Physics 2101
Section 3
March 17rd : Ch. 12

Announcements:

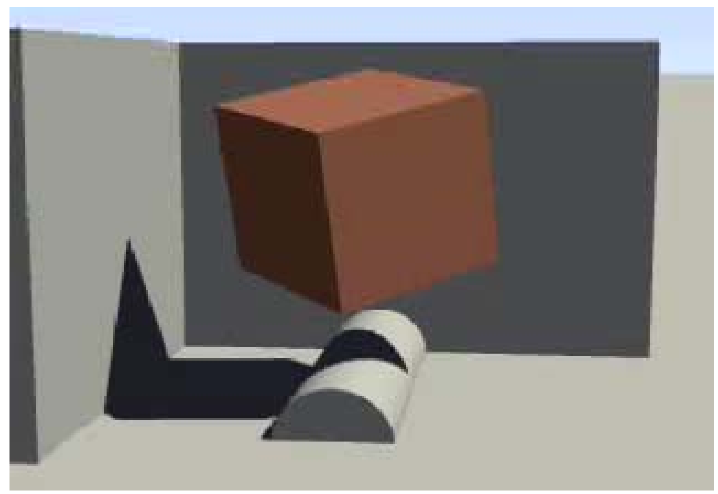
- **Next quiz on 19th**

Class Website:

http://www.phys.lsu.edu/classes/spring2010/phys2101-3/

http://www.phys.lsu.edu/~jzhang/teaching.html

Chapt 12:
Equilibrium and Elasticity



Equilibrium

Remember Newton's 2nd law:

$$\vec{F}_{net} = \sum \vec{F}_i = m\vec{a} = \frac{d\vec{P}}{dt}$$

$$\vec{\tau}_{net} = \sum \vec{\tau}_i = I\vec{\alpha} = \frac{d\vec{L}}{dt}$$

Equilibrium when:

$$\vec{a} = 0 = \vec{\alpha}$$

$$\frac{d\vec{P}}{dt} = 0 = \frac{d\vec{L}}{dt} \Rightarrow \begin{array}{l} \vec{P} = \text{constant} \\ \vec{L} = \text{constant} \end{array}$$

1) Linear Momentum of its center of mass is constant
2) Angular Momentum about <u>any point</u> is constant

Static Equilibrium when:

$$\vec{P} = 0 \quad \Rightarrow \quad \vec{v}_{com} = 0$$

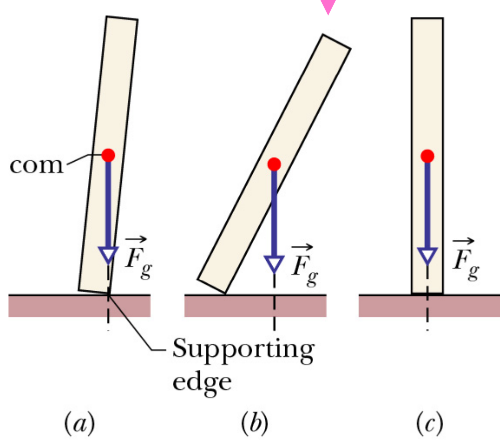
$$\vec{L} = 0 \quad \Rightarrow \quad \vec{\omega} = 0$$

