Physics 2101
Section 3
March 12rd : Ch. 10

Announcements:

- Mid-grades posted in PAW
- Quiz today
- I will be at the March APS meeting the week of 15-19th. Prof. Rich Kurtz will help me.

Class Website:

http://www.phys.lsu.edu/classes/spring2010/phys2101-3/

http://www.phys.lsu.edu/~jzhang/teaching.html

Forces of Rolling

1) If object is rolling with a_{com}=0 (i.e. no net forces), then $v_{com}=\omega R$ = constant (smooth roll)

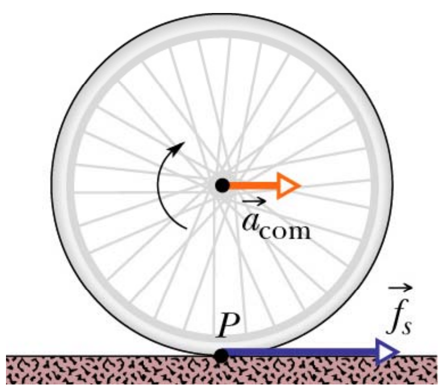
…**if constant speed**, it has no tendency to slide at point of contact - **no frictional forces**

$a_{com} = 0$ → $\alpha = 0$ → $\tau = 0$ → $f = 0$

$a_{com} = \alpha R$ $\quad$ $I_{com}\alpha = \tau_{net}$ $\quad$ $\tau_{net} = |R||f|\sin\theta$

2) If object is rolling with $a_{com} \neq 0$ (i.e. there are net forces) and <u>no slipping</u> occurs, then $\alpha \neq 0 \Rightarrow \tau \neq 0$

… static friction needed to supply torque !

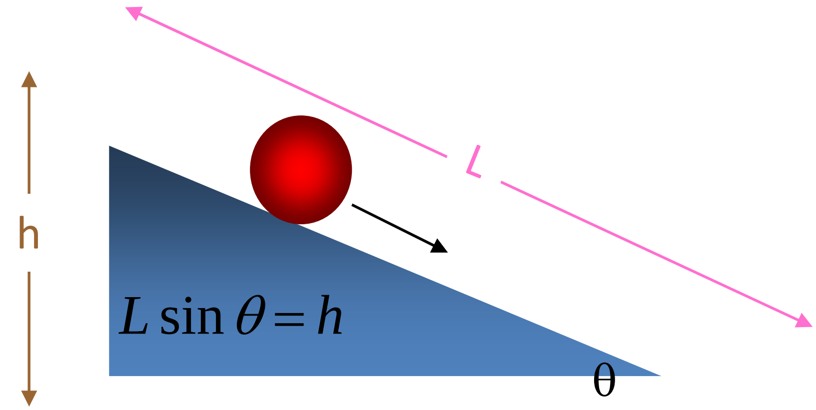


kinetic friction - during sliding ($a_{com} \neq \alpha R$)

static friction - smooth rolling ($a_{com} = \alpha R$)

Rolling down a ramp : Energy considerations

An object at rest, with mass **m** and radius **r**, roles from top of incline plane to bottom. What is **v** at bottom and **a** throughout.



$$\Delta E_{mech} = 0$$

$$\Delta KE_{tot} = -\Delta U$$

$$\left(KE_{rot+trans,COM}\right)_{final} - (0)_{init} = -\left[(0)_{final} - (mgh)_{init}\right]$$

$$\rightarrow \tfrac{1}{2} m v_{COM}^2 + \tfrac{1}{2} I_{COM}\,\omega^2 = mgL\sin\theta$$

→ Using $v_{com} = \omega r$

→ Using $v^2 = v_0^2 + 2aL$

$$\left|v_{COM}\right| = \sqrt{\frac{2gL\sin\theta}{\left(1 + \frac{I_{com}}{mR^2}\right)}}$$

