Physics 2101
Section 3
March 19th : Ch. 13

Announcements:

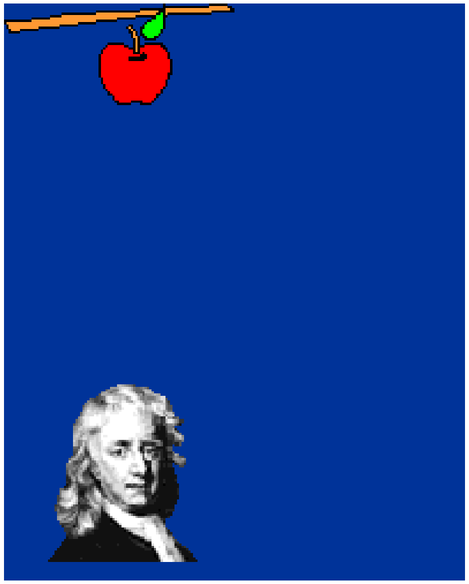
- **Quiz today.**

<u>Class Website</u>:

http://www.phys.lsu.edu/classes/spring2010/phys2101-3/

http://www.phys.lsu.edu/~jzhang/teaching.html

Chapt. 13: Gravitation



Isaac Newton (1687) : What keeps the moon in a nearly circular orbit about the earth?

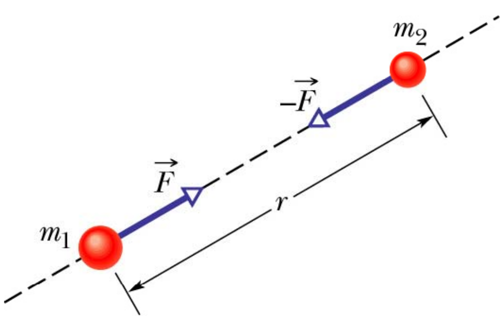
If falling objects accelerate, they must experience a force.

Force = Gravity

No contact !

Every body attracts every other body

Example: For 2 particles the magnitude of the attractive force between them is



$$|F| = G\frac{m_1 m_2}{r^2}$$

m_1 and m_2 are masses and r is distance between them and...

$$G = 6.67 \times 10^{-11}\, N \cdot m^2 / kg^2$$
$$= 6.67 \times 10^{-11}\, m^3 / kg \cdot s^2$$

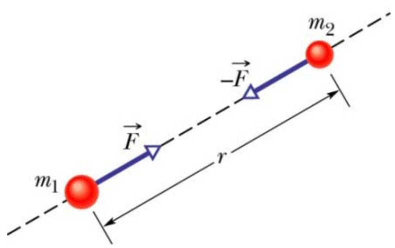
$$\vec{F}_{12} = -\vec{F}_{21}$$ Newton's third law

Gravitational Constant (≠ g, ≠ 9.8 m/s^2)

Gravitation: Notes

1) Three objects -- independent of each other
Newton's 3rd Law

2) Gravitational Force is a VECTOR - unit vector notation



$$\vec{F}_{12} = G\frac{m_1 m_2}{r_{12}^2}\hat{r}_{12}$$ Force on m_1 due to m_2 $$\hat{r}_{12} = \frac{\vec{r}_{12}}{|r_{12}|}$$

$$\vec{F}_{21} = G\frac{m_1 m_2}{r_{21}^2}\hat{r}_{21}$$ Force on m_2 due to m_1 $$\hat{r}_{21} = \frac{\vec{r}_{21}}{|r_{21}|} = -\hat{r}_{12}$$

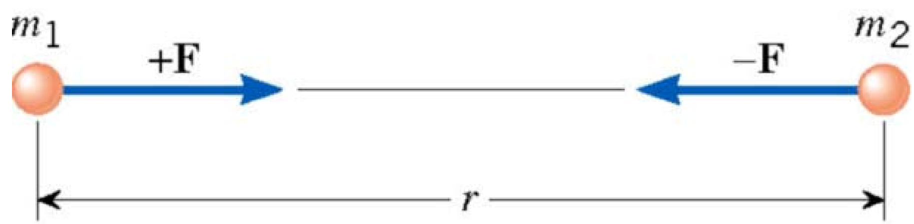
$\longrightarrow$ $$|\vec{F}_{21}| = |\vec{F}_{12}|$$ $$\vec{F}_{21} = -\vec{F}_{12}$$

3) Principle of <u>superposition</u>

$$\vec{F}_{1,net} = \vec{F}_{12} + \vec{F}_{13} + \vec{F}_{14} + \ldots + \vec{F}_{1n} = \sum_{i=1}^{n}\vec{F}_{1i}$$ VECTOR ADDITION!!