Stability

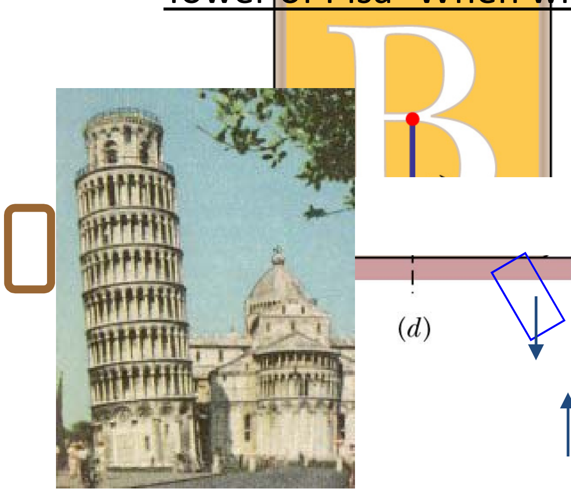
Unstable static equilibrium

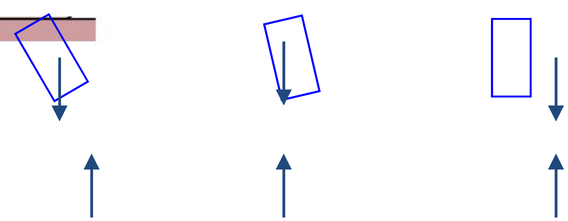
Non-equilibrium

Stable static equilibrium

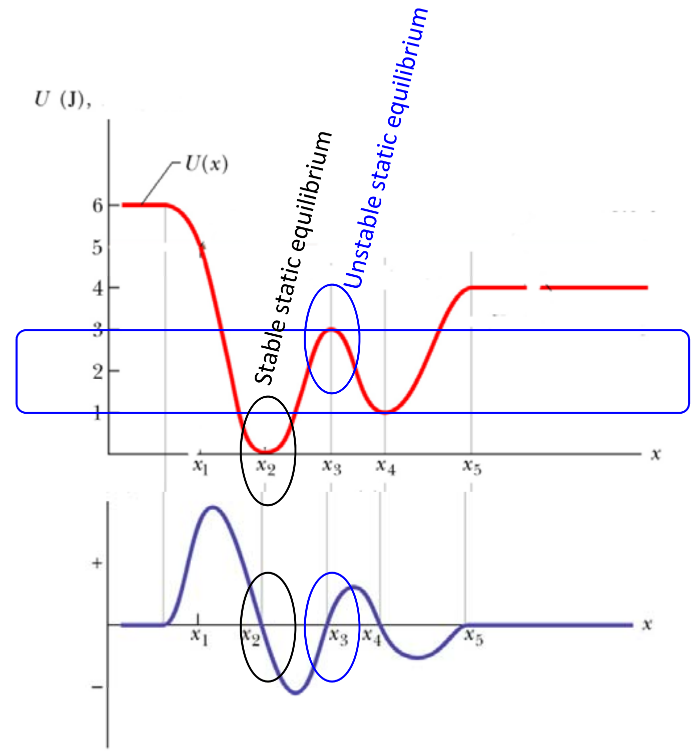


Tower of Pisa- When will it fall?



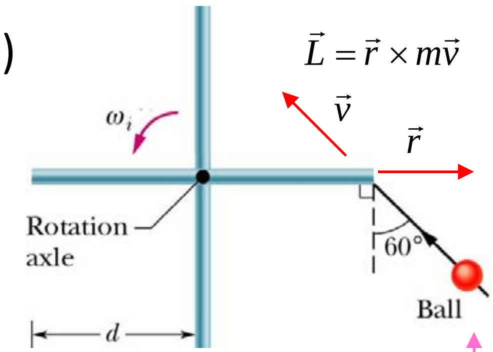


Is the driving force towards or away from stable static equilibrium?



$$F(x) = -\frac{dU(x)}{dx}$$

Sample Problem* 12-9 ( angular version)



Four thin, uniform rods, each with mass **M** and length **d**, are attached rigidly to a vertical axle to form a turnstile (ts). The turnstile rotates ccw with an initial angular velocity ω_i. A mud ball (mb) of mass **m = 1/3 M** and initial speed $\mathbf{v_i}$ is thrown along the path shown and sticks to one end of one rod. <u>What is the final angular velocity of the ball-turnstile system?</u>

To find ω_f of the combined system, what is conserved?

1) Is KE_{tot} conserved. NO!
2) Is E_{mech} conserved. NO!
3) Is $\vec{P}$ conserved. NO!
4) Is $\vec{L}$ conserved. YES!