Using 1-D kinematics (const accel) ⟹

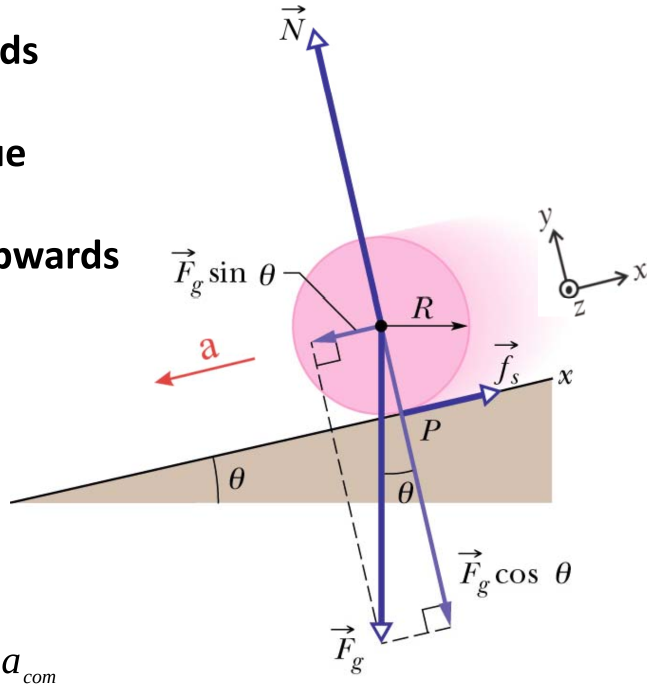
$$a_{com} = \frac{v^2}{2L} = \frac{g\sin\theta}{\left(1 + \frac{I_{com}}{mR^2}\right)}$$

Speed at Bottom

Constant Acceleration

Rolling down a ramp: acceleration

- **Acceleration downwards**
- **Friction provides torque**
- **Static friction points upwards**



