Physics 2101
Section 3
March 24th

Announcements:

- Review today
- Exam tonight at 6 PM, Lockett-6

Class Website:

http://www.phys.lsu.edu/classes/spring2010/phys2101-3/

http://www.phys.lsu.edu/~jzhang/teaching.html

Chap. 10: Rotation

$\vec{\theta}(t)\ \left(\Delta\vec{\theta}=\vec{\theta}_2-\vec{\theta}_1\right)$	$s=\lvert\theta\rvert r$	
$\vec{\omega}(t)=\dfrac{d\vec{\theta}(t)}{dt}$	$\lvert\vec{v}\rvert=\dfrac{d\lvert\theta\rvert}{dt}r=\lvert\omega\rvert r$	$\vec{v}=\vec{\omega}\times\vec{r}$
$\vec{\alpha}(t)=\dfrac{d\vec{\omega}(t)}{dt}=\dfrac{d^2\vec{\theta}(t)}{dt^2}$	$\lvert\vec{a}_{tot}\rvert^2=\lvert\vec{a}_r\rvert^2+\lvert\vec{a}_t\rvert^2$ $=\lvert\omega^2 r\rvert^2+\lvert\alpha r\rvert^2$	$\vec{a}_{tot}=\vec{a}_t+\vec{a}_r$ $\vec{a}_r=\vec{\omega}\times\left(\vec{\omega}\times\vec{r}\right)$ $\vec{a}_t=\vec{\alpha}\times\vec{r}$

Rotational Kinematics:
(ONLY IF α = constant)

$$\omega=\omega_0+\alpha t$$

$$\theta=\theta+\omega_0 t+\tfrac{1}{2}\alpha t$$

$$\omega^2=\omega_0^2+2\alpha\theta$$

Rotational Kinetic energy: $KE_{rot}=\tfrac{1}{2}I\omega^2$

Moment of inertia: $I\equiv\sum_{\text{all mass}} m_i r_i^2=\int r^2 dm$ $\quad I=I_{COM}+Mh^2$

Torgue: $\vec{\tau}=\vec{r}\times\vec{F}$ $\quad \lvert\vec{\tau}\rvert=\lvert\vec{r}\rvert\lvert\vec{F}\rvert\sin\vartheta$

Work done by a torgue: $W=\int_{\theta_1}^{\theta_2}\tau_{net}\,d\theta=\tau\left(\theta_f-\theta_i\right)=\Delta KE_{rot}=\tfrac{1}{2}I\left(\omega_f^2-\omega_i^2\right)$

Chap. 11: Rotational dynamics

2nd – Law for rotation: $$\vec{\tau}_{net} = \sum \vec{\tau}_i = I\vec{\alpha} \qquad \left\{\vec{F}_{net} = \sum \vec{F}_i = m\vec{a}\right\}$$

Rolling bodies: $$v_{com} = \omega R \quad a_{com} = \alpha R \qquad KE_{tot} = \frac{1}{2} I_{com}\omega^2 + \frac{1}{2} M v_{com}^2$$

The friction is always against the tendency of sliding

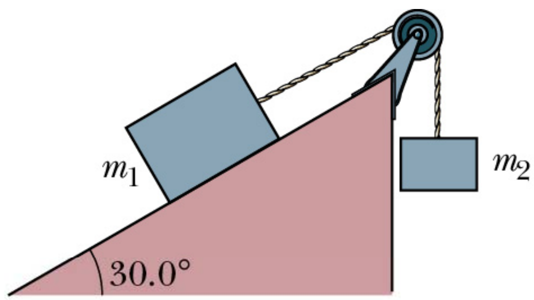
Angular momentum: $$\vec{\ell} = \vec{r} \times \vec{p} = m\left(\vec{r} \times \vec{v}\right) \quad \vec{\tau}_{net} = \sum \vec{\tau}_i = \frac{d\vec{\ell}}{dt}$$ (a particle)

$$\vec{L} = I\vec{\omega} \qquad \frac{d\vec{L}}{dt} = \vec{\tau}_{net} = I\vec{\alpha} = I\frac{d\vec{\omega}}{dt}$$ (a rigid body)

Angular momentum conservation: $$\vec{L}_i = \vec{L}_f \quad if \quad \vec{\tau}_{net} = 0; \quad I_i\omega_i = I_f\omega_f$$