4) A uniform spherical shell of matter attracts an object on the outside as if all the shell's mass were concentrated at its center (note: this defines the position)
height = R_E + h

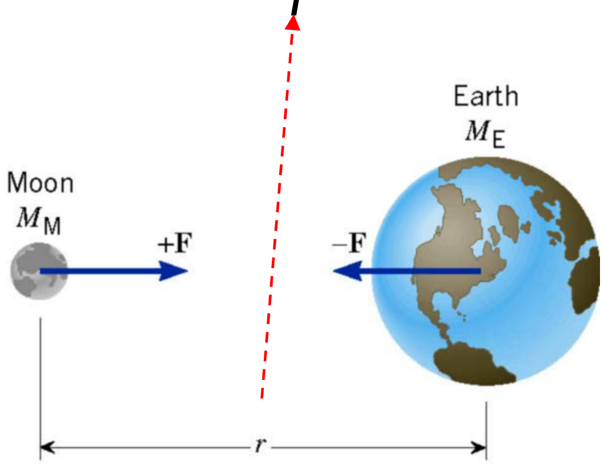
Newton's Law of Gravitation



$$F = G\frac{m_1 m_2}{r^2}$$

This is always attractive.

$$G = 6.67 \times 10^{-11}\,\frac{N \cdot m^2}{kg^2}$$

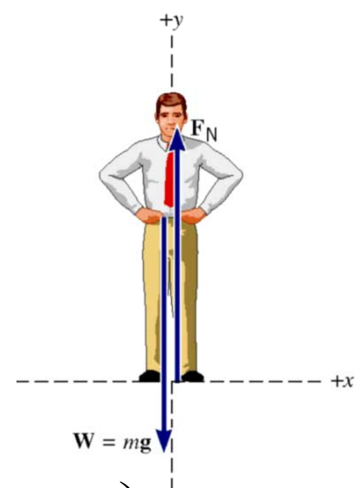


$$F = m_1 g = G\frac{m_1 m_2}{r^2}$$

$$= m_1\left(G\frac{m_{EARTH}}{r_{EARTH}^2}\right)$$

$$= m_1\left(6.67 \times 10^{-11}\,\frac{N \cdot m^2}{kg^2}\right)\frac{5.98 \times 10^{24}\,kg}{\left(6.38 \times 10^6\,m\right)^2}$$

$$= m_1\left(9.8\,\frac{m}{s^2}\right) = m_1 g$$



Where does "g" come from?

What is force of gravity at surface of earth?

Acceleration of Gravity at Hubble

What is the acceleration due to gravity at the Hubble Space Telescope? It orbits at an altitude of 600 km.

$$g_{HST} = G\frac{m_E}{r^2}$$

$$= \left(6.67\times10^{-11}\frac{N\cdot m^2}{kg^2}\right)\frac{5.98\times10^{24}\,kg}{\left(6.38\times10^6\,m + 6\times10^5\,m\right)^2}$$

$$= 8.19\frac{m}{s^2}$$

Why do astronauts float? => they are in **free-fall**

How fast are they going ?