Newton's 2nd Law

Linear version

$$\hat{x}: \quad f_s - F_g \sin\theta = -ma_{com}$$

$$\hat{y}: \quad N - F_g \cos\theta = 0$$

Angular version

$$\hat{z}: \quad I_{com}\alpha = \tau = Rf_s$$

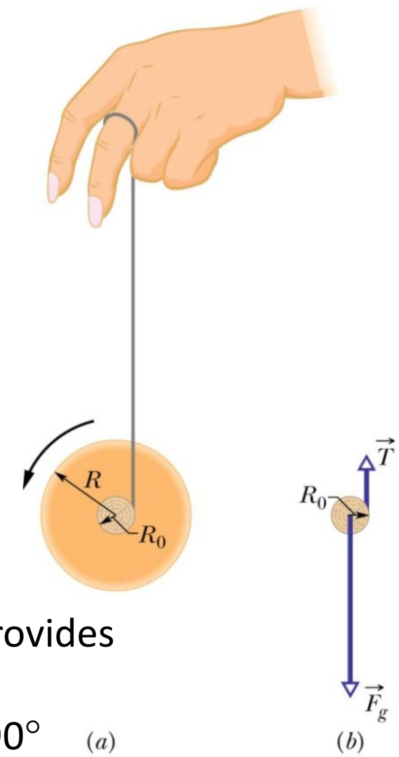
$$\rightarrow a_{com} = \alpha R$$

$$\alpha = \frac{a_{com}}{R} = \frac{Rf_s}{I_{com}}$$

$$a_{com} = \frac{R^2(mg\sin\theta - ma_{com})}{I_{com}}$$

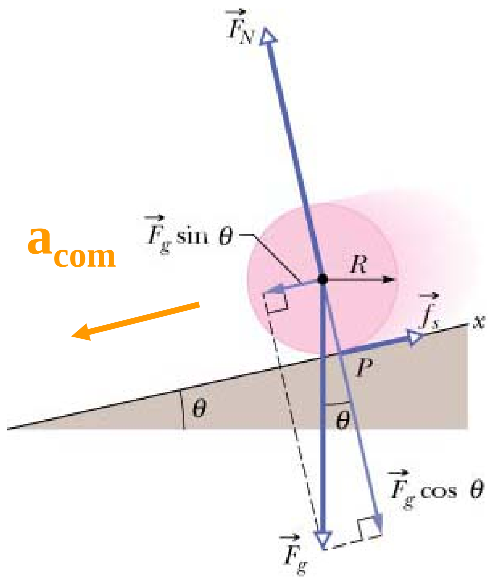
$$a_{com} = \frac{g\sin\theta}{1 + I_{com}/mR^2}$$

Yo-Yo



- Tension provides torque
- Here θ = 90°
- Axle: R ⇒ R_0

$$a_{com} = \frac{g}{1 + I_{com}/mR_0^2}$$



Rolling Down a Ramp

Consider a round uniform body of mass M and radius R rolling down an inclined plane of angle θ. We will calculate the acceleration a_{com} of the center of mass along the x-axis using Newton's second law for the translational and rotational motion.

Newton's second law for motion along the x-axis: $f_s - Mg\sin\theta = Ma_{\text{com}}$ (eq. 1)

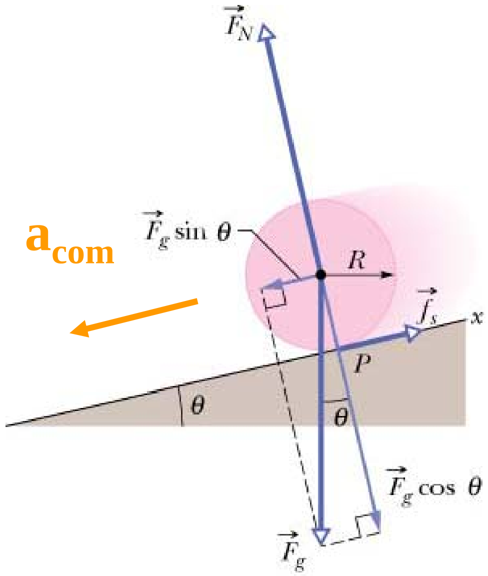
Newton's second law for rotation about the center of mass: $\tau = Rf_s = I_{\text{com}}\alpha$

$\alpha = -\dfrac{a_{\text{com}}}{R}$ We substitute α in the second equation and get $Rf_s = -I_{\text{com}}\dfrac{a_{\text{com}}}{R} \rightarrow$

$f_s = -I_{\text{com}}\dfrac{a_{\text{com}}}{R^2}$ (eq. 2). We substitute f_s from equation 2 into equation 1 $\rightarrow$

$$-I_{\text{com}}\frac{a_{\text{com}}}{R^2} - Mg\sin\theta = Ma_{\text{com}}$$

$$\boxed{a_{\text{com}} = -\frac{g\sin\theta}{1+\dfrac{I_{\text{com}}}{MR^2}}}$$



$$|a_{com}| = \frac{g\sin\theta}{1+\frac{I_{com}}{MR^2}}$$

Cylinder

$$I_1 = \frac{MR^2}{2}$$

$$a_1 = \frac{g\sin\theta}{1+I_1/MR^2}$$

$$a_1 = \frac{g\sin\theta}{1+MR^2/2MR^2}$$

$$a_1 = \frac{g\sin\theta}{1+1/2}$$

$$a_1 = \frac{2g\sin\theta}{3} = (0.67)g\sin\theta$$

Hoop

$$I_2 = MR^2$$

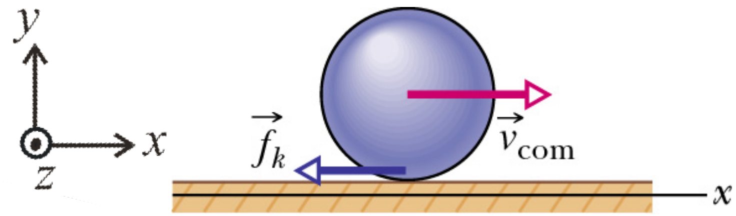
$$a_2 = \frac{g\sin\theta}{1+I_2/MR^2}$$