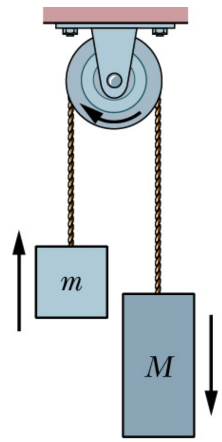
Summary: Effects of rotation

From before



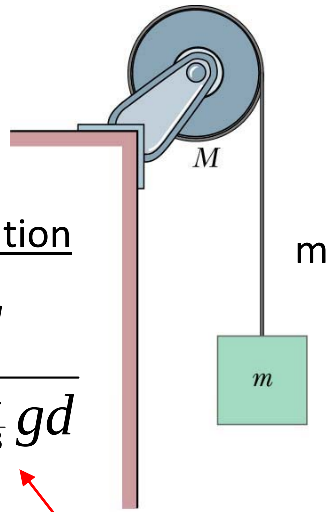
$$a = \frac{2g(m_2 - m_1 \sin\theta)}{M_w + 2(m_1 + m_2)}$$

Done via force/torque



$$|v| = \sqrt{\frac{4gd(M-m)}{M_W + 2(m+M)}}$$

Done via energy



m = M

With Rotation

$$a = \tfrac{2}{3}g$$

$$|v| = \sqrt{\tfrac{4}{3}gd}$$

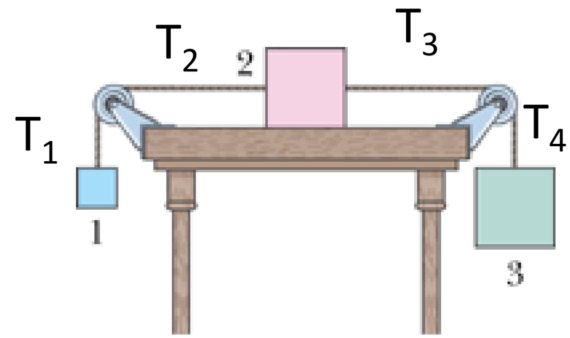
Done with both

remember this for later

In each of these cases: "translation" was separate from "rotation"

Pure translation + Pure rotation

SHW#6: Three masses M, 2M and 3M are shown in the figure with two pulley of moment of inertial I and radius r. The coefficient of friction is μ_k and the table is L m long.



$$T_1 - Mg = Ma$$

$$(T_2 - T_1)r = I\alpha = \frac{Ia}{r}$$

$$T_3 - T_2 - 2\mu_k Mg = 2Ma$$

$$(T_4 - T_3)r = \frac{Ia}{r}$$

$$3Mg - T_4 = 3Ma$$

$$T_2 = gM(3 - 2\mu_k) - a\left(\frac{I}{r^2} + 5M\right)$$

$$T_2 = \frac{gM\left[\frac{I}{r^2}(2 - \mu_k) + M(4 - \mu_k)\right]}{\left[\frac{I}{r^2} + 3M\right]}$$

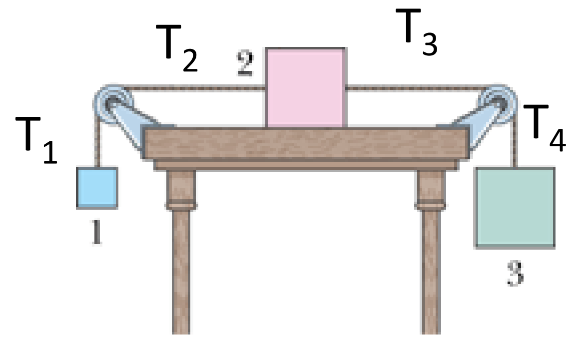
$$T_3 = -a\left(\frac{I}{r^2} + 3M\right) + 3Mg$$

$$T_3 = -Mg(1 - \mu_k) + 3Mg = Mg(2 + \mu_k)$$

$$a = \frac{Mg(1 - \mu_k)}{\left[\frac{I}{r^2} + 3M\right]}$$

Need $v = r\omega$ & $a = r\alpha$

SHW#6: Three masses M, 2M and 3M are shown in the figure with two pulley of moment of inertial I and radius r. The coefficient of friction is μ_k and the table is L m long.
Let us use Energy.



We can define zero so $E_{mech}(start) = 0$

After block 3 has moved a distance d we have

$$E_{mech} = 0 - 2\mu_k Mgd = \frac{6Mv^2}{2} + \frac{2I\omega^2}{2} - (3M - M)gd$$

$$-\mu_k Mgd = \frac{3Mv^2}{2} + \frac{Iv^2}{2r^2} - Mgd \quad (\text{use } v = r\omega)$$

