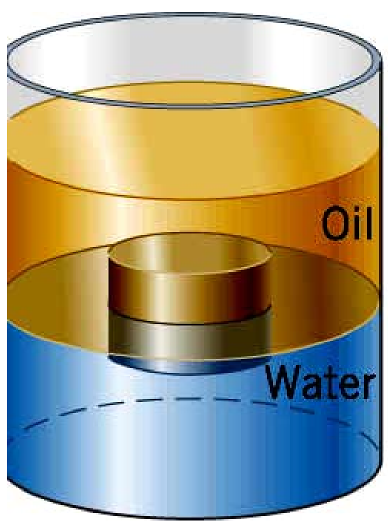
1. The cup with the ice cubes.
2. The cup without the ice cubes.
3. The two weigh the same.

Since the ice weighs exactly the same as the displaced fluid and the levels of the water are the same, the two weigh the same.

Consider an object that floats in water but sinks in oil. When the object floats in water half of it is submerged. If we slowly pour oil on the top of the water so it completely covers the object, the object

1. moves up.
2. stays in the same place.
3. moves down.

When the oil is poured over the object it displaces some oil. This means it feels a buoyant force from the oil in addition to the buoyant force from the water. Therefore it rises higher.



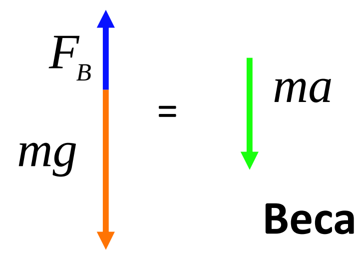
Question 14-1

You are on a distant planet where the acceleration due to gravity is half that on Earth. Would you float more easily in water on that planet?

1. Yes, you will float higher
2. Floating would not be changed.
3. No, you will float deeper.

Archimedes' Principle : Apparent weight in a fluid

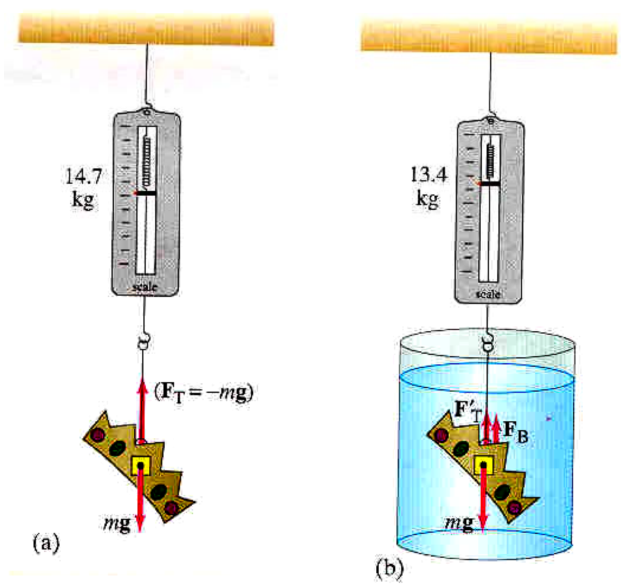
Weigh in air, then weigh in water. From this, and knowing ρ_{water} you can get object's ρ.



$$Weight_{apparent} = Weight_{actual} - F_B$$

Because of buoyant force, apparent weight in water is less than actual weight

Archemedes' : Is the King's crown gold??? (Hiero III 306-215 BC)



$$Weight_{apparent} = Weight_{actual} - F_B$$
$$= \rho_{crown} Vg - \rho_{water} Vg$$

Is it gold???

$$\frac{W_{actual}}{W_{actual} - W_{apparent}} = \frac{\rho_{crown} Vg}{\rho_{water} Vg} = \frac{\rho_{crown}}{\rho_{water}}$$

Specific gravity

$$\left(\frac{14.7 \cdot kg}{14.7 \cdot kg - 13.4 \cdot kg} \right) = 11.3 \quad ???$$

Specific gravities:	
Gold (Au)	19.3
Lead (Pb)	11.3

$$13.4 = 14.7 - F_B$$

$$\rho_{water} Vg = 1.3N \qquad V = 1.3 \times 10^{-4} m^3$$

$$\rho_{crown} = \frac{14.7}{Vg} = 11300 \, {kg}/{m^3}$$

Specific Gravity

$$specific\ gravity = \frac{\rho}{\rho_{WATER}}$$

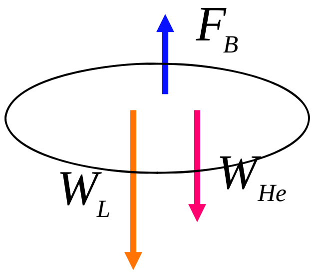
Rather than using the large SI unit it is sometimes convenient to use specific gravity