$$a_2 = \frac{g\sin\theta}{1+MR^2/MR^2}$$

$$a_2 = \frac{g\sin\theta}{1+1}$$

$$a_2 = \frac{g\sin\theta}{2} = (0.5)g\sin\theta$$

Prob. 11-64 **NON-smooth rolling motion**



A bowler throws a bowling ball of radius **R** along a lane. The ball slides on the lane, with initial speed $\mathbf{v_{com,0}}$ and initial angular speed $\omega_0 = 0$. The coefficient of kinetic friction between the ball and the lane is μ_k. The kinetic frictional force $\mathbf{f_k}$ acting on the ball while producing a torque that causes an angular acceleration of the ball. When the speed $\mathbf{v_{com}}$ has decreased enough and the angular speed ω has increased enough, the ball stops sliding and then rolls smoothly.

a) [After it stops sliding] What is the $\mathbf{v_{com}}$ in terms of ω ?

Smooth rolling means $v_{com} = R\omega$

b) During the sliding, what is the ball's <u>linear</u> acceleration?

From 2nd law: $\hat{x}: \; -f_k = ma_{com}$ But $f_k = \mu_k N = \mu_k mg$ So $a_{com} = -f_k/m = -\mu_k g$

(linear) $\hat{y}: \; N - mg = 0$

c) During the sliding, what is the ball's <u>angular</u> acceleration?

From 2nd law: $\vec{\tau} = Rf_k(-\hat{z})$ But $f_k = \mu_k N = \mu_k mg$ So $I\alpha = Rf_k = R(\mu_k mg)$

(angular) $I\alpha(-\hat{z}) = \vec{\tau} = Rf_k(-\hat{z})$ $\quad \alpha = R\mu_k mg / I$

d) What is the speed of the ball when smooth rolling begins?

When does $v_{com} = R\omega$? $\quad v_{com} = v_0 + a_{com}t \quad v_{com} = v_0 - \mu_k gt$

From kinematics: $\omega = \omega_0 + \alpha t \quad t = \omega/\alpha = I\omega / R\mu_k mg$

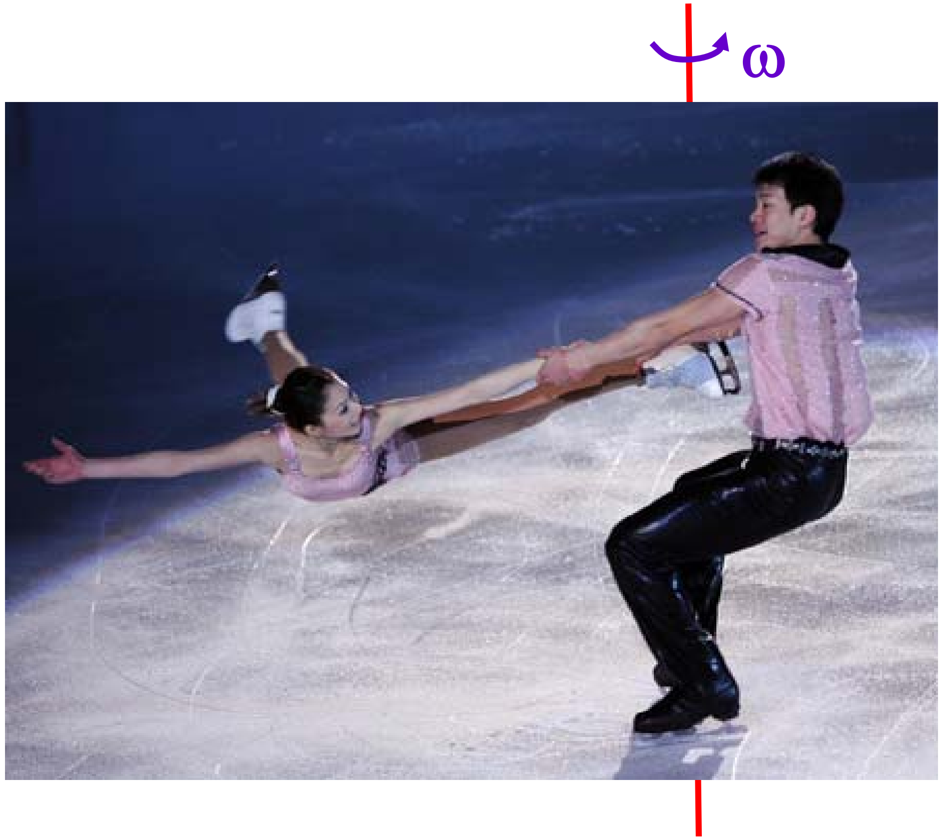
$$v_{com} = v_0 - \mu_k g \left(\frac{I\left(v_{com}/R \right)}{R\mu_k mg} \right)$$

