Physics 2101
Section 3
March 15th : Ch. 11

Announcements:

Class Website:

http://www.phys.lsu.edu/classes/spring2010/phys2101-3/

http://www.phys.lsu.edu/~jzhang/teaching.html

Conservation of Angular Momentum

Translational motion: Conservation Law of Linear Momentum

(closed, isolated system = no net external forces)

$$\Delta\vec{P} = 0$$

Rotational motion: If the net external torque acting on a system is zero, the angular momentum of the system remains constant, no matter what happens within the system

$$\Delta\vec{L} = 0$$

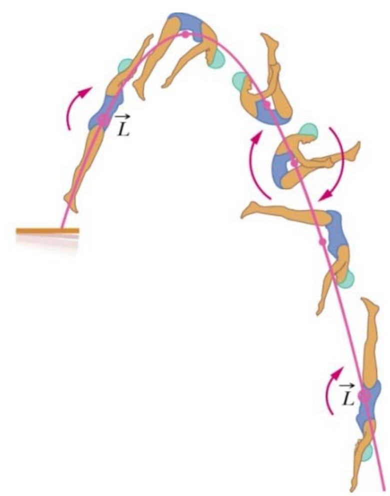
$$\vec{L}_i = \vec{L}_f$$

- Total angular momentum of system at all times is equal

- Vector {conserved in all three directions, x-y-z}

$$I_i\omega_i = I_f\omega_f$$

-Initially **rigid body** redistributes mass relative to rotational axis



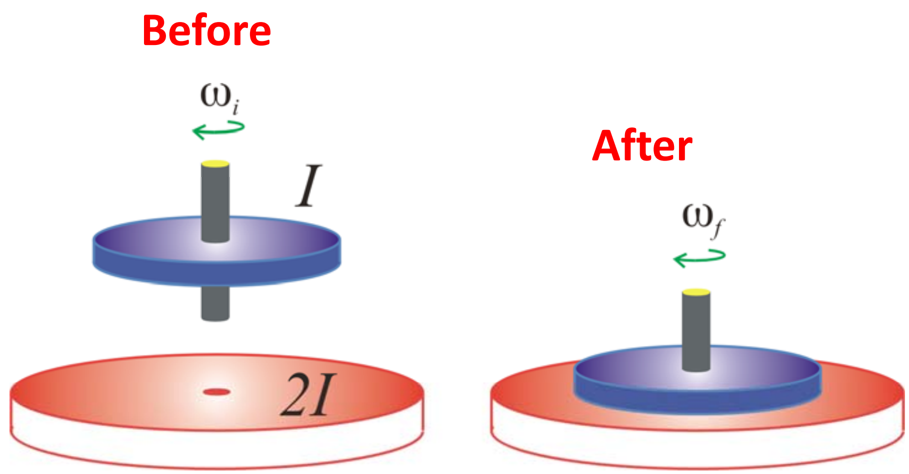
Linear and angular relations

Force	$\vec{F}$	$\vec{\tau} = \vec{r} \times \vec{F}$	Torque
Linear momentum (one)	$m\vec{v} = \vec{p}$	$\vec{l} = \vec{r} \times \vec{p}$	Angular momentum (one)
Linear momentum (system)	$\sum \vec{p} = \vec{P}$	$\vec{L} = \sum \vec{l}$	Angular momentum (system)
Linear momentum (system)	$M\vec{v}_{com} = \vec{P}$	$\vec{L} = I\vec{\omega}$	Angular momentum (system, fixed axis)
Newton's second law (system)	$\frac{d\vec{P}}{dt} = \vec{F}_{net}$	$\vec{\tau}_{net} = \frac{d\vec{L}}{dt}$	Newton's second law (system)
	$Ma_{com} = \vec{F}_{net}$	$\vec{\tau}_{net} = I\vec{\alpha}$	
Conservation Law (closed,isolated)	$0 = \Delta\vec{P}$	$\Delta\vec{L} = 0$	Conservation Law (closed,isolated)

$\vec{\tau}_{net} = d\vec{L}/dt$ **has no meaning unless the net torque $\vec{\tau}_{net}$ and the total rotational momentum $\vec{L}$, are defined with respect to the same origin**

Example #3 : Clutch

A wheel is rotating freely with angular speed ω_i on a shaft whose rotational inertia is negligible. A second wheel, <u>initially at rest</u> and with **twice** the rotational inertia of the first, is suddenly coupled to the same shaft.