Material	ρ (kg/m³)	Specific gravity
air	1.21	0.001
water	1000	1
Al	2700	2.7
Fe	7800	7.8
Cu	8960	8.9
Pb	11400	11.4
Os	22400	22.4

Helium Blimps

Length 192 feet   Width 50 feet   Height 59.5 feet
Volume 202,700 cubic feet **(5740 m^3)**
Maximum Speed 50 mph   Cruise Speed 30 mph
Powerplant: Two 210 hp fuel-injected, air-cooled piston engines

What is the maximum load weight of blimp (W_L) in order to fly?



The "fluid" blimp is in is: air

$$\rho_{He} = 0.179 \cdot kg/m^3 \quad \& \quad \rho_{air} = 1.21 \cdot kg/m^3$$

At static equilibrium $\sum F = 0: \quad W_{He} + W_L = F_B$

$$W_L = F_B - W_{He}$$

$$W_L = m_{air} g - m_{He} g$$

$$= \rho_{air} V_{ship} g - \rho_{He} V_{ship} g$$

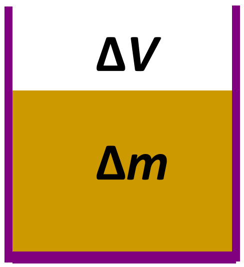
$$= V_{ship} g (\rho_{air} - \rho_{He})$$

$$= (5740 \cdot m^3)(9.8 \cdot m/s^2)(1.21 - 0.179 \cdot kg/m^3)$$

$$= \underline{58 \cdot kN = 13{,}000 \cdot lbs}$$

"Maximum Gross Weight 12,840 pounds"

Summary of Chapter 14
Study Guide



$$\rho = \frac{\Delta m}{\Delta V}$$

Fluids

As the name implies, a fluid is defined as a substance that can flow. Fluids conform to the boundaries of any container in which they are placed. A fluid cannot exert a force tangential to its surface. It can only exert a force perpendicular to its surface. Liquids and gases are classified together as fluids to contrast them with solids. In crystalline solids the constituent atoms are organized in a rigid three-dimensional regular array known as the "lattice."

Density :

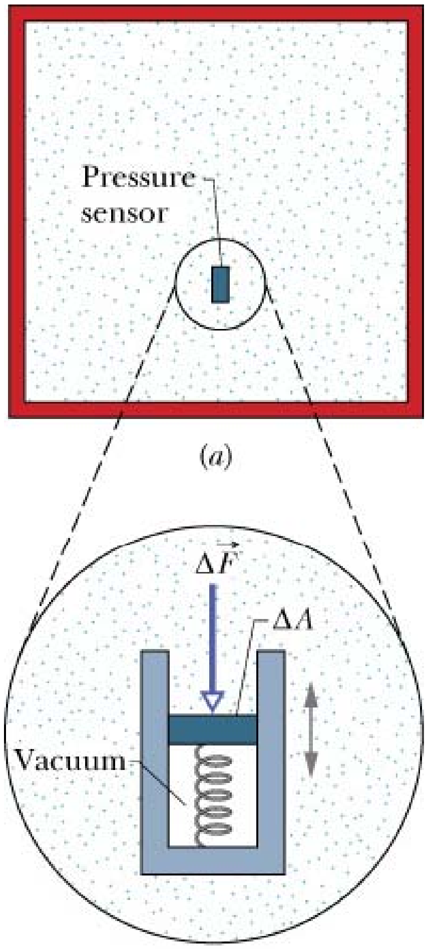
Consider the fluid shown in the figure. It has a mass Δm and volume ΔV. The density (symbol ρ) is defined as the ratio of the mass over the volume: $\rho = \frac{\Delta m}{\Delta V}$.