$$v_{com} = \frac{v_0}{\left(1 + I/mR^2\right)}$$

e) How long does the ball slide?

$$t = \frac{v_0 - v_{com}}{\mu_k g}$$

Torque & Angular Momentum



Conservation of:

- **Linear momentum (closed, isolated system)**
- **KE (elastic collisions)**
- **Mechanical energy (only conservative forces)**
- **Total energy**

- **Angular momentum (closed, isolated system)**

Angular Momentum
(single particle)

Linear momentum

when particle passes through point A

it has linear momentum: $\vec{p} = m\vec{v}$

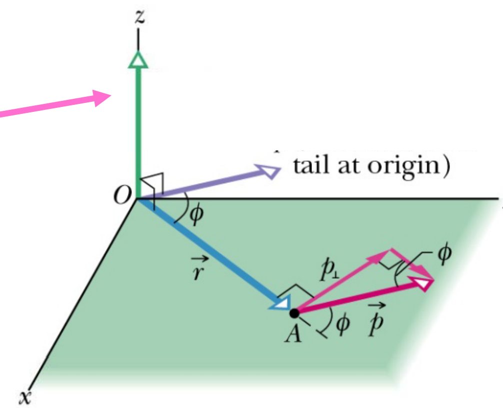
with components in x-y plane

Angular momentum

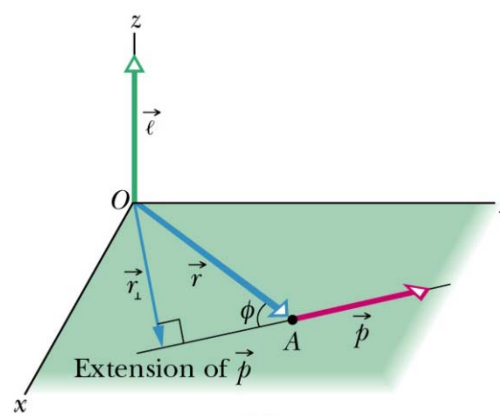
with respect to point O the angular momentum is defined as:

$$\vec{\ell} = \vec{r} \times \vec{p} = m(\vec{r} \times \vec{v})$$

where $\vec{r}$ is the position vector of the particle with respect to O.