Using Conservation of angular momentum about axis:

$$\left[L_i = L_f\right]_{\text{about axle}}$$

$$L_{ts,i} + L_{mb,i} = L_{ts,f} + L_{mb,f} \longrightarrow L_{ts,i} = I_{ts}\omega_i \quad \& \quad L_{ts,f} = I_{ts}\omega_f$$

$$\left[I_{ts}\omega_i\right]_{ts,i} + L_{mb,i} = \left[I_{ts}\omega_f\right]_{ts,f} + L_{mb,f} \longrightarrow I_{ts} = 4\left(\tfrac{1}{12}Md^2 + M\left(\tfrac{1}{2}d\right)^2\right) = \tfrac{4}{3}Md^2$$

$$\left[\tfrac{4}{3}Md^2\omega_i\right]_{ts,i} + L_{mb,i} = \left[\tfrac{4}{3}Md^2\omega_f\right]_{ts,f} + L_{mb,f} \longrightarrow L_{mb,f} = I_{mb}\omega_f = \left(md^2\right)\omega_f = \tfrac{1}{3}Md^2\omega_f$$

$$\left[\tfrac{4}{3}Md^2\omega_i\right]_{ts,i} + L_{mb,i} = \left[\tfrac{4}{3}Md^2\omega_f\right]_{ts,f} + \left[\tfrac{1}{3}Md^2\omega_f\right]_{mb,f} \longrightarrow L_{mb,i} = rmv_i \sin\vartheta = d\left(\tfrac{1}{3}M\right)v_i \sin\left(90^\circ + 60^\circ\right) = \tfrac{1}{6}Mdv_i$$

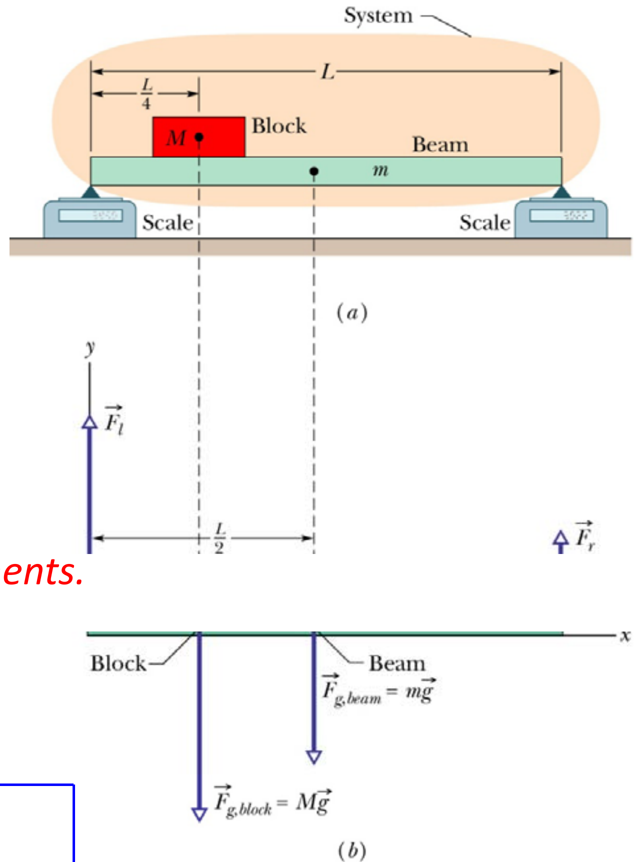
$$\left[\tfrac{4}{3}Md^2\omega_i\right]_{ts,i} + \left[\tfrac{1}{6}Mdv_i\right]_{mb,i} = \left[\tfrac{4}{3}Md^2\omega_f\right]_{ts,f} + \left[\tfrac{1}{3}Md^2\omega_f\right]_{mb,f} = \tfrac{5}{3}Md^2\omega_f$$

$$\omega_f = \left(\tfrac{4}{3}Md^2\omega_i + \tfrac{1}{6}Mdv_i\right) \Big/ \tfrac{5}{3}Md^2 = \tfrac{4}{5}\omega_i + {v_i}\big/{10d} = \omega_f$$

Problem #1

*A uniform beam of length **L** and mass **m** is at rest with its ends on two scales. A uniform block, with mass **M** is rest on the beam, with its center a distance **L/4** from the beam's end.*

What do the scales read?



1) Draw a free-body diagram showing all forces acting on body and the points at which these forces act.

2) Draw a convenient coordinate system and resolve forces into components.

3) Using letters to represent unknowns, write down equations for: $\sum F_x=0$, $\sum F_y=0$, and $\sum F_z=0$

Here only F_y components: $F_l + F_r - Mg - mg = 0$

We have one equation and two unknowns are F_l and F_r.