We will see in chapter 5 that

$$\frac{v^2}{r} = g_{HST}$$

so

$$v = \sqrt{g_{HST}r} = 7550\frac{m}{s}\left(\frac{1mph}{0.447m/s}\right) = 17{,}000mph$$

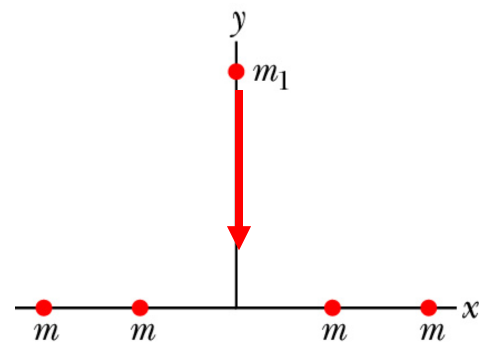
What is the time for one orbit?

$$T = \frac{2\pi r_{ORBIT}}{v} = \left(\frac{2\pi\left(6.98\times10^6\,m\right)}{7550m/s}\right) = 5800s = 1.6hours$$

Checkpoint

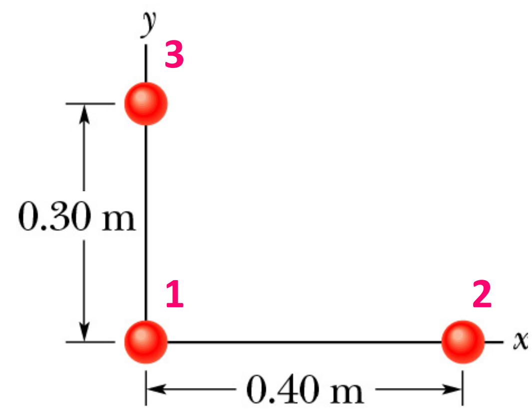
What are the gravitational forces on the particle of mass m_1 due to the other particles of mass m.

What is the direction of the **net** gravitational force on the particle of mass m_1 due to the other particles.



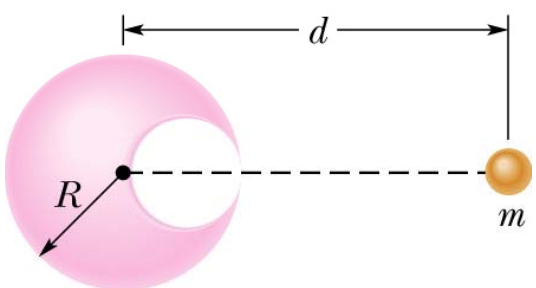
Problem 12-8

Three 5 kg spheres are located in the xy-plane. What is the magnitude and direction of the net gravitational force on the sphere at the origin due to the other two spheres?



Problem 14-13

With what gravitational force does the hollowed-out sphere attract a small sphere of mass m?

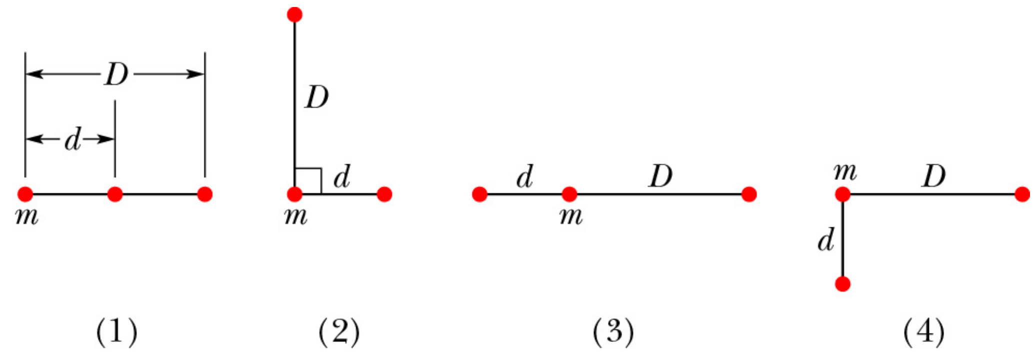


Negative mass = hollow

The figure shows four arrangements of three particles of equal mass.

Checkpoint

Rank the arrangements according to the magnitude of the net gravitational force on the particle labeled m, greatest first.



Announcements Oct. 28, 2008 Tues (34th)

- I'll pass the tests back at the end (let me know when there's 5 min left).
- I'll go over the test and any HW questions on Friday (optional)
- I've already posted some of HW8. I'll add a few more later this week, depending on how far we're at.
- Oh, not to add salt to the wounds, but TEST #3 will tentatively be Nov. 21 (FRIDAY)

Class material

→ Today: Chapt. 13 - Gravitational Potential Energy and Satellites

→ Tomorrow (Wed) we'll finish up Chapt. 13 and start Chapt. 14.