(a)



(b)

- Units kg·m²/s
- components in z-direction only !
  (⊥ to x-y plane)
- **just as in $\vec{\tau}$, $\vec{\ell}$ has meaning only with respect to a specified point (axis)**

Newton's 2nd Law in Angular Form
(single particle)

Relationship between force and linear momentum

Newton's 2nd Law in linear form $\vec{F}_{net} = \sum \vec{F}_i = m\vec{a} = \frac{d\vec{p}}{dt}$

Relationship between torque and angular momentum

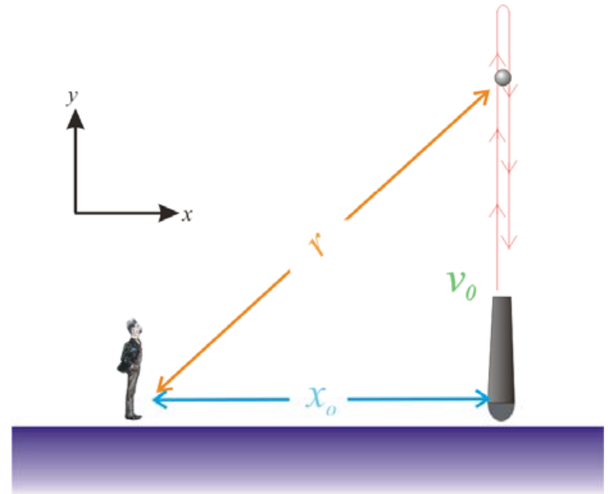
$$\vec{\tau}_{net} = \sum \vec{\tau}_i = \frac{d\vec{\ell}}{dt}$$

The vector sum of all the torques acting on a particle is equal to the time rate change of the angular momentum of that particle.

$\vec{\tau}_{net} = d\vec{\ell}/dt$ **has no meaning unless the net torque $\vec{\tau}_{net}$ and the rotational momentum $\vec{\ell}$, are defined with respect to the same origin**

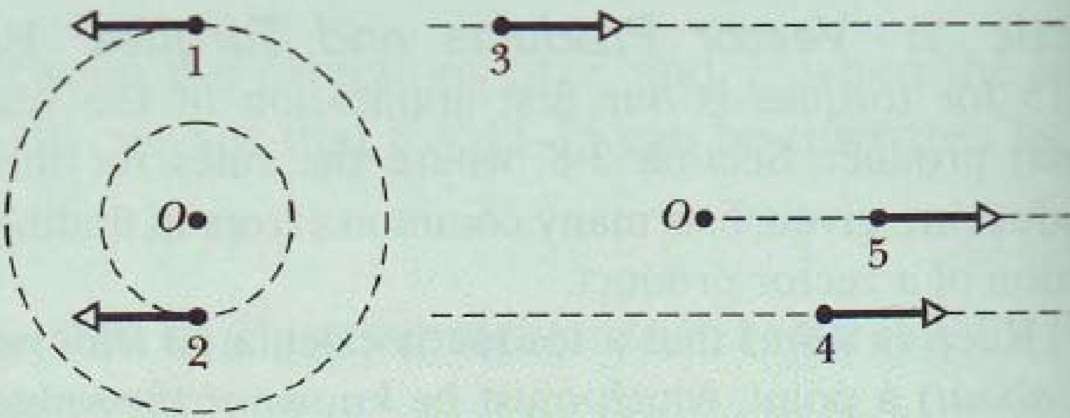
Example #1

A cannon ball, with mass **m**, is shot directly upward with velocity $\mathbf{v_0}$. What is the angular momentum and torque of the ball about a point a distance $\mathbf{x_0}$ from the cannon?



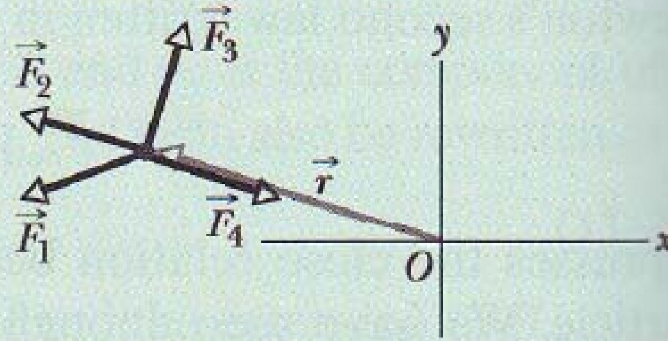
[redacted]

CHECKPOINT 4 In part *a* of the figure, particles 1 and 2 move around point O in opposite directions, in circles with radii 2 m and 4 m. In part *b*, particles 3 and 4 travel in the same direction, along straight lines at perpendicular distances of 4 m and 2 m from point O. Particle 5 moves directly away from O. All five particles have the same mass and the same constant speed. (a) Rank the particles according to the magnitudes of their angular momentum about point O, greatest first. (b) Which particles have negative angular momentum about point O?