4) For $\sum \tau = 0$ equation, choose any axis perpendicular to the xy plane. But choose judiciously/wisely! Pay careful attention to determining lever arm and sign! [for xy-plane, ccw is positive & cw is negative]

If we choose left-hand end of beam to calculate torques, the balancing equation $\sum \tau = 0$ is:

$$(0)(F_l) - (L/4)(Mg) - (L/2)(mg) + (L)(F_r) = 0$$

4) Solve equations for unknowns.

$$F_r = \tfrac{1}{4} Mg + \tfrac{1}{2} mg$$

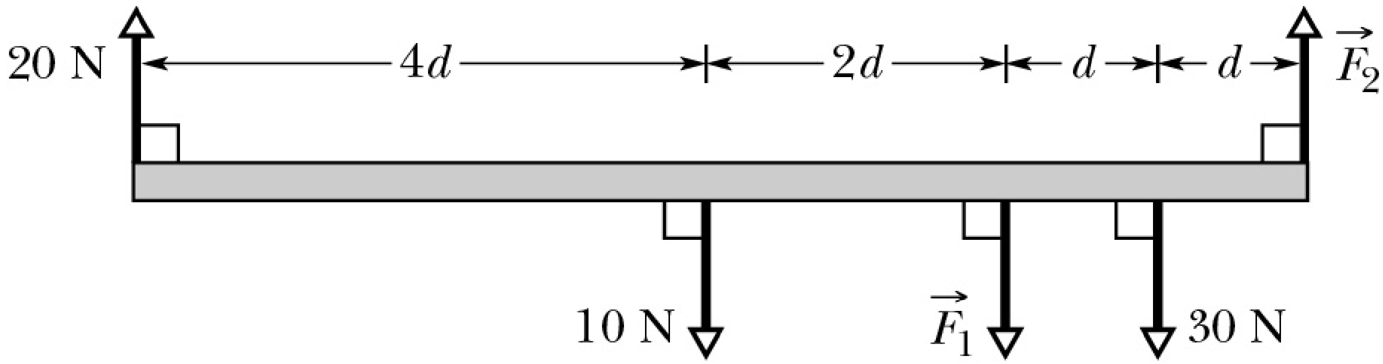
$$F_l = Mg + mg - F_r$$
$$= \tfrac{3}{4} Mg + \tfrac{1}{2} mg$$

Checkpoint 12-1

The figure gives an <u>overhead</u> view of a uniform rod in static equilibrium.

a) *Can you find the magnitudes of F_1 and F_2 by balancing the forces?*

b) *If you wish to find the magnitude of F_2 by using a single equation, where should you place a rotational axis?*



1) Draw a free-body diagram showing all forces acting on body and the points at which these forces act.

2) Draw a convenient coordinate system and resolve forces into components.

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$\sum F_x=0$, $\sum F_y=0$, and $\sum F_z=0$

4) For $\sum \tau = 0$ equation, choose any axis perpendicular to the xy plane. But choose judiciously!
Pay careful attention to determining lever arm and sign! [for xy-plane, ccw is positive & cw is negative]

5) Solve equations for unknowns.

Again: Requirement of Equilibrium

$$\vec{F}_{net} = \frac{d\vec{P}}{dt} = 0$$

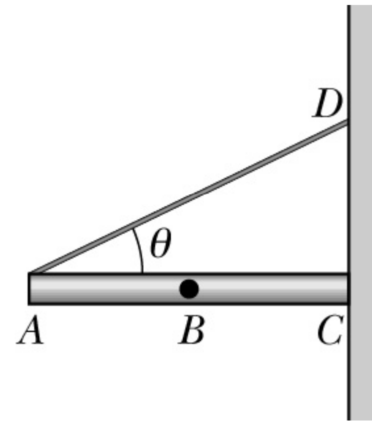
$$\vec{\tau}_{net} = \frac{d\vec{L}}{dt} = 0$$

1) *Vector sum of all external forces that act on the body must be zero*
2) *Vector sum of all external torques that act on the body, measured about* ***any*** *possible point, must also be zero.*

Balance of forces	Balance of torques
$F_{net,x} = 0$	$\tau_{net,x} = 0$
$F_{net,y} = 0$	$\tau_{net,y} = 0$
$F_{net,z} = 0$	$\tau_{net,z} = 0$