SI unit: kg/m^3

If the fluid is homogeneous, the above equation has the form

$$\rho = \frac{m}{V}.$$

Pressure



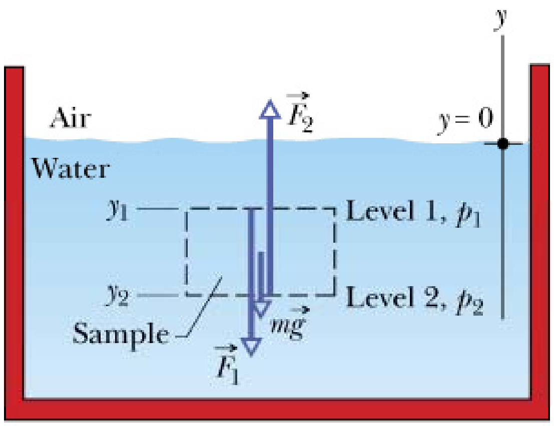
$$p = \frac{F}{A}$$

Consider the device shown in the insert of the figure, which is immersed in a fluid-filled vessel. The device can measure the normal force F exerted on its piston from the compression of the spring attached to the piston. We assume that the piston has an area A. The pressure p exerted by the fluid on the piston is defined as $p = \frac{F}{A}$.

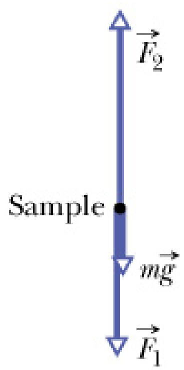
The SI unit for pressure, $\frac{\text{N}}{\text{m}^2}$, is known as the pascal (symbol: Pa). Other units are the atmosphere (atm), the torr, and the lb/in^2. The atm is defined as the average pressure of the atmosphere at sea level:

$$1 \text{ atm} = 1.01 \times 10^5 \text{ Pa} = 760 \text{ Torr} = 14.7 \text{ lb/in}^2.$$

Experimentally it is found that the pressure p at any point inside the fluid has the same value regardless of the orientation of the cylinder. The assumption is made that the fluid is at rest.



(a)



(b)

$$(p_2 - p_1) = \rho g (y_1 - y_2)$$

$$p = p_0 + \rho g h$$

Fluids at Rest

Consider the tank shown in the figure. It contains a fluid of density ρ at rest. We will determine the pressure difference $p_2 - p_1$ between point 2 and point 1 whose y-coordinates are y_2 and y_1, respectively. Consider a part of the fluid in the form of a cylinder indicated by the dashed lines in the figure. This is our "system" and it is at equilibrium. The equilibrium condition is:

$F_{y,net} = F_2 - F_1 - mg = 0$. Here F_2 and F_1 are the forces exerted by the rest of the fluid on the bottom and top faces of the cylinder, respectively. Each face has an area A:

$$F_1 = p_1 A, \quad F_2 = p_2 A, \quad m = \rho V = \rho A (y_1 - y_2).$$

If we substitute into the equilibrium condition we get:

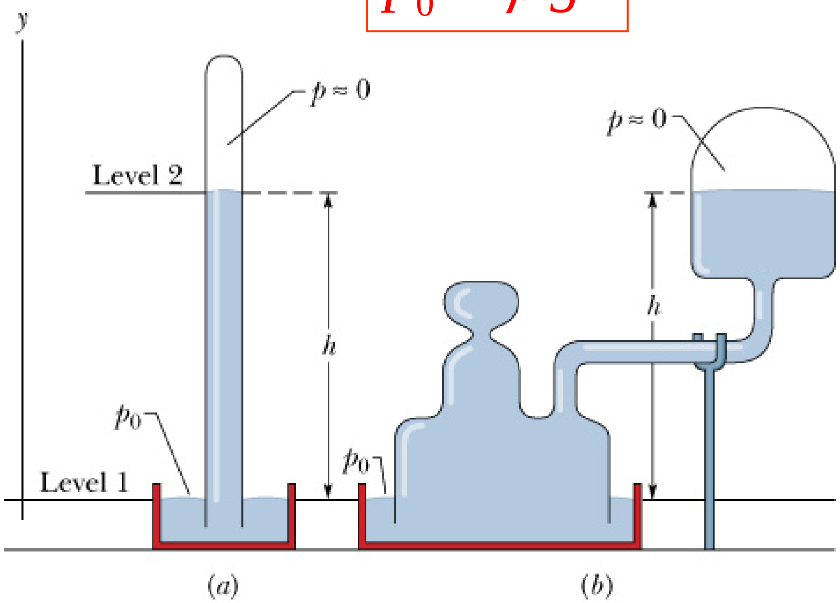
$$p_2 A - p_1 A - \rho g A (y_1 - y_2) = 0 \rightarrow (p_2 - p_1) = \rho g (y_1 - y_2).$$

If we take $y_1 = 0$ and $h = -y_2$ then $p_1 = p_0$ and $p_2 = p$.