(*a*) (*b*)

CHECKPOINT 5 The figure shows the position vector $\vec{r}$ of a particle at a certain instant, and four choices for the direction of a force that is to accelerate the particle. All four choices lie in the xy plane. (a) Rank the choices according to the magnitude of the time rate of change ($d\vec{\ell}/dt$) they produce in the angular momentum of the particle about point O, greatest first. (b) Which choice results in a negative rate of change about O?



[redacted]

Newton's 2nd Law in Angular Form

System of particles

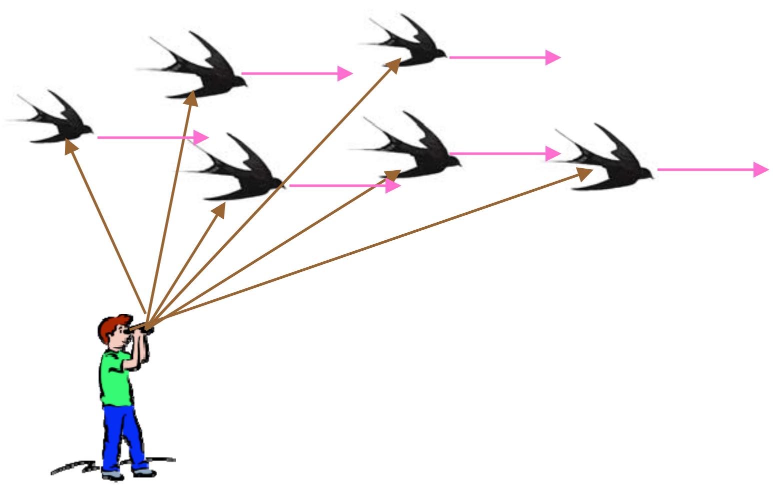
Total angular momentum

$$\vec{L} = \vec{l}_1 + \vec{l}_2 + \vec{l}_3 + \ldots + \vec{l}_n = \sum_{i=1}^{n} \vec{l}_i$$

Relationship between torque and angular momentum for system of particles

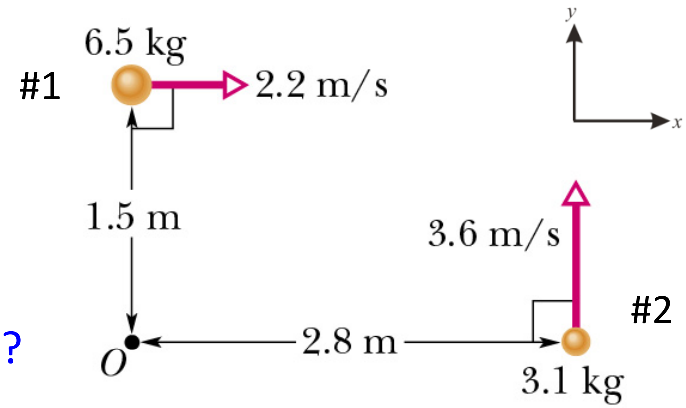
$$\vec{\tau}_{net} = \frac{d\vec{L}}{dt}$$

$\vec{\tau}_{net}$ represents the net **external** torque.



