Physics 2101
Section 3
March 10rd : Ch. 10

Announcements:

- Exam #2 grade posted
- Next Quiz is March 12
- I will be at the March APS meeting the week of 15-19th. Prof. Rich Kurtz will help me.

Class Website:

http://www.phys.lsu.edu/classes/spring2010/phys2101-3/

http://www.phys.lsu.edu/~jzhang/teaching.html

Rotational Work and Energy

We can compare linear variables with rotational variables

x	⟷	θ
v	⟷	ω
a	⟷	α
Δt	⟷	Δt
F	⟷	τ
m	⟷	I

$$s = r\theta$$

$$v_T = r\omega$$

$$a_T = r\alpha$$

The same can be done for work and energy:

For translational systems

$$W = F \cdot x$$

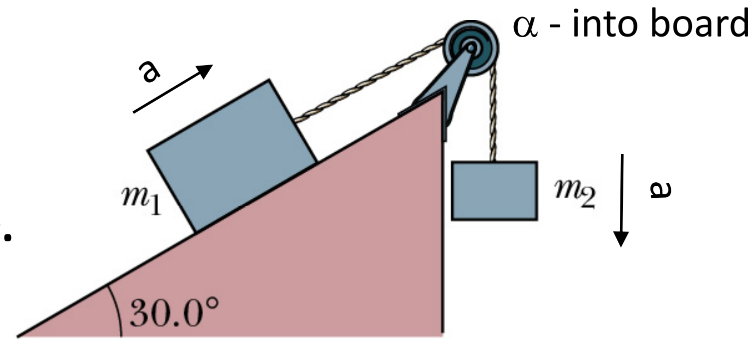
$$KE = \frac{1}{2} m v^2$$

For rotational systems

$$W = \tau \cdot \theta$$

$$KE = \frac{1}{2} I \omega^2$$

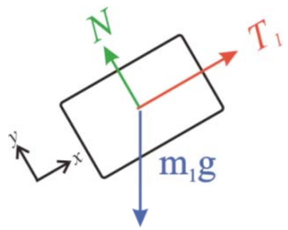
Example #2



Massless cord wrapped around a pulley of radius *r* and mass M_W (frictionless surface/bearings) and I_W=1/2$M_W r^2$.

What is angular acceleration, α, of pulley (disc)?

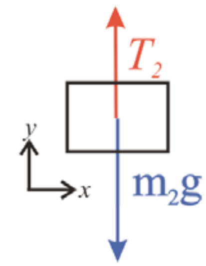
1) What are forces on m_1?



$$\hat{x}: \; T_1 - m_1 g\sin\theta = m_1 a$$
$$\hat{y}: \; N - m_1 g\cos\theta = 0$$
$$\Rightarrow \; T_1 = m_1(a + g\sin\theta)$$

2) What are forces on m_2?

$$\hat{y}: \; T_2 - m_2 g = -m_2 a$$
$$\Rightarrow \; T_2 = m_2(g - a)$$

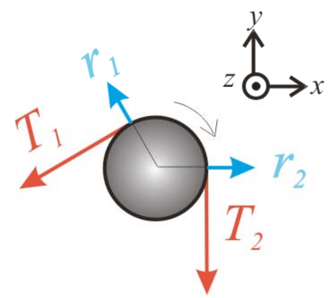


NOTE: T_1 & T_2 are NOT equal

3) What are torques about wheel?

$$\vec{\tau}_{net} = \vec{\tau}_1 + \vec{\tau}_2$$

$$\vec{\tau}_{net} = rT_1\sin(90^\circ) + rT_2\sin(-90^\circ)$$
$$= R(T_1 - T_2)(+\hat{z})$$



NOTE: angular acceleration vector is in negative-z direction

4) Solve for α ?

$$I_W\alpha = r[(T_2) - (T_1)]$$
$$I_W\alpha = \left(\tfrac{1}{2}M_W r^2\right)\alpha = r\left[\left(m_2(g-a)\right) - \left(m_1(a + g\sin\theta)\right)\right]$$

$$here \quad a_t = \alpha r \quad \Rightarrow \quad \left(\tfrac{1}{2}M_W R^2\right)\alpha = rg(m_2 - m_1\sin\theta) - r(\alpha r)(m_2 + m_1)$$