$$v^2 = \frac{2Mgd(1-\mu_k)}{3M + \frac{I}{r^2}}:$$

$$v^2 = 2ad: \quad a = \frac{Mg(1-\mu_k)}{3M + \frac{I}{r^2}}$$

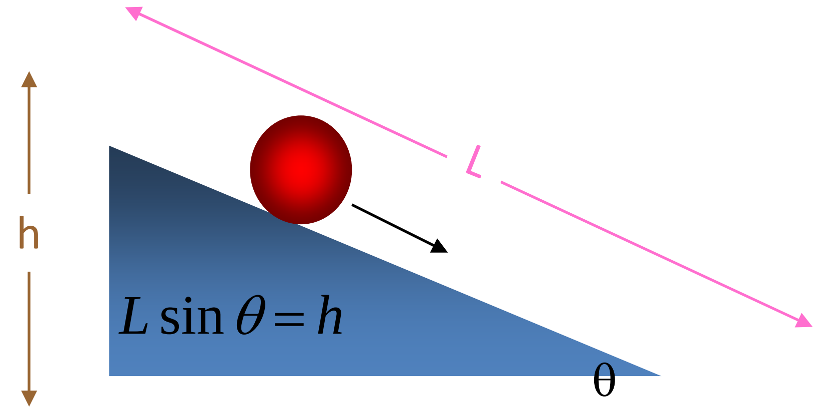
Same as before

$$a = \frac{Mg(1-\mu_k)}{\left[\frac{I}{r^2} + 3M\right]}$$

$$\frac{v_f + v_i}{2} t = d: \quad t = \sqrt{\frac{2d\left(3M + \frac{I}{r^2}\right)}{Mgd(1-\mu_k)}}$$

Rolling down a ramp : Energy considerations

An object at rest, with mass **m** and radius **r**, roles from top of incline plane to bottom. What is **v** at bottom and **a** throughout.



$$\Delta E_{mech} = 0$$

$$\Delta KE_{tot} = -\Delta U$$

$$\left(KE_{rot+trans,COM}\right)_{final} - (0)_{init} = -\left[(0)_{final} - (mgh)_{init}\right]$$

$$\rightarrow \tfrac{1}{2} m v_{COM}^2 + \tfrac{1}{2} I_{COM}\,\omega^2 = mgL\sin\theta$$

$\rightarrow$ Using $v_{com} = \omega r$

$\rightarrow$ Using $v^2 = v_0^2 + 2aL$

$$\left|v_{COM}\right| = \sqrt{\frac{2gL\sin\theta}{\left(1 + \frac{I_{com}}{mR^2}\right)}}$$

Using 1-D kinematics (const accel) $\Longrightarrow$

$$a_{com} = \frac{v^2}{2L} = \frac{g\sin\theta}{\left(1 + \frac{I_{com}}{mR^2}\right)}$$

Speed at Bottom

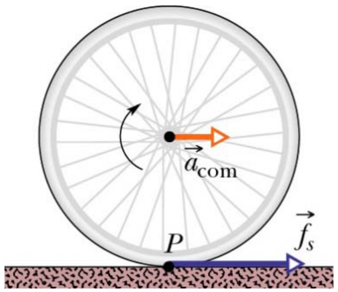
Constant Acceleration

Friction and Rolling

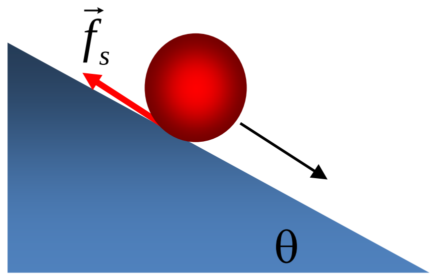
The static friction is always opposing the tendency to slide

1) If $a_{com} = 0$, it has no tendency to slide at point of contact - **no frictional force**

2) If $a_{com} > 0$ (i.e. there are net forces) and <u>no slipping</u> occurs, $\tau \neq 0$ provided by **static friction force**

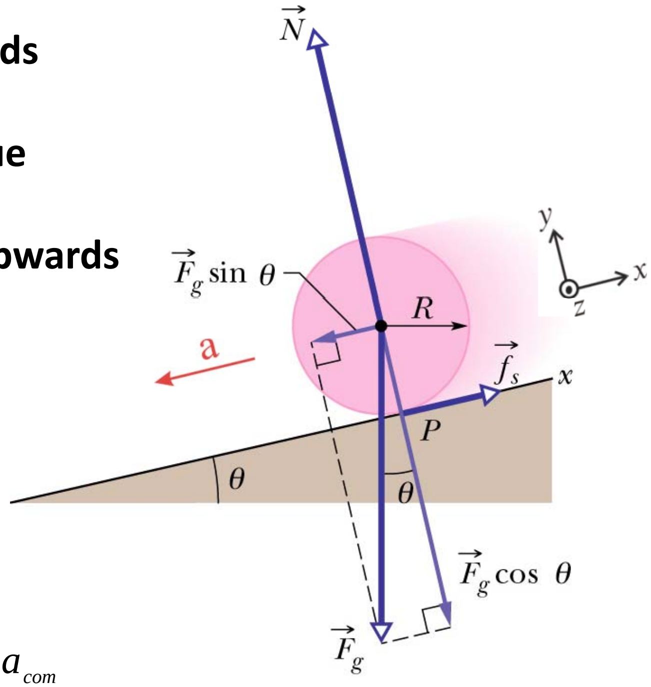


3) If $a_{com} > 0$, and <u>no slipping</u> occurs, $\tau \neq 0$ provided by weight and **static friction force**