The equation above takes the form $p = p_0 + \rho g h$.

Note: The difference $p - p_0$ is known as "**gauge pressure.**"

$$p_0 = \rho g h$$



The Mercury Barometer

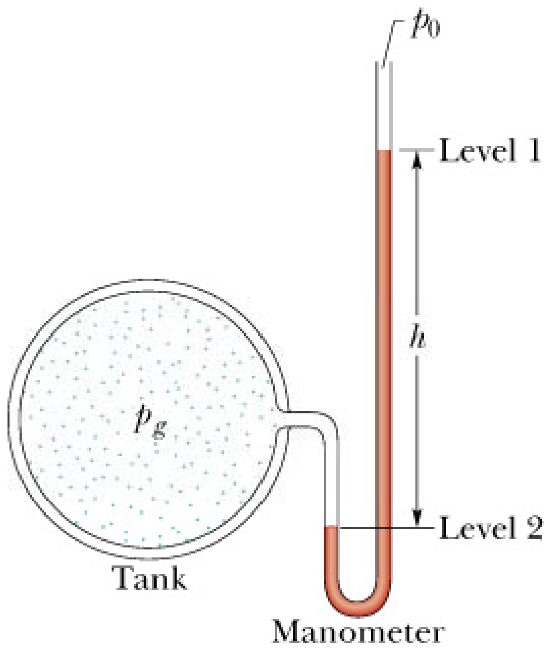
The mercury barometer shown in fig. a was constructed for the first time by Evangelista Toricelli. It consists of a glass tube of length approximately equal to 1 meter. The tube is filled with mercury and then it is inverted with its open end immersed in a dish filled also with mercury.

Toricelli observed that the mercury column drops so that its length is equal to h. The space in the tube above the mercury can be considered as empty.

If we take $y_1 = 0$ and $y_2 = h$ then $p_1 = p_0$ and

$(p_2 - p_1) = \rho g (y_1 - y_2) \rightarrow p_0 = \rho g h.$

We note that the height h does not depend on the cross-sectional area A of the tube. This is illustrated in fig. b. The average height of the mercury column at sea level is equal to 760 mm.



$$p_g = \rho g h$$

The Open - Tube Manometer

The open-tube manometer consists of a U-tube that contains a liquid. One end is connected to the vessel for which we wish to measure the gauge pressure. The other end is open to the atmosphere.

At level 1: $y_1 = 0$ and $p_1 = p_0$

At level 2: $y_2 = -h$ and $p_2 = p$

$p_2 = p_1 + \rho g h \rightarrow p - p_0 = \rho g h \rightarrow$

$p_g = \rho g h$

If we measure the length h and if we assume that g is known, we can determine p_g.

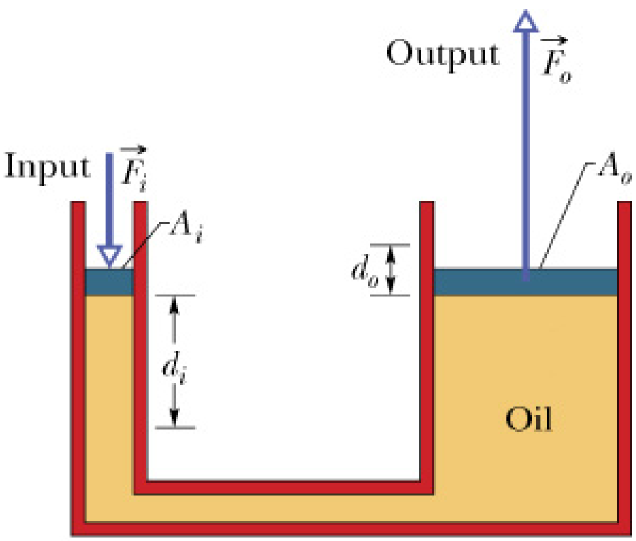
The gauge pressure can take either positive or negative values.