What is the angular speed of the the resultant combination of the shaft and two wheels?

$$L_i = L_f$$

$$(I\omega_i)_{\substack{\text{small}\\\text{wheel}}} + (2I_i(0))_{\substack{\text{large}\\\text{wheel}}} = (I\omega_f)_{\substack{\text{small}\\\text{wheel}}} + (2I\omega_f)_{\substack{\text{large}\\\text{wheel}}}$$

$$I\omega_i = 3I\omega_f$$

$$\omega_f = \tfrac{1}{3}\omega_i$$

Pure inelastic "Hit-'n-stick"

What fraction of the original rotational kinetic energy is lost?

Fraction of KE lost :

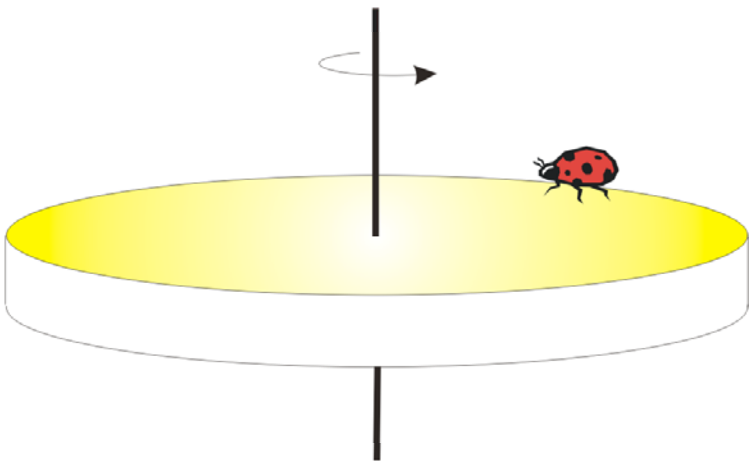
$$\left(\frac{KE_i - KE_f}{KE_i}\right)\% = \left(\frac{\frac{1}{2}I_{initial}\omega_i^2 - \frac{1}{2}I_{final}\omega_f^2}{\frac{1}{2}I_{initial}\omega_i^2}\right)\% = \left(\frac{\frac{1}{2}I\omega_i^2 - \frac{1}{2}(3I)\left(\frac{1}{3}\omega_i\right)^2}{\frac{1}{2}I\omega_i^2}\right)\%$$

$$= \left(\frac{1-(3)\left(\frac{1}{3}\right)^2}{1}\right)\% = \left(\tfrac{2}{3}\right)\% = 66.7\%$$

Question Question 4

A beetle rides the rim of a rotating merry-go-round. If the beetle crawls towards the center of the disk, what happens to the rotational inertia of beetle-disk system?