Example #2

(Problem 11-27)



Two particles are moving as shown.

a) What is their total moment of inertia about point O?
b) What is their total angular momentum about point O?
c) What is their net torque about point O?

a) The total moment of inertia about point O is found from:

$$I_{tot} = \sum I_i = \sum m_i r_i^2 = (6.5 \cdot kg)(1.5 \cdot m)^2 + (3.1 \cdot kg)(2.8 \cdot m)^2 = 38.9 \cdot kg \cdot m^2$$

b) The total angular momentum about point 0 is:

$$\vec{L}_{tot} = \sum \vec{l}_i = \sum m_i \left(\vec{r}_i \times \vec{v}_i\right) = m_1 \left(r_1 v_1 \sin\theta_1\right)(-\hat{z}) + m_2 \left(r_2 v_2 \sin\theta_2\right)(+\hat{z})$$

$$\vec{L}_{tot} = \left[-(6.5 \cdot kg)(1.5 \cdot m)(2.2 \cdot m/s) + (3.1 \cdot kg)(2.8 \cdot m)(3.6 \cdot m/s)\right]\hat{z}$$

$$= 9.8 \cdot kg \cdot m^2/s \cdot \hat{z}$$

c) The net torque about point 0 is:

$$\vec{\tau}_{net} = \frac{d\vec{L}_{tot}}{dt} = 0$$

closed, isolated system

Angular Momentum of a **Rigid Body** Rotating about a **Fixed axis**

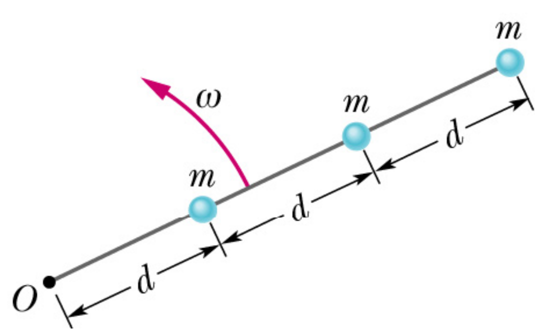
$$\frac{d\vec{L}}{dt} = \vec{\tau}_{net} = I\vec{\alpha} = I\frac{d\vec{\omega}}{dt}$$

$$\vec{L} = I\vec{\omega}$$

Example #3 (Problem 11-39)

Three particles are fastened to each other with massless string and rotate around point O.

What is the total angular momentum about point O?

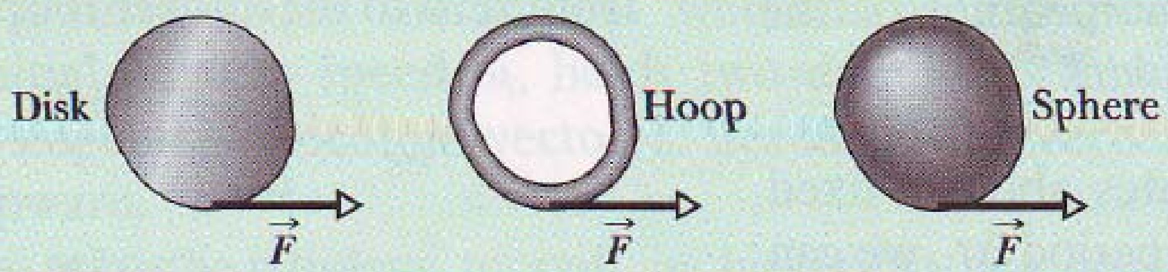


$$I_{tot} = m(d)^2 + m(2d)^2 + m(3d)^2$$
$$= 14md^2$$

$$\vec{L}_{tot} = I\omega = 14m\omega d^2 \cdot \hat{z}$$

[kg·m²/s] ✓

CHECKPOINT 6 In the figure, a disk, a hoop, and a solid sphere are made to spin about fixed central axes (like a top) by means of strings wrapped around them, with the strings producing the same constant tangential force $\vec{F}$ on all three objects. The three objects have the same mass and radius, and they are initially stationary. Rank the objects according to (a) their angular momentum about their central axes and (b) their angular speed, greatest first, when the strings have been pulled for a certain time t.



[redacted]