For motion only in x-y plane

Checkpoint 12-4 *A stationary 5 kg rod AC is held against a wall by a rope and friction between the rod and the wall. The uniform rod is 1 m long and $\theta = 30°$.*



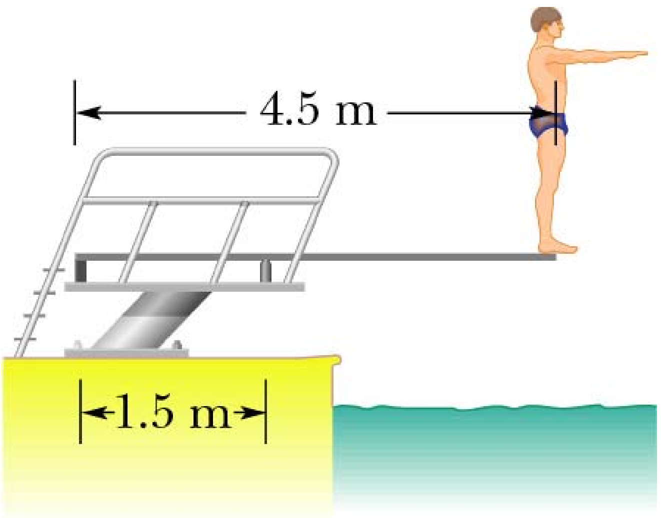
(a) *If you are to find the magnitude of the force T on the rod from the rope with a single equation, at what labeled point should a rotational axis be placed?*

(b) *About this axis, what is the sign of the torque due to the rod's weight and the tension?*

[redacted]

Problem 12-7

A diver of weight 580 N stands on the end of a 4.5 m diving board of negligible mass. The board is attached to two pedestals 1.5 m apart. What is the magnitude and direction of the force on the board at the left and right pedestals?



1) Draw a free-body diagram showing all forces acting on body and the points at which these forces act.

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4) For $\sum\tau = 0$ equation, choose any axis perpendicular to the xy plane. But choose judiciously! Pay careful attention to determining lever arm and sign! [for xy-plane, ccw is positive & cw is negative]

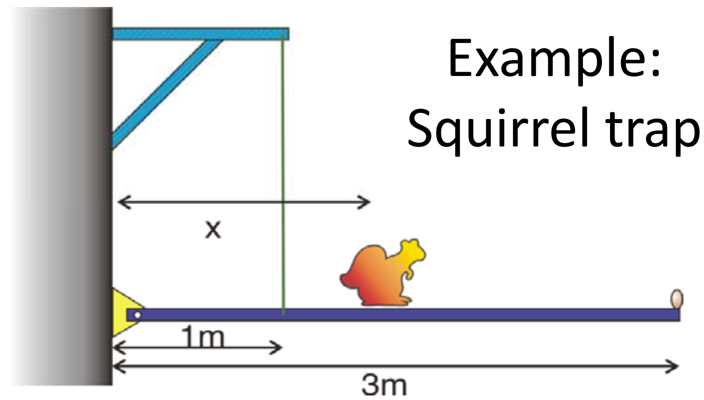
5) Solve equations for unknowns.

Example: Squirrel trap

A small aluminum bar is attached to the side of a building with a hinge. The bar is held in the horizontal via a fish line with 14 lb test. The bar has a mass of 3.2 kg.

a) Will the 14 lb (62.4 N) hold the bar?

b) A nut is placed on the end and a squirrel of mass 0.6 kg is tries to climb out and get the nut. Does the squirrel get to the nut before the string breaks? If not, how far does he get?

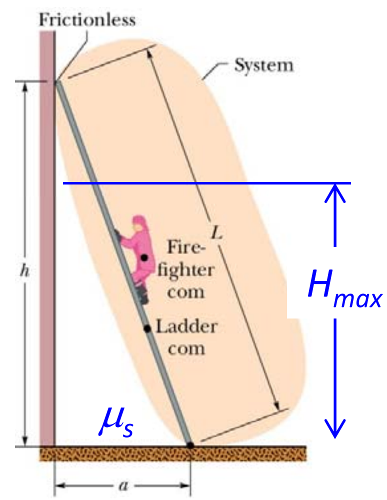


[redacted]