1. ***increases***
2. ***decreases***
3. ***stays the same***



$$I_{tot} = I_{disk} + I_{bug}$$
$$= I_{disk} + m_{bug} r^2$$

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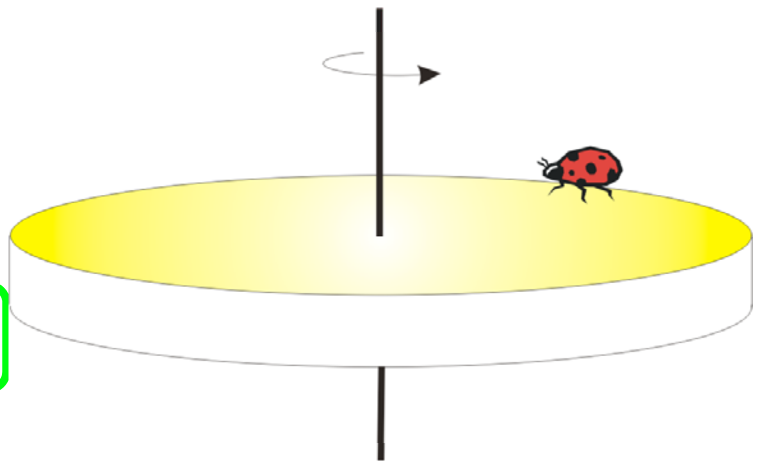

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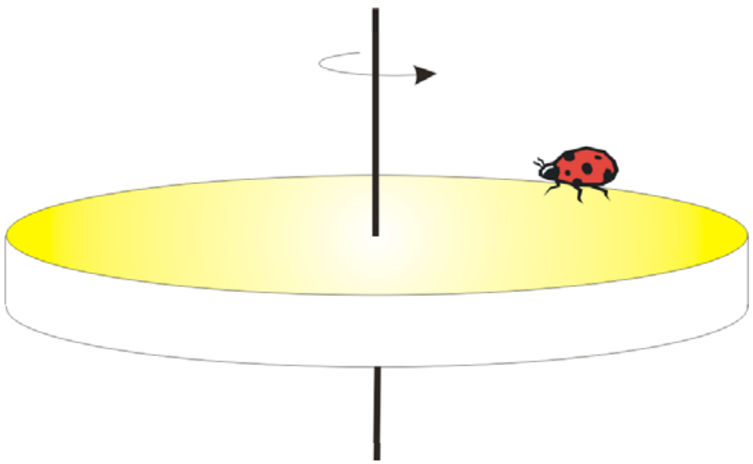
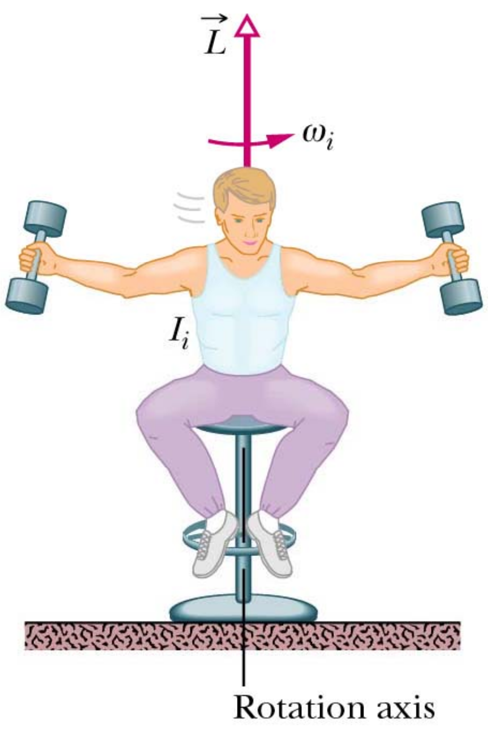


Demo: Conservation of Angular Momentum

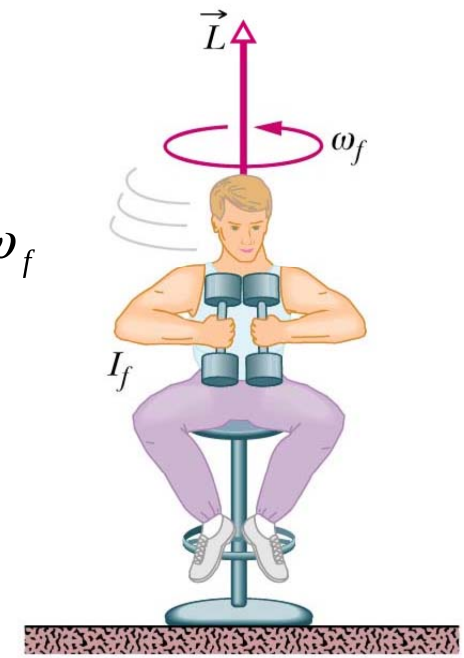
No external torques act



$$L_i = L_f$$

$$I_i \omega_i = I_f \omega_f$$

$$\left(I_{person} + 2MR^2\right)_i \omega_i = \left(I_{person} + 2Mr^2\right)_f \omega_f$$