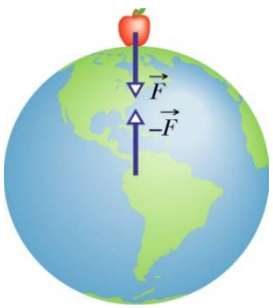
→ Thurs: More Fluids

→ On Friday, I'll be here to go over Test #2 and also any other questions you have (HW, class…)

Gravitational Force

Force on 1 due to 2: $$\vec{F}_{12} = G\frac{m_1 m_2}{r_{12}^2}\hat{r}_{12}$$ $$G = 6.67\times 10^{-11}\,N\cdot m^2/kg^2$$

Force on 1 due to many: $$\vec{F}_{1,net} = \vec{F}_{12} + \vec{F}_{13} + \vec{F}_{14} + \ldots + \vec{F}_{1n} = \sum_{i=1}^{n}\vec{F}_{1i}$$ VECTOR ADDITION!!



On Earth

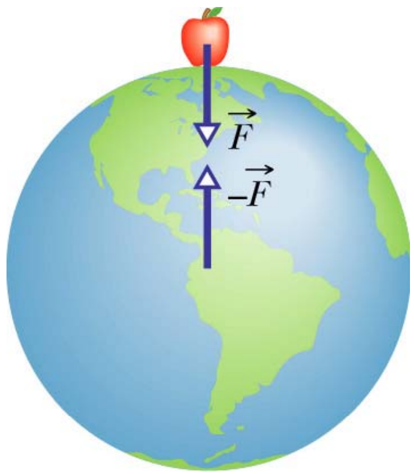
$$\left|F_g\right| = \left(G\frac{M_E}{R_E^2}\right)m_{apple} = m_{apple}\,g$$

Net force points towards center of earth

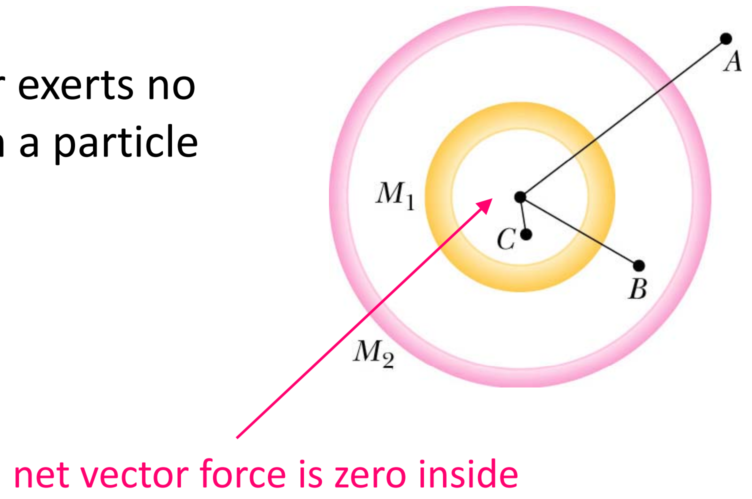
$$GM_E \cong 4\times 10^{14}\cdot m^3/s^2$$
$$R_E = 6,380\cdot km$$
$$\Rightarrow \frac{GM_E}{R_E^2} \cong 9.8\cdot m/s^2 = g$$

Gravity and spheres

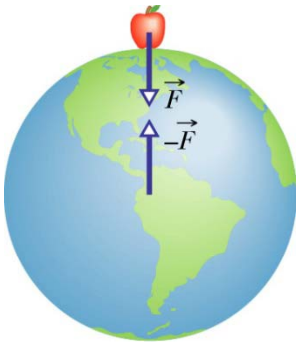
A uniform spherical shell of matter attracts a particle that is outside the shell as if all the shell's mass were concentrated at its center.



A uniform shell of matter exerts no net gravitational force on a particle located inside it.