Pascal's Principle and the Hydraulic Lever

$$F_o = F_i \frac{A_o}{A_i}$$

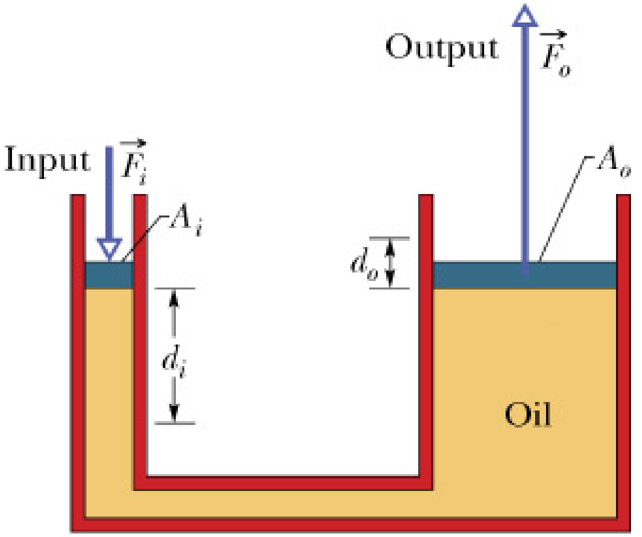
Pascal's principle can be formulated as follows:

A change in the pressure applied to an enclosed incompressible liquid is transmitted undiminished to every portion of the fluid and to the walls of the container.



Consider the enclosed vessel shown in the figure, which contains a liquid. A force F_i is applied downward to the left piston of area A_i. As a result, an upward force F_o appears on the right piston, which has area A_o. Force F_i produces a change in pressure $\Delta p = \frac{F_i}{A_i}$. This change will also appear on the right piston. Thus we have:

$$\Delta p = \frac{F_i}{A_i} = \frac{F_o}{A_o} \rightarrow F_o = F_i \frac{A_o}{A_i} \qquad \text{If } A_o > A_i \rightarrow F_o > F_i$$



The Hydraulic Lever; Energy Considerations

The hydraulic lever shown in the figure is filled with an incompressible liquid. We assume that under the action of force F_i the piston to the left travels downward by a distance d_i. At the same time the piston to the right travels upward by a distance d_o. During the motion we assume that a volume V of the liquid is displaced at both pistons:

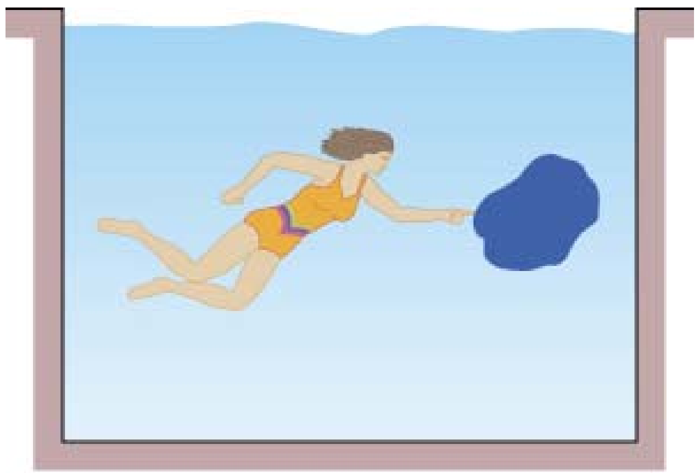
$$V = A_i d_i = A_f d_f \rightarrow d_o = d_i \frac{A_i}{A_o}$$

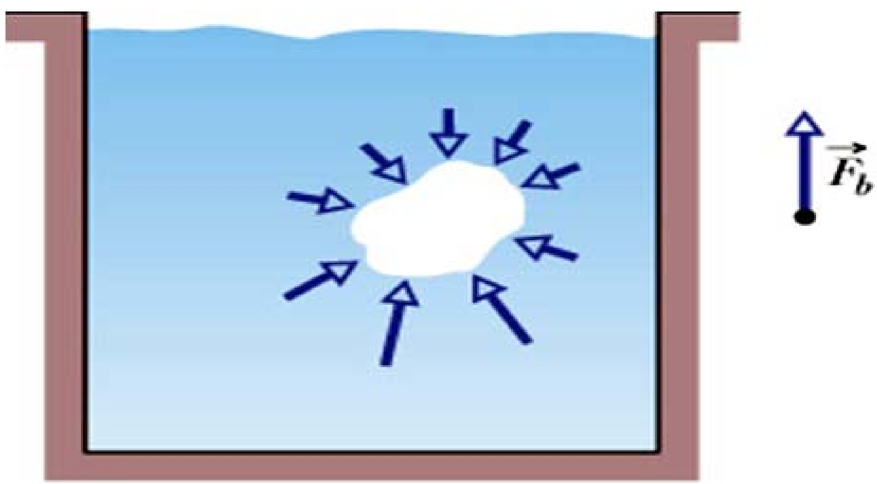
Note : Since $A_o > A_i \rightarrow d_o < d_i$.