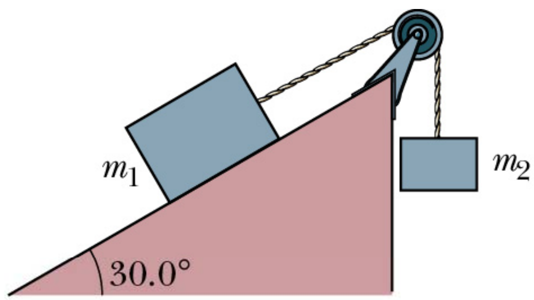
$$\alpha = \frac{2g}{r}\left[\frac{m_2 - m_1\sin\theta}{M_w + 2(m_1 + m_2)}\right]$$

a) If M_w = 0, same as problem 5-43

b) If θ =90º, same as problem 11-55 (HW # 9)

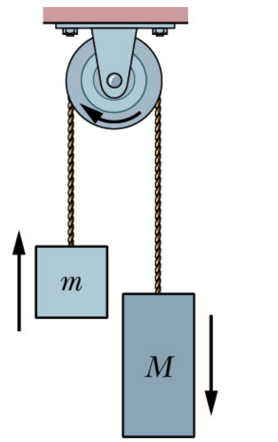
Summary: Effects of rotation

From before



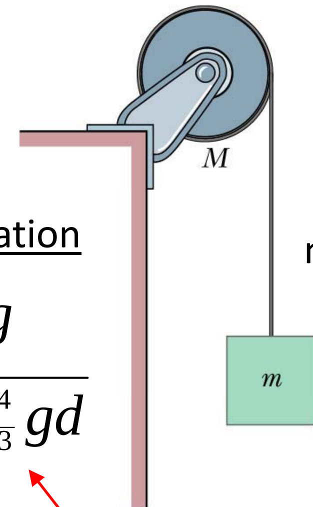
$$a = \frac{2g(m_2 - m_1 \sin\theta)}{M_w + 2(m_1 + m_2)}$$

Done via force/torque



$$|v| = \sqrt{\frac{4gd(M-m)}{M_W + 2(m+M)}}$$

Done via energy



m = M

<u>With Rotation</u>

$$a = \tfrac{2}{3} g$$

$$|v| = \sqrt{\tfrac{4}{3} gd}$$

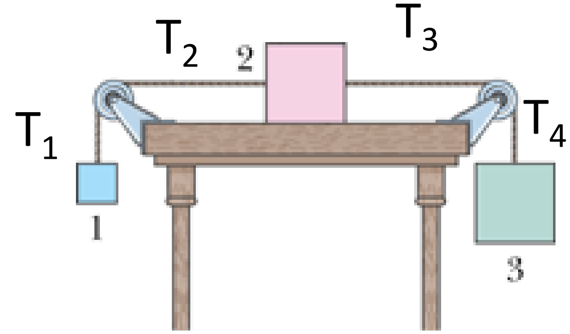
Done with both

remember this for later

In each of these cases: "translation" was separate from "rotation"

Pure translation + Pure rotation

SHW#6: Three masses M, 2M and 3M are shown in the figure with two pulley of moment of inertial I and radius r.
(a) What is the linear Acceleration?



$$T_1 - Mg = Ma$$

$$(T_2 - T_1)r = I\alpha = \frac{Ia}{r}$$

$$T_3 - T_2 = 2Ma$$

$$(T_4 - T_3)r = \frac{Ia}{r}$$

$$3Mg - T_4 = 3Ma$$

Solve for T_4, then T_3, ------

If I did it right

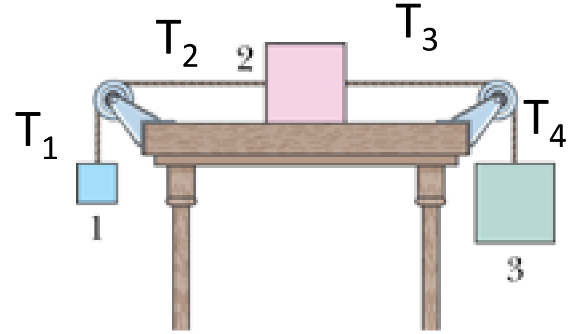
$$a = \frac{Mg}{\left[\frac{I}{r^2} + 3M\right]}$$