Gravitation and the earth



$$|F_g| = \left(G\frac{M_E}{R_E^2}\right)m_{apple} = m_{apple}\,g$$

Net force points towards center of earth

<u>g differs around the earth</u> (equator-9.780 & north pole-9.832 m/s^2)

1) <u>Earth is not a perfect sphere - height</u> (R_E is not constant):
   - On Mount Everest (8.8 km) g=9.77 m/s^2 (0.2% smaller)
   - At Equator earth bulges by 21 km

2) <u>Earth is not uniform density</u>: "gravity irregularities" (10^{-6}-10^{-7})g
   gravimeters can measure down to 10^{-9}g

2) <u>Earth is rotating</u>: centripetal force makes apparent weight change

At poles:

$$W - mg = 0$$
$$W = mg$$

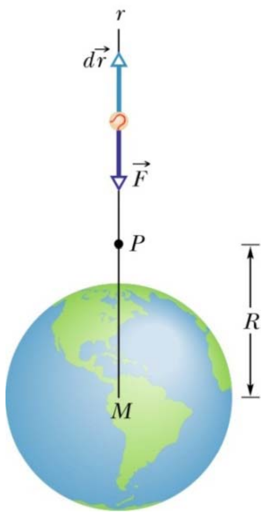
At equator:

$$W - mg = -m\left(\frac{v^2}{R_E}\right)$$
$$W = m\left(g - \frac{v^2}{R_E}\right)$$

Weight is less (0.3%)

Gravitational potential energy

From Section 8.3 $\Rightarrow$ $-\Delta U = W_{\text{done by force}}$ $\Rightarrow$ Conservative force-path independent



$$\Delta U_g = -W_{\text{done}} = -\int_{x_i}^{x_f} \vec{F}_g \bullet d\vec{x}$$

At Earth's surface, F_g~const.

$$\Delta U_g = -W_{\text{done}} = -m(-g)\int_{y_i}^{y_f} dy = mg\Delta y$$

$$W = \int_{r_i}^{r_f} \left(-G\frac{mM}{r^2}\right)dr = -GmM \int_{r_i}^{r_f} \left(\frac{1}{r^2}\right)dr$$

If we define U = 0 at ∞, then the work done by taking mass m from R to ∞

$$U_\infty - U(r) = -W = -GmM\left[0 - \left(-\frac{1}{R}\right)\right]$$

$$U(r) = -\frac{GmM}{r}$$

$$F(r) = -\frac{dU(r)}{dr}$$

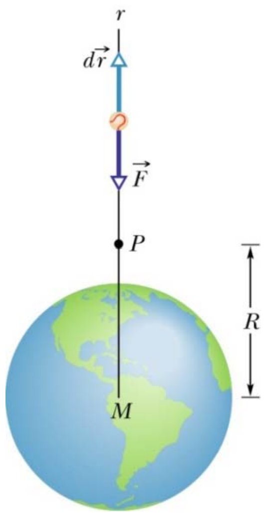
$$-\frac{d}{dr}\left(-\frac{GmM}{r}\right) = -\frac{GmM}{r^2}$$

Note:

1) As before, Grav. Pot. Energy decreases as separation decreases (more negative)
2) Path independent
3) MUST HAVE AT LEAST TWO PARTICLES TO POTENTIAL ENERGY (& force)
4) Knowing potential, you can get force....

Gravitational potential energy

$$\Delta U_g = -W_{\text{done}} = -\int_{x_i}^{x_f} \vec{F}_g \bullet d\vec{x}$$ $\Rightarrow$ Conservative force-path independent



If we define U = 0 at ∞, then the work done by taking mass m from R to ∞

$$U(r) = -G\frac{m_1 m_2}{|r_{12}|}$$

$$\vec{F}_{12} = G\frac{m_1 m_2}{r_{12}^2}\hat{r}_{12}$$

Note: Potential energy increases as you the separation gets larger:

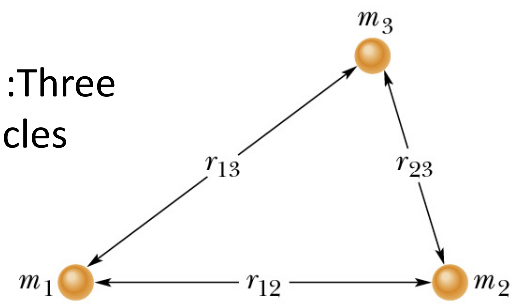
$$\text{If } r\uparrow \text{ then } \left(\frac{GmM}{r}\right)\downarrow \text{ and U gets less negative (larger)}$$