Rolling down a ramp: acceleration

- **Acceleration downwards**
- **Friction provides torque**
- **Static friction points upwards**



Newton's 2nd Law

Linear version

$$\hat{x}: \quad f_s - F_g \sin\theta = -ma_{com}$$

$$\hat{y}: \quad N - F_g \cos\theta = 0$$

Angular version

$$\hat{z}: \quad I_{com}\alpha = \tau = Rf_s$$

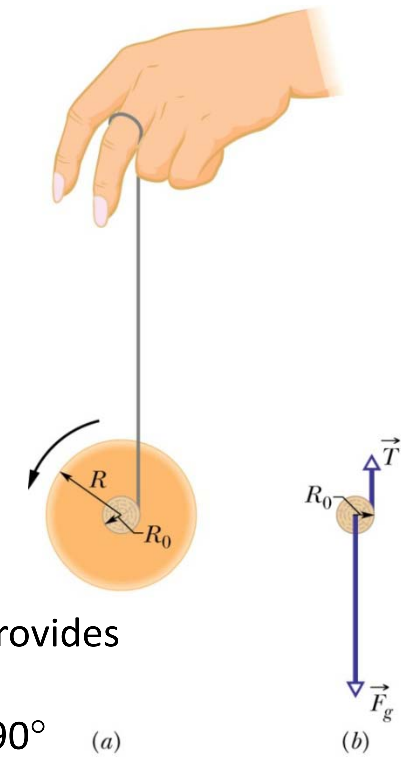
$$\rightarrow a_{com} = \alpha R$$

$$\alpha = \frac{a_{com}}{R} = \frac{Rf_s}{I_{com}}$$

$$a_{com} = \frac{R^2(mg\sin\theta - ma_{com})}{I_{com}}$$

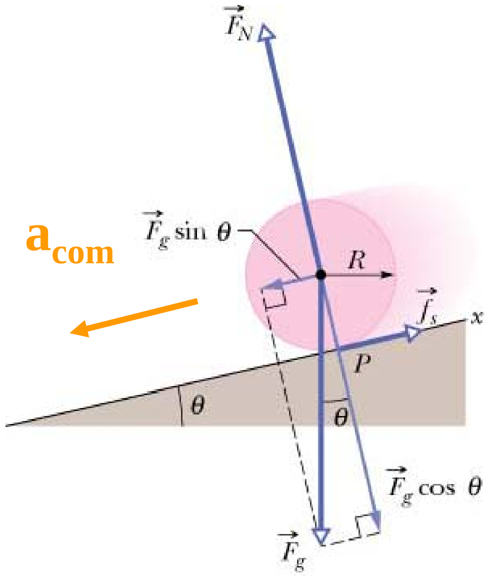
$$a_{com} = \frac{g\sin\theta}{1 + I_{com}/mR^2}$$

Yo-Yo



- Tension provides torque
- Here θ = 90°
- Axle: R ⇒ R_0

$$a_{com} = \frac{g}{1 + I_{com}/mR_0^2}$$



$$|a_{com}| = \frac{g\sin\theta}{1+\dfrac{I_{com}}{MR^2}}$$

Cylinder

$$I_1 = \frac{MR^2}{2}$$

$$a_1 = \frac{g\sin\theta}{1+I_1/MR^2}$$

$$a_1 = \frac{g\sin\theta}{1+MR^2/2MR^2}$$

$$a_1 = \frac{g\sin\theta}{1+1/2}$$

$$a_1 = \frac{2g\sin\theta}{3} = (0.67)g\sin\theta$$

Hoop

$$I_2 = MR^2$$

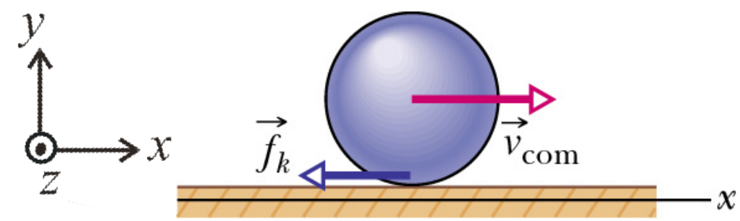
$$a_2 = \frac{g\sin\theta}{1+I_2/MR^2}$$