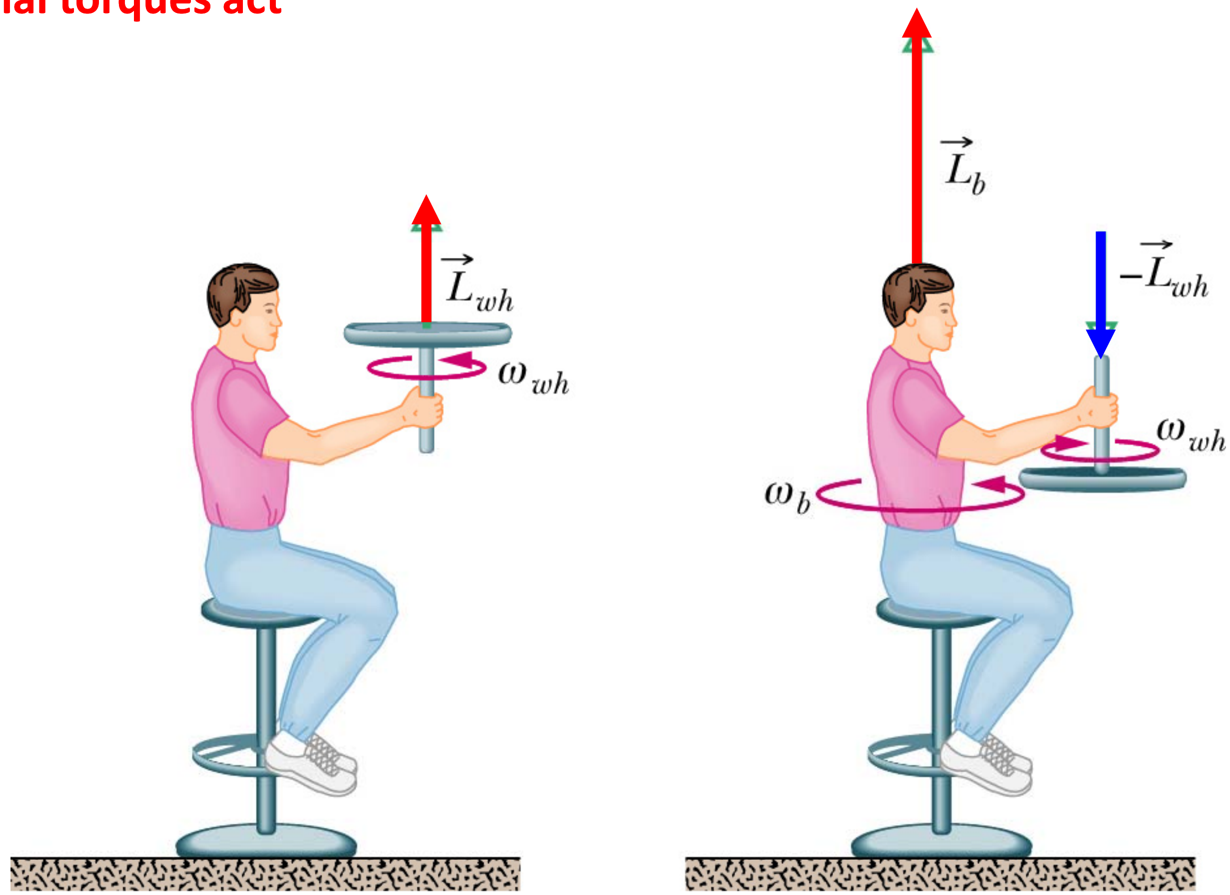


$$if \quad \Delta r \downarrow \quad then \quad \omega_f > \omega_i$$

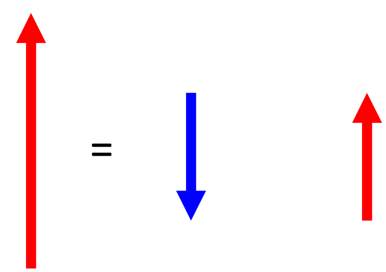
$$if \quad \Delta r \uparrow \quad then \quad \omega_f < \omega_i$$

Demo: Conservation of Angular Momentum

No external torques act



$\mathbf{L}_{initial}$= 

$\mathbf{L}_{final}$= +  =

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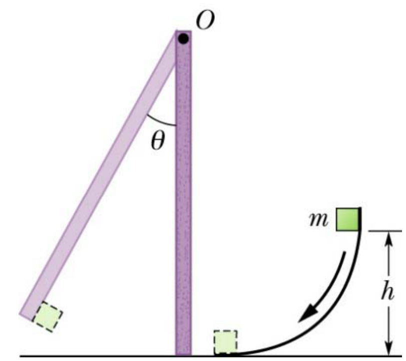
Problem 11-66 A particle of mass m slides down the frictionless surface through height h and collides with the uniform vertical rod (of mass M and length d), sticking to it. The rod pivots about point O through the angle θ before momentarily stopping. Find θ.



What do we know? Where are we trying to get to?

What do we know about the ramp? What does this give us?

Conservation of Mechanical Energy going down ramp!

$$(KE_{init} + U_{init}) = (KE_{final} + U_{final})$$

$$(0 + mgh) = (\tfrac{1}{2} mv_{final}^2 + 0)$$

This gives us the speed of the mass right before hitting the rod:

$$v_{m,bottom} = \sqrt{2gh}$$

There's a collision! What kind? What do we know about this collision? What is constant?

Only Conservation of <u>Angular Momentum</u> holds!

(choose a point wisely)

$$\vec{L}_{initial} = \vec{L}_{final}$$

$$(\vec{r} \times \vec{p}_{m,0} + 0) = (\vec{r} \times \vec{p}_{m,f} + I_{rod}\omega)$$