Sample Problem *A ladder of length **L** and mass **m** leans against a slick (frictionless) wall. Its upper end is at a height **h** above the payment on which the lower end rests (the payment is not fricionless). The ladder's center of mass is **L/3** from the lower end. A firefighter of mass **M** climbs the ladder until her center of mass is **L/2** from the lower end.*

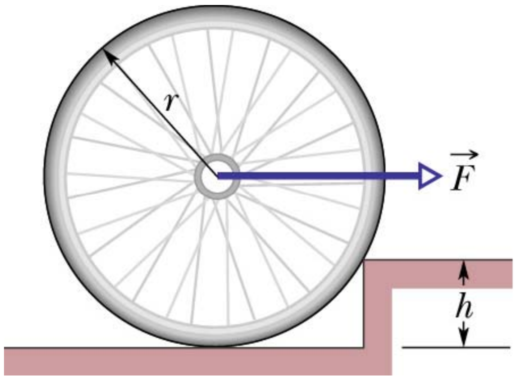
a) What are the magnitudes of the forces exerted on the ladder?

b) Now assume that the wall is again frictionless and the pavement has a coefficient of friction μ_s . What is the maximum height the firefighter climbs (H_{max})



Problem 12-21

*What magnitude of force **F** applied horizontally at the axle of the wheel is necessary to raise the wheel over an obstacle of height **h**? The wheel's radius is **r** and its mass is **m**.*



Problem 12-10: A Man is trying to get his car out of the mud on the shoulder of a road. He ties one end of a rope tightly around the front bumper and the other end tightly around a utility pole a distance L away. He then pushes sideways on the rope at its midpoing with a force F, displacing the center of the rope h from its previous position, and the car barely moves. (a) What force does the rope exert on the car?

QuickTime™ and a
decompressor
are needed to see this picture.

Elasticity

- *All material is made up of atoms.*
- *Atoms are connected with "springs" (~ obey Hooke's law* $\vec{F} = -k\vec{x}$ or $U_{sp} = \frac{1}{2}kx^2$
- *Real materials are "elastic" up to a point - can change shape by pulling/pushing/twisting*

Elastic = Capable of returning to an initial form or state after deformation

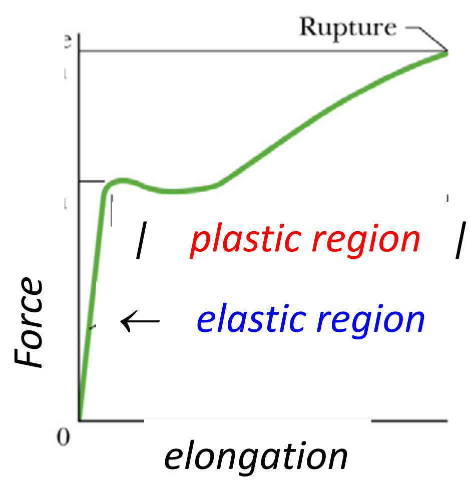
start ⇄ deform ⇄ final

Plastic = Capable of undergoing continuous, permanent deformation without rupture or relaxation

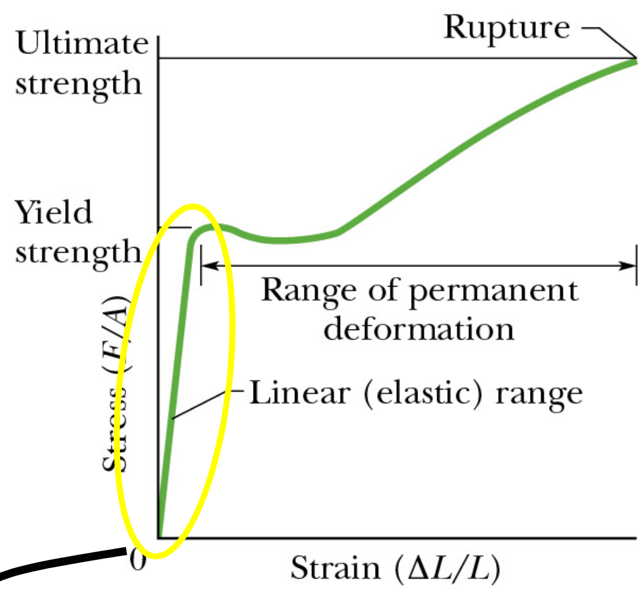
start → deform → final

In elastic region, the elongation is linearly proportional to the force

(Hooke's Law)