Check if I=0

$$F = 3Mg - Mg = 6Ma$$

$$a = \frac{g}{3}$$

SHW#6: Three masses M, 2M and 3M are shown in the figure with two pulley of moment of inertial I and radius r. The coefficient of friction is μ_k and the table is L m long.



$$T_1 - Mg = Ma$$

$$(T_2 - T_1)r = I\alpha = \frac{Ia}{r}$$

$$T_3 - T_2 - 2\mu_k Mg = 2Ma$$

$$(T_4 - T_3)r = \frac{Ia}{r}$$

$$3Mg - T_4 = 3Ma$$

$$T_2 = gM(3 - 2\mu_k) - a\left(\frac{I}{r^2} + 5M\right)$$

$$T_2 = \frac{gM\left[\frac{I}{r^2}(2-\mu_k) + M(4-\mu_k)\right]}{\left[\frac{I}{r^2} + 3M\right]}$$

$$T_3 = -a\left(\frac{I}{r^2} + 3M\right) + 3Mg$$

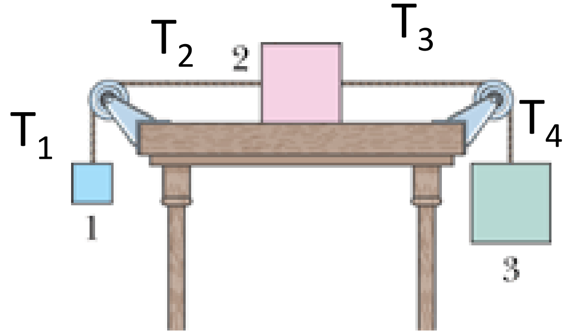
$$T_3 = -Mg(1-\mu_k) + 3Mg = Mg(2+\mu_k)$$

$$a = \frac{Mg(1-\mu_k)}{\left[\frac{I}{r^2} + 3M\right]}$$

Need $v = r\omega$ & $a = r\alpha$

SHW#6: Three masses M, 2M and 3M are shown in the figure with two pulley of moment of inertial I and radius r. The coefficient of friction is μ_k and the table is L m long.

Let us use Energy.



We can define zero so $E_{mech}(start) = 0$

After block 3 has moved a distance d we have

$$E_{mech} = 0 - 2\mu_k Mgd = \frac{6Mv^2}{2} + \frac{2I\omega^2}{2} - (3M - M)gd$$

$$-\mu_k Mgd = \frac{3Mv^2}{2} + \frac{Iv^2}{2r^2} - Mgd \qquad (\text{use } v = r\omega)$$

$$v^2 = \frac{2Mgd(1-\mu_k)}{3M + \frac{I}{r^2}}:$$

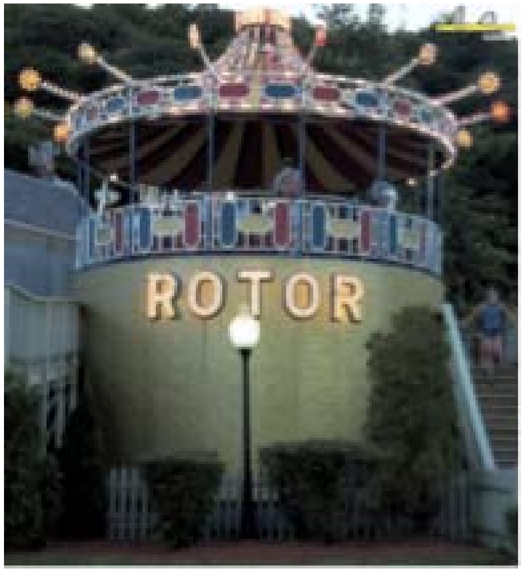
$$v^2 = 2ad: \quad a = \frac{Mg(1-\mu_k)}{3M + \frac{I}{r^2}}$$