$$a_2 = \frac{g\sin\theta}{1+MR^2/MR^2}$$

$$a_2 = \frac{g\sin\theta}{1+1}$$

$$a_2 = \frac{g\sin\theta}{2} = (0.5)g\sin\theta$$

Prob. 11-64 **NON-smooth rolling motion**



A bowler throws a bowling ball of radius **R** along a lane. The ball slides on the lane, with initial speed $\mathbf{v}_{com,0}$ and initial angular speed $\omega_0 = 0$. The coefficient of kinetic friction between the ball and the lane is μ_k. The kinetic frictional force $\mathbf{f}_k$ acting on the ball while producing a torque that causes an angular acceleration of the ball. When the speed $\mathbf{v}_{com}$ has decreased enough and the angular speed ω has increased enough, the ball stops sliding and then rolls smoothly.

a) [After it stops sliding] What is the $\mathbf{v}_{com}$ in terms of ω ?

Smooth rolling means $v_{com} = R\omega$

b) During the sliding, what is the ball's linear acceleration?

From 2nd law: (linear)
$\hat{x}: \; -f_k = ma_{com}$
$\hat{y}: \; N - mg = 0$

But $f_k = \mu_k N = \mu_k mg$

So $a_{com} = -f_k/m = -\mu_k g$

c) During the sliding, what is the ball's angular acceleration?

From 2nd law: (angular)
$\vec{\tau} = Rf_k(-\hat{z})$
$I\alpha(-\hat{z}) = \vec{\tau} = Rf_k(-\hat{z})$

But $f_k = \mu_k N = \mu_k mg$

So $I\alpha = Rf_k = R(\mu_k mg)$
$\alpha = R\mu_k mg / I$

d) What is the speed of the ball when smooth rolling begins?

When does $v_{com} = R\omega$?

$v_{com} = v_0 + a_{com}t$ $\qquad v_{com} = v_0 - \mu_k g t$

From kinematics: $\omega = \omega_0 + \alpha t$ $\qquad t = \omega/\alpha = I\omega / R\mu_k mg$

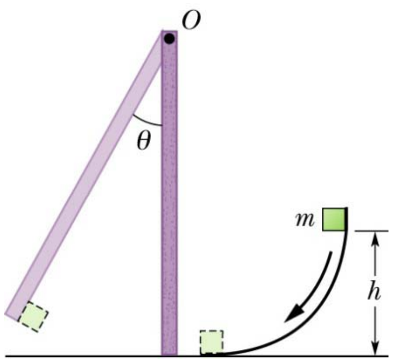
$$v_{com} = v_0 - \mu_k g \left(\frac{I\left(v_{com}/R \right)}{R\mu_k mg} \right)$$

$$v_{com} = \frac{v_0}{\left(1 + I/mR^2\right)}$$

e) How long does the ball slide?

$$t = \frac{v_0 - v_{com}}{\mu_k g}$$

Problem 11-66 A particle of mass m slides down the frictionless surface through height h and collides with the uniform vertical rod (of mass M and length d), sticking to it. The rod pivots about point O through the angle θ before momentarily stopping. Find θ.



What do we know? Where are we trying to get to?

What do we know about the ramp? What does this give us?

Conservation of Mechanical Energy going down ramp!

$$(KE_{init} + U_{init}) = (KE_{final} + U_{final})$$

$$(0 + mgh) = (\tfrac{1}{2} m v_{final}^2 + 0)$$

This gives us the speed of the mass right before hitting the rod:

$$v_{m,bottom} = \sqrt{2gh}$$

There's a collision! What kind? What do we know about this collision? What is constant?

Only Conservation of Angular Momentum holds!

(choose a point wisely)

$$\vec{L}_{initial} = \vec{L}_{final}$$

$$(\vec{r} \times \vec{p}_{m,0} + 0) = (\vec{r} \times \vec{p}_{m,f} + I_{rod}\omega)$$

$$\left(d(mv_{m,0})\right) = \left(md^2(\omega_{bottom}) + \left(\tfrac{1}{3}Md^2\right)\omega_{bottom}\right)$$

This gives the angular speed of the rod/mass just after hitting:

$$\omega_{bottom} = \frac{mv_{m,0}}{dm + \frac{1}{3}Md} = \frac{m\sqrt{2gh}}{dm + \frac{1}{3}Md}$$

Now what? After the collision, what holds?

$$(KE_{init} + U_{init}) = (KE_{final} + U_{final})$$

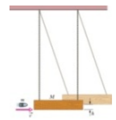
Now Conservation of Mechanical energy holds again!