Elasticity: valid only in linear region



Ultimate Strength S_u ***- breaking point***

Yield Strength S_y ***- elastic-to-plastic crossover***

$$\text{stress} = \text{modulus} \times \text{strain} \qquad \{F = k\Delta x\}$$

Force/unit area [pressure]

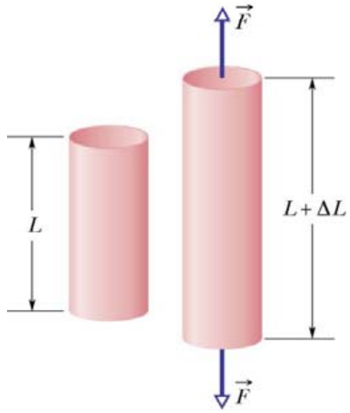
"Elasticity" [N/m²]

"Unit deformation"

$$\frac{\textit{Change in length}}{\textit{original length}}$$ *[unit-less]*

Young's modulus

Tension and Compression



$$\frac{F}{A} = E\,\frac{\Delta L}{L}$$

Force per unit area [Pressure]

Relative length change : unit-less

E is Young's modulus

E > 0

Young's Modulus with stress and strain tensor

$$\underbrace{\sigma_{xx}}_{\text{stress}} = E\underbrace{\varepsilon_{xx}}_{\text{strain}} \Rightarrow E = \frac{\sigma_{xx}}{\varepsilon_{xx}}$$

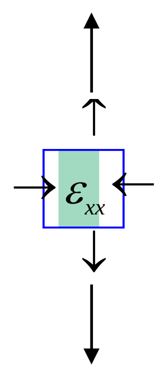
$$\{F = k\Delta x\}$$

	$E\times10^9$ N/m^2	$S_u\times10^6$ N/m^2
steel	200 GPa	400 MPa
Al	70 GPa	110 MPa
concrete	30 GPa	40 MPa*
		(*compression)

Al: tensile = compressive strength

Concrete: tensile = (compressive strength)/10

Poisson's Ratio :



$$\varepsilon_{yy} = \varepsilon_{zz}$$

Ratio of transverse to longitudinal strain

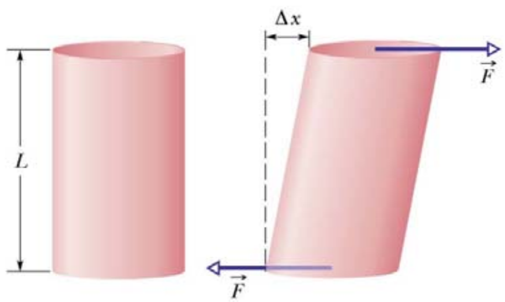
$$\nu = -\frac{\varepsilon_{yy}}{\varepsilon_{xx}}$$

$$\nu = \frac{1}{2} \quad \textit{Incompressible}$$

$$\frac{\Delta V}{V} = 0 = \varepsilon_{xx} + \varepsilon_{yy} + \varepsilon_{zz}$$

Shear and Bulk modulus

Shear - Bending or Rotating



$$\frac{F}{A} = G\frac{\Delta x}{L}$$

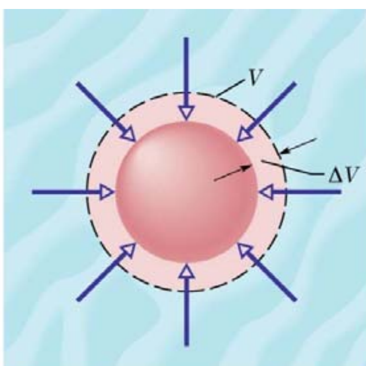
G is shear modulus

Young's and Shear Moduli are related via Poisson's ratio

$$G = \frac{E}{2(1+\nu)}$$

That means: G = 0.3 - 0.5 E

Hydraulic Stress



$$\frac{F}{A} = p = B\frac{\Delta V}{V}$$

B is bulk modulus

_________(B > 0)

Compressibility = 1/B

In general: Compressibility gasses > liquids > solids

If isotropic, $\sigma_{xx} = \sigma_{yy} = \sigma_{zz} = \sigma$

Young's and Bulk Moduli are related via Poisson's ratio

$$B = \frac{E}{3(1-2\nu)} = \rho_{density} v^2_{speed-of-sound}$$

Problem

Tension in a piano wire

A 1.6 m long steel piano wire has a diameter of 2.0 mm. How great is the tension in the wire if it stretches 3.0 mm when tightened?