The output work $W_o = F_o d_o = \left(F_i \frac{A_o}{A_i} \right)\left(d_i \frac{A_i}{A_o} \right)$.

Thus $W_o = F_i d_i = W_i$. The work done on the left piston by F_i is equal to the work done by the piston to the right in lifting a load placed on it.

With a hydraulic lever a given force F_i applied over a distance d_i can be transformed into a larger force F_o applied over a smaller distance d_o.





$$F_b = \rho_f gV$$

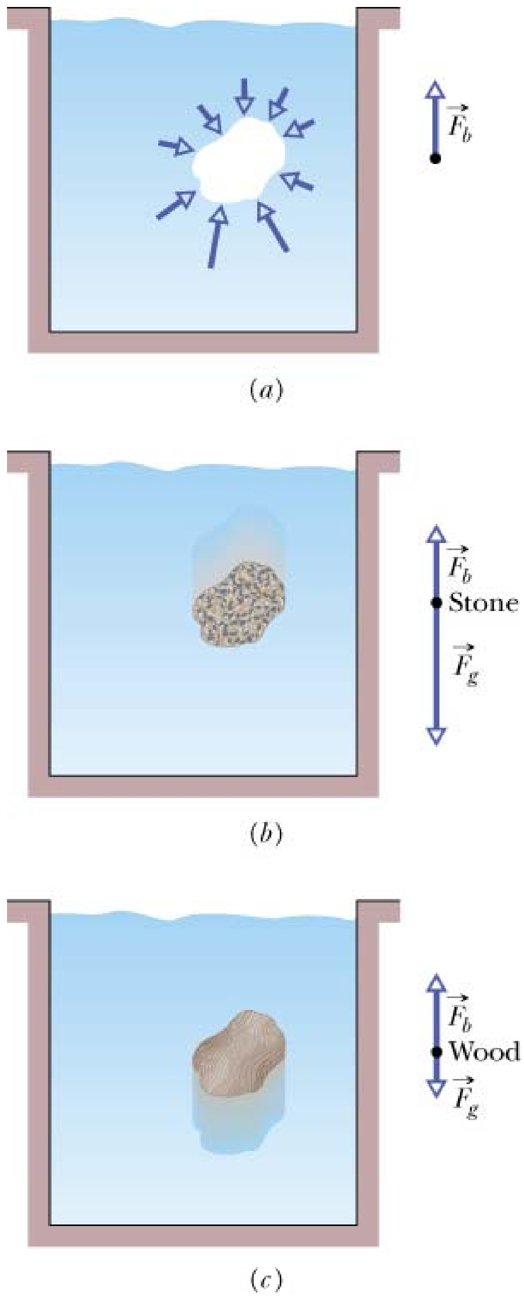
Buoyant Force

Consider a very thin plastic bag that is filled with water. The bag is at equilibrium thus the net force acting on it must be zero. In addition to the gravitational force $\vec{F}_g$ there exists a second force $\vec{F}_b$ known as "**buoyant force,**" which balances $\vec{F}_g$: $F_b = F_g = m_f g$.

Here m_f is the mass of the water in the bag.

If V is the bag volume we have $m_f = \rho_f gV$.

Thus the magnitude of the buoyant force $F_b = \rho_f gV$. $\vec{F}_b$ exists because the pressure on the bag exerted by the surrounding water increases with depth. The vector sum of all the forces points upward, as shown in the figure.



Archimedes' Principle

Consider the three figures to the left. They show three objects that have the same volume (V) and shape but are made of different materials. The first is made of water, the second of stone, and the third of wood. The buoyant force F_b in all cases is the same: $F_b = \rho_f gV$. This result is summarized in what is known as "**Archimedes' Principle.**"

When a body is fully or partially submerged in a fluid a buoyant force $\vec{F}_b$ is exerted on the body by the surrounding fluid. This force is directed upward and its magnitude is equal to the weight m_fg of the fluid that has been displaced by the body.

We note that the submerged body in fig. *a* is at equilibrium with $F_g = F_b$. In fig. *b* $F_g > F_b$ and the stone accelerates downward. In fig. *c* $F_b > F_g$ and the wood accelerates upward.