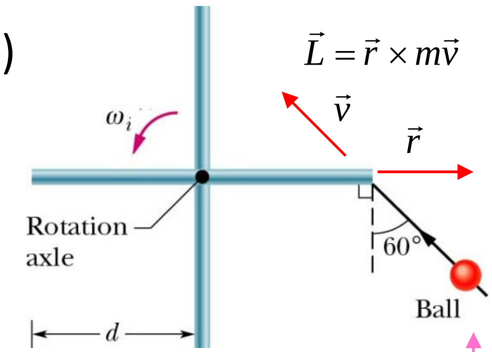
$$\left(\tfrac{1}{2} I_{tot}\omega_{bottom}^2 + 0\right) = \left(0 + mgd(1-\cos\theta_{final}) + Mg(\tfrac{1}{2}d)(1-\cos\theta_{final})\right)$$

Solve for θ:

$$\theta = \cos^{-1}\left[1 - \frac{m^2 h}{d\left(m + \frac{1}{2}M\right)\left(m + \frac{1}{3}M\right)}\right]$$

Sample Problem* 12-9

( angular version)



Four thin, uniform rods, each with mass **M** and length **d**, are attached rigidly to a vertical axle to form a turnstile (ts). The turnstile rotates ccw with an initial angular velocity ω_i. A mud ball (mb) of mass **m = 1/3 M** and initial speed v_i is thrown along the path shown and sticks to one end of one rod. What is the final angular velocity of the ball-turnstile system?

To find ω_f of the combined system, what is conserved?

1) Is KE_{tot} conserved. NO!
2) Is E_{mech} conserved. NO!
3) Is $\vec{P}$ conserved. NO!
4) Is $\vec{L}$ conserved. YES!

Using Conservation of angular momentum about axis:

$$\left[L_i = L_f\right]_{about\ axle}$$

$$L_{ts,i} + L_{mb,i} = L_{ts,f} + L_{mb,f}$$

$$L_{ts,i} = I_{ts}\omega_i \quad \& \quad L_{ts,f} = I_{ts}\omega_f$$

$$\left[I_{ts}\omega_i\right]_{ts,i} + L_{mb,i} = \left[I_{ts}\omega_f\right]_{ts,f} + L_{mb,f}$$

$$I_{ts} = 4\left(\tfrac{1}{12}Md^2 + M\left(\tfrac{1}{2}d\right)^2\right) = \tfrac{4}{3}Md^2$$

$$\left[\tfrac{4}{3}Md^2\omega_i\right]_{ts,i} + L_{mb,i} = \left[\tfrac{4}{3}Md^2\omega_f\right]_{ts,f} + L_{mb,f}$$

$$L_{mb,f} = I_{mb}\omega_f = \left(md^2\right)\omega_f = \tfrac{1}{3}Md^2\omega_f$$

$$\left[\tfrac{4}{3}Md^2\omega_i\right]_{ts,i} + L_{mb,i} = \left[\tfrac{4}{3}Md^2\omega_f\right]_{ts,f} + \left[\tfrac{1}{3}Md^2\omega_f\right]_{mb,f}$$

$$L_{mb,i} = rmv_i \sin\vartheta = d\left(\tfrac{1}{3}M\right)v_i \sin\left(90^\circ + 60^\circ\right) = \tfrac{1}{6}Mdv_i$$

$$\left[\tfrac{4}{3}Md^2\omega_i\right]_{ts,i} + \left[\tfrac{1}{6}Mdv_i\right]_{mb,i} = \left[\tfrac{4}{3}Md^2\omega_f\right]_{ts,f} + \left[\tfrac{1}{3}Md^2\omega_f\right]_{mb,f} = \tfrac{5}{3}Md^2\omega_f$$

$$\omega_f = \left(\tfrac{4}{3}Md^2\omega_i + \tfrac{1}{6}Mdv_i\right) \Big/ \tfrac{5}{3}Md^2 = \tfrac{4}{5}\omega_i + v_i/10d = \omega_f$$

Chap. 12-13: Equilibrium, Gravitation

Static equilibrium:

$$\vec{F}_{net} = 0 \text{ (balance of forces)}$$

$$\vec{\tau}_{net} = 0 \text{ (balance of torgue)}$$

Virtual rotation for torgues

Gravitation law:

$$\vec{F}_{12} = G\frac{m_1 m_2}{r_{12}^2}\hat{r}_{12} \qquad \vec{F}_{1,net} = \vec{F}_{12} + \vec{F}_{13} + \vec{F}_{14} + \ldots + \vec{F}_{1n} = \sum_{i=1}^{n} \vec{F}_{1i}$$

$$F = ma_g; \quad a_g = G\frac{M}{r^2}$$

Conservative force

Gravitational potential energy:

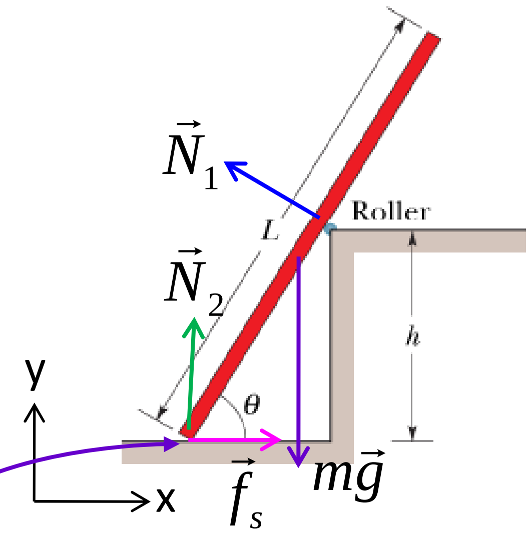
$$U(r) = -G\frac{m_1 m_2}{|r_{12}|}; \quad \Delta U_{fi} = -W = -\int \vec{F}_g \cdot d\vec{r}$$

$$U_{total} = -G\sum_{i<j}\frac{m_i m_j}{|r_{ij}|} \quad \text{(many objects)}$$

Kepler's Laws:

$$\frac{dA}{dt} = \frac{L}{2m} = \text{constant} \quad (2^{\text{nd}}\text{-Law}) \qquad T^2 = \left(\frac{4\pi^2}{GM}\right) r^3 \quad (3^{\text{rd}}\text{ - Law})$$

Problem 37: A uniform plank, with a length *L* of 6.10 m and a weight of *mg* = 445 N, rests on the ground and against frictionless roller at the top of a wall of height *h* = 3.05 m. The plank remains in equilibrium for any value of $\theta \geq 70°$ but slips if $\theta < 70°$. Find the coefficient of static friction between the plank and the ground.



What do we know: θ = 70° is critical angle: $f_s = \mu_s N_2$

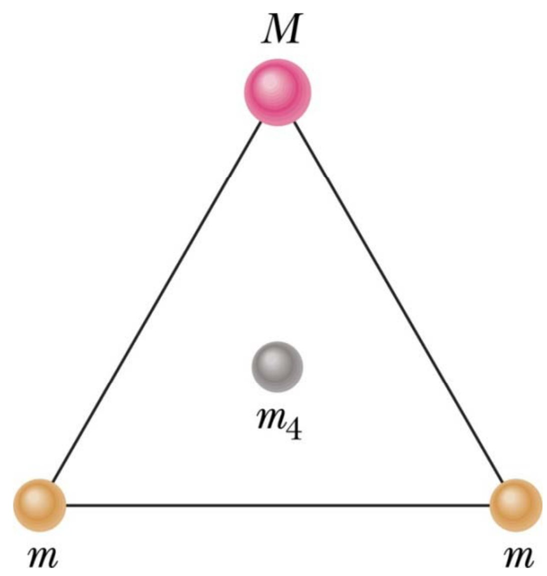
$$\sum \vec{F}_i = 0 \quad \sum \vec{F}_{ix} = 0: \quad f_s - N_1 \sin\theta = 0 \qquad (1)$$

$$\sum \vec{F}_{iy} = 0: \quad N_2 + N_1 \cos\theta - mg = 0 \qquad (2)$$

Choose a convenient VIRTURAL rotation axis

$$\sum \vec{\tau}_i = 0 \, N_1 \cdot \frac{h}{\sin\theta} - mg \cdot \frac{L}{2}\cos\theta = 0 \qquad (3)$$

Ch. 13 # 11: Two spheres of mass m and a third of mass M form an equilateral triangle, and a forth of mass m_4 is at the center of the triangle. If the net force on the center is zero what is M? If m_4 is doubled what is the answer?



The angle at each corner is 60^0, so the distance to the center mass is

$$r = \frac{l}{2}\cos 30^0 = \frac{l}{\sqrt{3}}$$

All the x forces cancel, the downward force on m_4 *is*

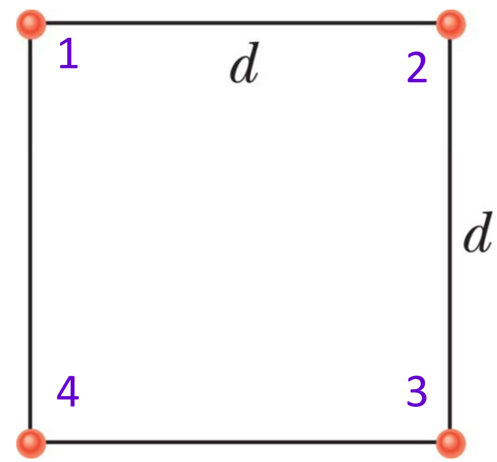
$$F_y(net) = 2\left(\frac{Gm_4m}{r^2}\right)\sin 30^0 = 3\left(\frac{Gm_4m}{l^2}\right)$$