From the relationship between stress and strain we know:

$$\frac{F}{A} = E\frac{\Delta L}{L} \qquad \Rightarrow \qquad F = E\frac{\Delta L}{L}A$$

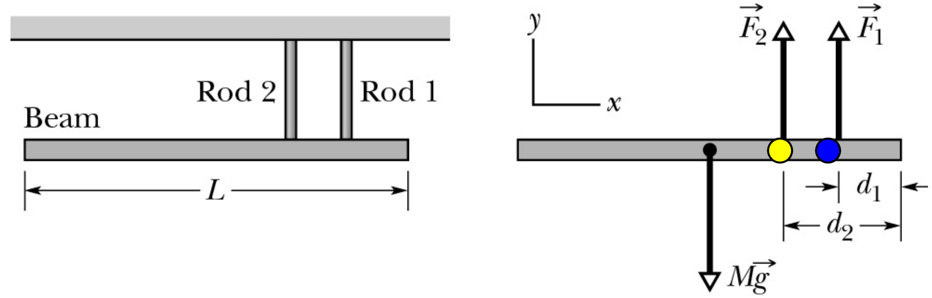
From which we can solve for the tension (F):

$$F = E\frac{\Delta L}{L}A$$

$$= \left(200 \times 10^{9}\ \text{N/m}^{2}\right)\left(\frac{0.003\ \text{m}}{1.6\ \text{m}}\right)\left(\pi(0.001\ \text{m})^{2}\right)$$

$$= 1200\ \text{N}$$

A uniform horizontal beam of mass M and length L is supported by two uniform vertical metal rods, each with area cross-sectional area A and Young's modulus E



(b)

(a) Find $\vec{F}_1$ *and* $\vec{F}_2$

Balancing the static forces yields

$$\sum F_x = 0 \quad \rightarrow\rightarrow \quad 0 = 0 \qquad (1)$$

$$\sum F_y = 0 \quad \rightarrow\rightarrow \quad F_2 + F_1 - Mg = 0 \qquad (2)$$

Balancing the torques around both the point where Rod 1 attaches to the Beam (i.e. yellow circle) and where Rod 2 attaches to the Beam (i.e. blue circle)

$$\sum \tau_{Rod\ 1-end} = 0 \quad \rightarrow\rightarrow \quad -(d_2 - d_1)F_2 + (L/2 - d_1)Mg = 0 \qquad (3)$$

$$\sum \tau_{Rod\ 2-end} = 0 \quad \rightarrow\rightarrow \quad -(d_2 - d_1)F_1 - (L/2 - d_2)Mg = 0 \qquad (4)$$

From the (3) and (4) we have:

$$F_2 = \frac{(L/2 - d_1)Mg}{(d_2 - d_1)}$$

$$F_1 = -\frac{(L/2 - d_2)Mg}{(d_2 - d_1)}$$

- Note that F_1 is negative so it points downward (Rod 1 pushes on the Beam)
- Note that these forces obey equation (2)

b) What are the magnitudes of strain in Rod 1 and Rod 2

The stress on Rod 1 and Rod 2 are:

$$\frac{F_2}{A} = \frac{(L/2 - d_1)Mg}{A(d_2 - d_1)}$$

$$\frac{F_1}{A} = -\frac{(L/2 - d_2)Mg}{A(d_2 - d_1)}$$

The strains in Rod 1 and Rod 2 are:

under tension (pos)

$$\frac{\Delta l_2}{l} = \frac{F_2}{AE} = \frac{(L/2 - d_1)Mg}{EA(d_2 - d_1)}$$

$$\frac{\Delta l_1}{l} = \frac{F_1}{AE} = -\frac{(L/2 - d_2)Mg}{EA(d_2 - d_1)}$$

under compression (neg)