$$\left(d(mv_{m,0})\right) = \left(md^2(\omega_{bottom}) + \left(\tfrac{1}{3}Md^2\right)\omega_{bottom}\right)$$

This gives the angular speed of the rod/mass just after hitting:

$$\omega_{bottom} = \frac{mv_{m,0}}{dm + \frac{1}{3}Md} = \frac{m\sqrt{2gh}}{dm + \frac{1}{3}Md}$$

Now what? After the collision, what holds?

$$(KE_{init} + U_{init}) = (KE_{final} + U_{final})$$

Now Conservation of Mechanical energy holds again!

$$\left(\tfrac{1}{2} I_{tot}\omega_{bottom}^2 + 0\right) = \left(0 + mgd(1 - \cos\theta_{final}) + Mg(\tfrac{1}{2}d)(1 - \cos\theta_{final})\right)$$

Solve for θ:

$$\theta = \cos^{-1}\left[1 - \frac{m^2 h}{d\left(m + \frac{1}{2}M\right)\left(m + \frac{1}{3}M\right)}\right]$$

Solution Problem 11-66

(d) The center-of-mass of the sliding ball decelerates from $v_{com,0}$ to v_{com} during time t according to Eq. 2-11: $v_{com} = v_{com,0} - \mu g t$. During this time, the angular speed of the ball increases (in magnitude) from zero to $|\omega|$ according to Eq. 10-12:

$$|\omega| = |\alpha| t = \frac{5\mu g t}{2R} = \frac{v_{com}}{R}$$

where we have made use of our part (a) result in the last equality. We have two equations involving v_{com}, so we eliminate that variable and find

$$t = \frac{2v_{com,0}}{7\mu g} = \frac{2(8.5 \text{ m/s})}{7(0.21)(9.8 \text{ m/s}^2)} = 1.2 \text{ s}.$$