The downward force must equal the upward force by M, so

$$3\left(\frac{Gm_4m}{l^2}\right) = \left(\frac{Gm_4M}{\left(\frac{l}{\sqrt{3}}\right)^2}\right)$$

$$m = M$$

Ch. 13 #46: Four masses M are located on each corner of a square with side d. If d is reduced by ½ what is the change in the potential energy?



$$U_{total} = -G\sum_{i<j}\frac{m_i m_j}{|r_{ij}|} \quad \text{(many objects)}$$

$$= -G\left(\frac{m_1 m_2}{r_{12}} + \frac{m_1 m_3}{r_{13}} + \frac{m_1 m_4}{r_{14}} + \frac{m_2 m_3}{r_{23}} + \frac{m_2 m_4}{r_{24}} + \frac{m_3 m_4}{r_{34}}\right)$$

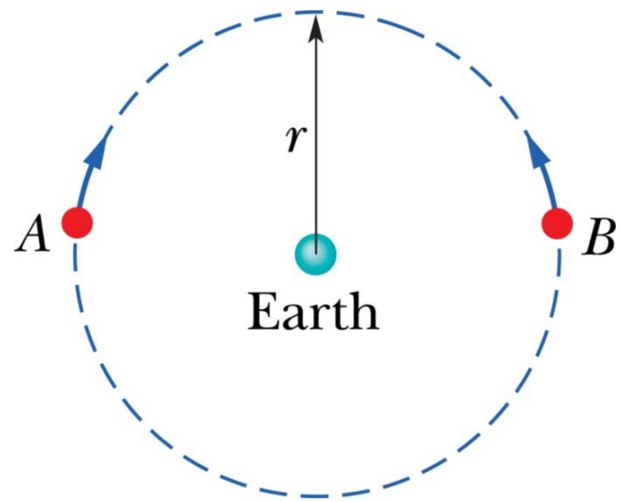
$$= -G\left(\frac{m^2}{d} + \frac{m^2}{\sqrt{2}d} + \frac{m^2}{d} + \frac{m^2}{d} + \frac{m^2}{\sqrt{2}d} + \frac{m^2}{d}\right)$$

$$= -G\left(\frac{4m^2}{d} + \frac{\sqrt{2}m^2}{d}\right) = -(4+\sqrt{2})\frac{m^2}{d}$$

$$\text{If } d \to \frac{1}{2}d; \quad U_{total} = -2(4+\sqrt{2})\frac{m^2}{d}$$

Double in magnitude!

Ch. 13 #64: Two satellites of the same mass are orbiting the earth at a radius r. But they are going in opposite directions and collide. (a) Find the total mechanical energy for A and B and the earth before the collision. (b) If the collision is completely inelastic (mass =2m) what is the mechanical energy right after the collision? (c) Just after the collision is the wreckage falling directly toward Earth's center or orbiting around the Earth?



(a) From equation 13-40 we see that the energy of each satellite is

$$-\frac{GM_E m}{2r}$$

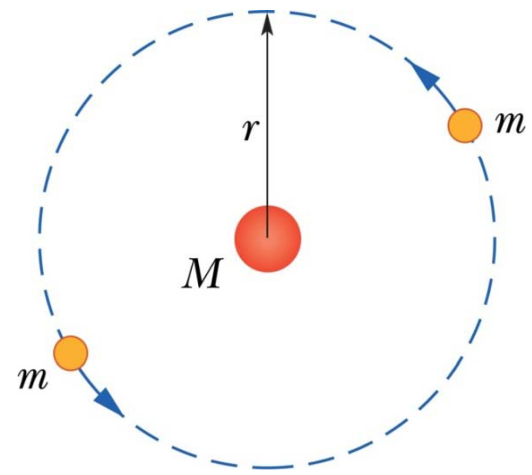
Total energy: $E_A + E_B = -\frac{GM_E m}{r}$

(b) The speed of the wreckage is zero--right.

$$E = -\frac{GM_E(2m)}{2r}$$

(c) It will fall straight down!!!!

Ch. 13 # 93: A triple-star system is shown in the picture. The two orbiting stars are always at opposite ends of the diameter of the orbit. Find the period?



The magnitude of the net gravitational force on one of the smaller stars (mass m) is

$$\frac{GMm}{r^2} + \frac{Gmm}{(2r)^2} = \frac{Gm}{r^2}\left[M + \frac{m}{4}\right]$$

This supplies the centripetal force needed for the motion of the star.

$$\frac{Gm}{r^2}\left[M + \frac{m}{4}\right] = m\frac{v^2}{r}$$

$$v = \frac{2\pi r}{T}$$

$$T = \frac{2\pi r^{3/2}}{\sqrt{G\left[M + m/4\right]}}$$