Gravitational Potential Energy

Scalar - just add up total potential energy
(BE CAREFUL: don't double count)

$$U_{tot} = -G\left(\frac{m_1 m_2}{r_{12}} + \frac{m_1 m_3}{r_{13}} + \frac{m_2 m_3}{r_{23}}\right)$$

System :Three particles



Note: don't need direction, just distance

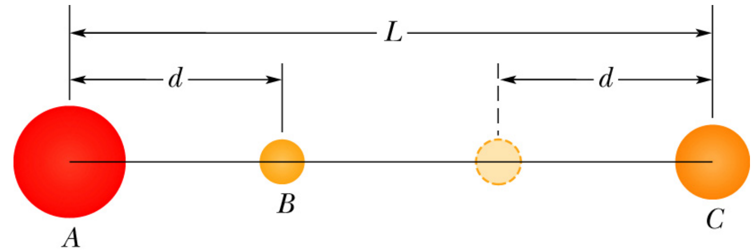
Problem 14-31

Gravitational Potential Energy

Three spheres with mass m_A, m_B, and m_C. You move sphere B from left to right.

How much work do you do on sphere B ?

How much work is done by the gravitational force?



[redacted]

Escape speed: *Conservation of Mechanical Energy*

Escape speed: minimum speed (v_{excape}) required to send a mass m, from mass M and position R, to infinity, while coming to rest at infinity.

At infinity: E_{mech}=0 because U=0 and KE = 0
Thus any other place we have:

$$E_{mech} = (KE + U_g) = 0 \quad \Rightarrow \quad E_{mech} = \left(\frac{1}{2}mv^2 - \frac{GmM}{R}\right) = 0 \quad \Rightarrow \quad v_{escape} = \sqrt{\frac{2GM}{R}}$$

<u>Escape speed:</u>

Earth = 11.2 km/s (25,000 mi/hr)
Moon = 2.38 km/s
Sun = 618 km/s

Problem 10-39
A projectile is fired vertically from the Earth's surface with an initial speed of 10 km/s (22,500 mi/hr)

Neglecting air drag, how far above the surface of Earth will it go?

$$(KE_i + U_i) = (KE_f + U_f)$$

$$\left(\frac{1}{2}mv_i^2 - G\frac{mM_E}{r_i}\right) = \left(0 - G\frac{mM_E}{r_f}\right)$$

$$R_E = 6380 \cdot km$$

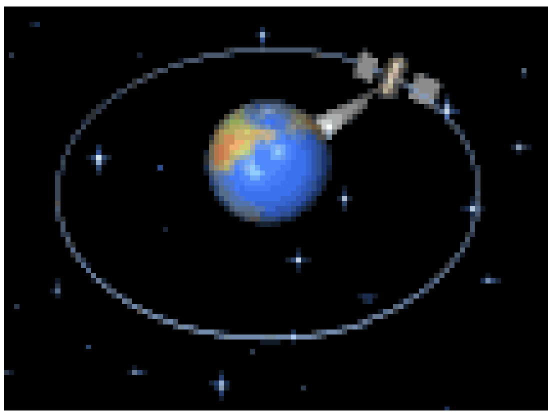
$$GM_E = 4 \times 10^{14} \cdot m^3/s^2$$

Satellites: weather, spy, moon

Geosynchronous Satellite: One that stays above same point on earth (only at equator)

TV, weather, communications……

How high must it be?