(e) The skid length of the ball is (using Eq. 2-15)

$$\Delta x = v_{com,0}t - \frac{1}{2}(\mu g)t^2 = (8.5 \text{ m/s})(1.2 \text{ s}) - \frac{1}{2}(0.21)(9.8 \text{ m/s}^2)(1.2 \text{ s})^2 = 8.6 \text{ m}.$$

(f) The center of mass velocity at the time found in part (d) is

$$v_{com} = v_{com,0} - \mu g t = 8.5 \text{ m/s} - (0.21)(9.8 \text{ m/s}^2)(1.2 \text{ s}) = 6.1 \text{ m/s}.$$

Solution Problem 11-66

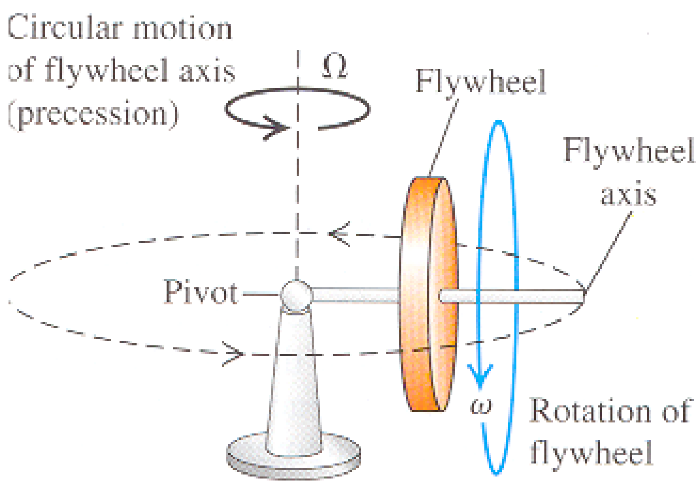
which means the system has kinetic energy $\left(I_{\text{rod}} + md^2\right)\omega^2/2$ which will turn into potential energy in the final position, where the block has reached a height H (relative to the lowest point) and the center of mass of the stick has increased its height by $H/2$. From trigonometric considerations, we note that $H = d(1 - \cos\theta)$, so we have

$$\frac{1}{2}\left(I_{\text{rod}} + md^2\right)\omega^2 = mgH + Mg\frac{H}{2} \;\Rightarrow\; \frac{1}{2}\frac{m^2d^2(2gh)}{(Md^2/3) + md^2} = \left(m + \frac{M}{2}\right)gd\,(1 - \cos\theta)$$

from which we obtain

$$\begin{aligned}\theta &= \cos^{-1}\left(1 - \frac{m^2h}{(m + M/2)\,(m + M/3)}\right) = \cos^{-1}\left(1 - \frac{h/d}{(1 + M/2m)\,(1 + M/3m)}\right)\\ &= \cos^{-1}\left(1 - \frac{(20\text{ cm}/\,40\text{ cm})}{(1+1)(1+2/3)}\right) = \cos^{-1}(0.85)\\ &= 32^\circ.\end{aligned}$$

Precession: gyroscopes



<u>Remember:</u>
-Angular velocity
-Torque
-Angular momentum
→ ALL VECTORS

Prob 12-15

A yo-yo has a rotational inertia of $\mathbf{I_{com}}$ and mass of **m**. Its axle radius is $\mathbf{R_0}$ and string's length is **h**. The yo-yo is thrown so that its initial speed down the string is $\mathbf{v_0}$.

a) How long does it take to reach the end of the string?

1-D kinematics given a_{com}

$$-h = \Delta y = -v_0 t - \tfrac{1}{2} a_{com} t^2 \;\Rightarrow\; solve\ for\ t\ (quadradic\ equation)$$

b) As it reaches the end of the string, what is its <u>total</u> KE?

Conservation of mechanical energy

$$KE_f = KE_i + U = \tfrac{1}{2} m v_{com,0}^2 + \tfrac{1}{2} I_{com} \left(\frac{v_{com,0}}{R_0}\right)^2 + mgh$$

c) As it reaches the end of the string, what is its linear speed?

1-D kinematics given a_{com}

$$-|v_{com}| = -v_0 - a_{com} t \;\Rightarrow\; solve\ for\ |v_{com}|$$

d) As it reaches the end of the string, what is its translational KE?

Knowing $|v_{com}|$

$$KE_{trans} = \frac{1}{2} m v_{com}^2$$

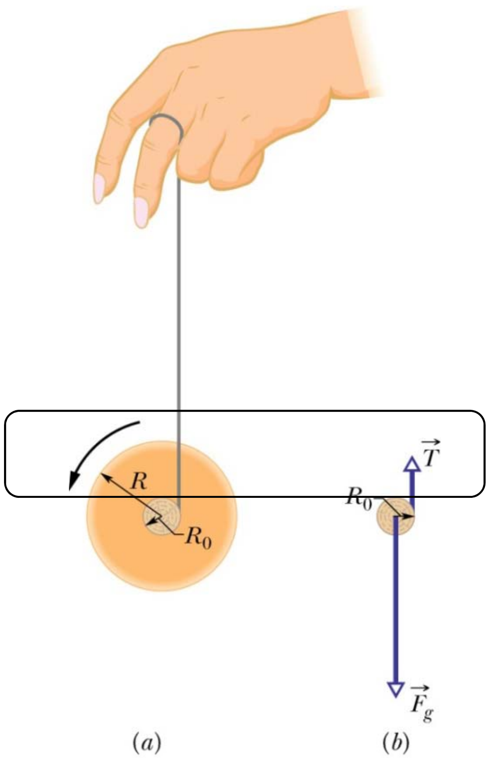
e) As it reaches the end of the string, what is its angular speed?

Knowing $|v_{com}|$

$$\omega = \frac{v_{com}}{R}$$

f) As it reaches the end of the string, what is its rotational KE?

Two ways: $KE_{rot} = \frac{1}{2} I_{com} \omega^2 \quad or \quad KE_{rot} = K_{Ef,tot} - KE_{trans}$

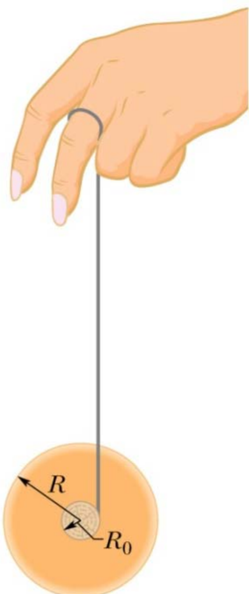


$$a_{com} = \frac{g}{1 + I_{com}/mR_0^2}$$

downwards

Which way will it roll??

1



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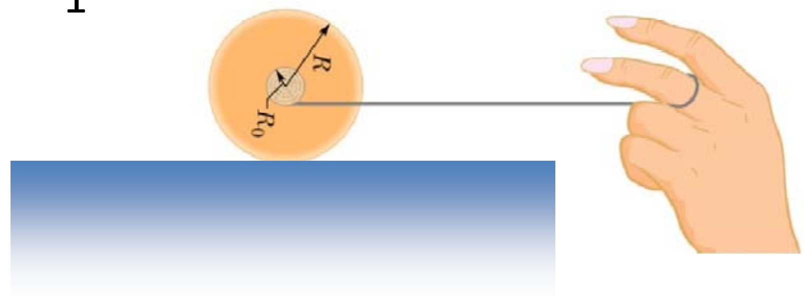

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