Only force is gravity: $$-\frac{Gm_{sat}M_E}{r_{sat}^2} = -m_{sat}\frac{v^2}{r_{sat}}$$

for synchronous orbit, period of satellite and earth must be the same $$v = \frac{2\pi r}{T} \qquad T = 1\ day$$

$$-\frac{Gm_{sat}M_E}{r_{sat}^2} = -m_{sat}\frac{\left(\frac{2\pi r_{sat}}{T}\right)^2}{r_{sat}} \quad \Rightarrow \quad r_{sat}^3 = \left(\frac{GM_E}{4\pi^2}\right)T^2$$

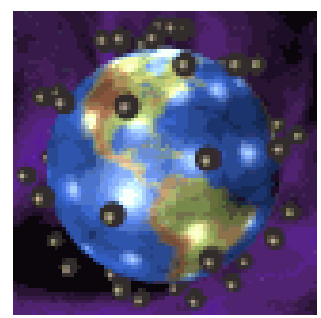
NOTE: $r^3 \propto T^2$

knowing T = 86,400 s , $\Rightarrow r_{sat} = 42{,}300\ km$

subtracting radius of earth: $\Rightarrow$ height above earth surface $= 35{,}000\ km \sim 6R_E$

Geosynchronous Satellite = 22,500 miles high

Spy Satellite (polar orbit) = 400 miles high



Space Shuttle 186 miles high

Satellites: Orbits and Energy

Newton's equation: F = ma
F is gravitational force
a is centripetal acceleration

$$\frac{GmM}{r^2} = m\frac{v^2}{r}$$

$$-\frac{1}{2}U_g = \frac{GmM}{2r} = m\frac{v^2}{2} = \tfrac{1}{2}mv^2 = KE$$

$$-\frac{1}{2}U_g = KE$$

$$KE = \frac{GMm}{2r}$$

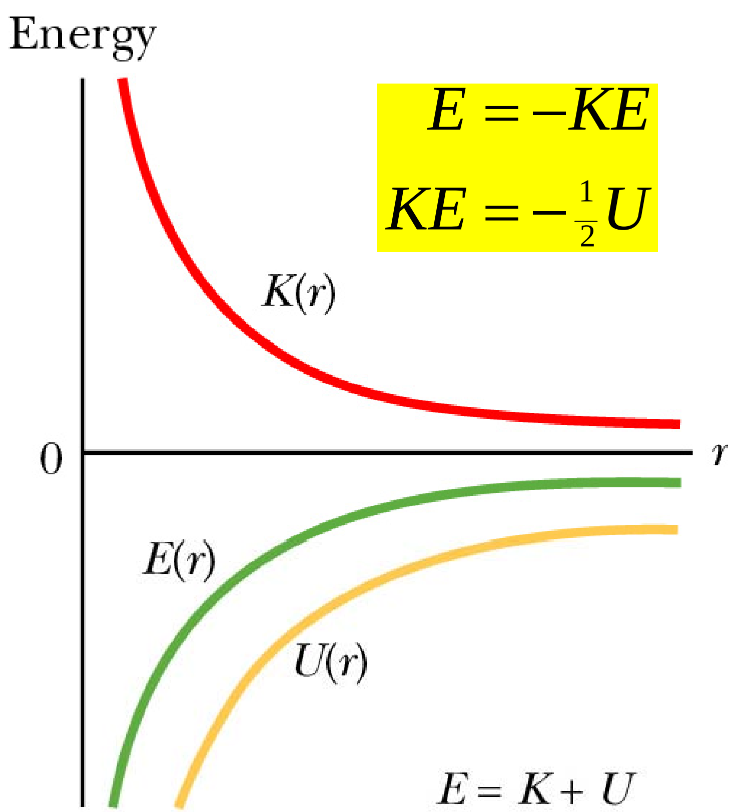
Mechanical Energy

$$E = U + KE = U + \left(-\tfrac{1}{2}U\right) = \tfrac{1}{2}U$$

$$E = -\frac{GMm}{2r}$$

$$E = -KE$$

Satellites: Energy graph



<u>Mechanical Energy</u>

$$E_{mech} = U + KE$$

$$= -\frac{GMm}{2r}$$

$$KE = \frac{GMm}{2r}$$

$$U = -\frac{GMm}{r}$$

Problem 10-35

a) What is the escape speed on a spherical asteroid whose radius is 500 km and whose gravitational acceleration at the surface is is 3 m/s^2 ?

b) How far from the surface will the particle go if it leaves the asteroid's surface with a radial speed of 1000 m/s?

c) With what speed will an object hit the asteroid if it is dropped from 1000 km above the surface?