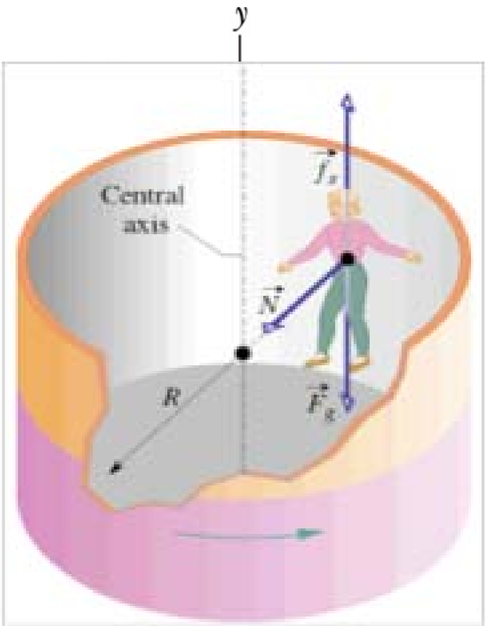
Same as before

$$a = \frac{Mg(1-\mu_k)}{\left[\frac{I}{r^2} + 3M\right]}$$

$$\frac{v_f + v_i}{2}t = d: \quad t = \sqrt{\frac{2d\left(3M + \frac{I}{r^2}\right)}{Mgd(1-\mu_k)}}$$

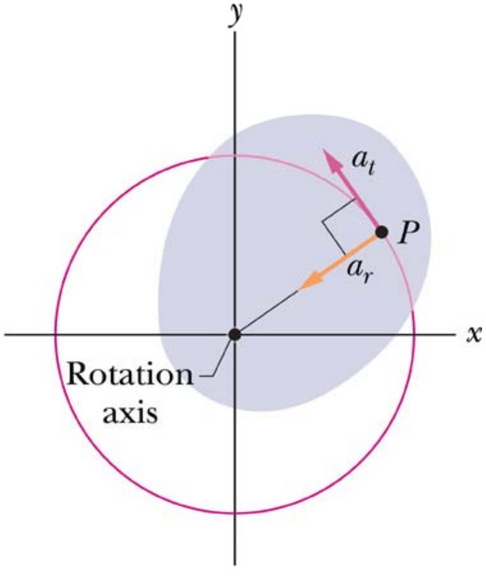
SHW#6-problem #4: A M kg woman is in a rotor with a r m radius, laying against the inner surface. The friction between the woman's sweater and the surface is μ_s. (a) The rotor begins spinning with a constant angular acceleration , and when the rotor reaches enough angular speed, the door drops but the woman stays sticking to the wall. (a) What is the minimum angular velocity that will allow the woman not to fall down when the door drops? (b) What's the maximum angular acceleration that will not make the woman slide along the wall when the rotor is speeding up to the angular velocity calculated in (a)?





$$a_t = \alpha r$$

$$a_r = \frac{v^2}{r}$$



(b)

(a) Calculate Normal Force

$$F_N = Ma_r = \frac{Mv^2}{r} \text{ gives Frictional Force}$$

$$F_f = \mu_s \frac{Mv^2}{r} > Mg$$

(b) Calculate Tangential Force

$$F_T = Ma_t = \frac{M\alpha}{r} < \mu_s Mr\omega^2 = \mu_s Mr(\alpha t)^2$$

Chapter 11

Rolling, Torque, and Angular Momentum

In this chapter we will cover the following topics:

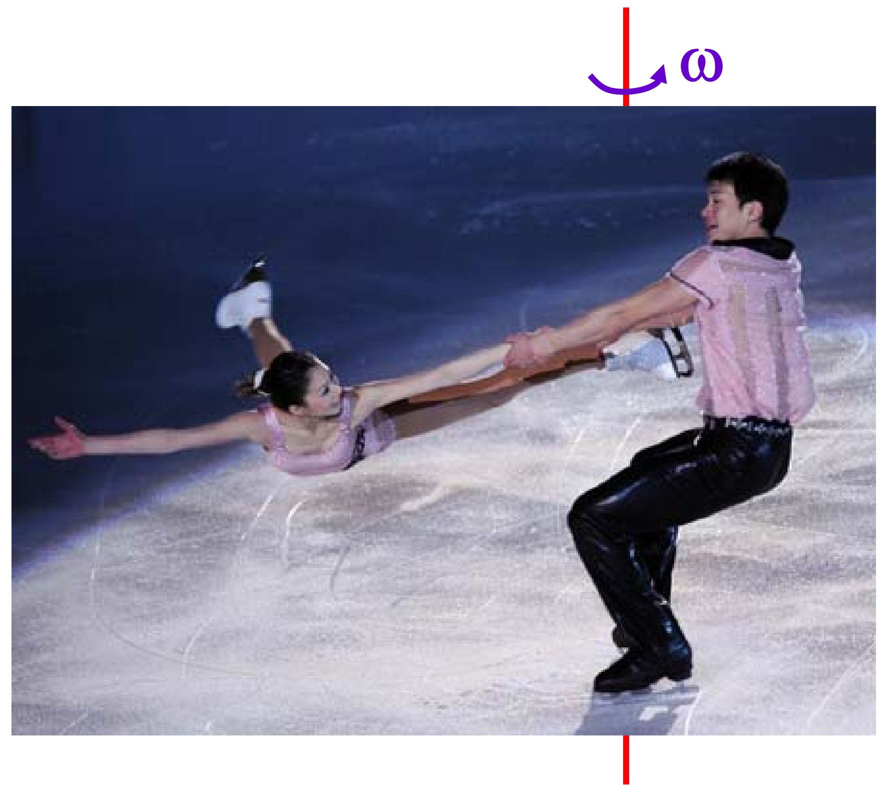
- Rolling of circular objects and its relationship with friction
- Redefinition of torque as a vector to describe rotational problems that are more complicated than the rotation of a rigid body about a fixed axis
- Angular momentum of single particles and systems of particles
- Newton's second law for rotational motion
- Conservation of angular momentum
- Applications of the conservation of angular momentum

Conservation of Angular Momentum

Describe their motion:

$$\theta(t) \rightarrow \omega(t) \rightarrow \alpha(t)$$

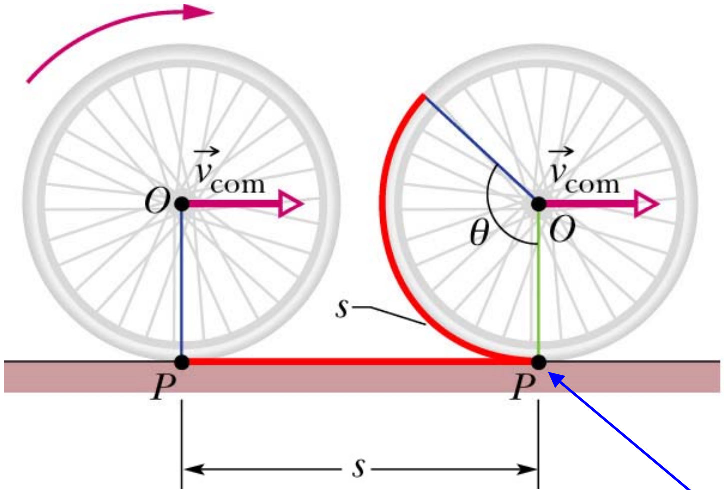
$$I \rightarrow \frac{I\omega^2}{2} \rightarrow Torque$$



Angular Momentum = $I\omega$

$$I_i\omega_i = I_f\omega_f$$

Understanding rolling with wheels



Wheel moving forward with constant speed v_{com}

$$s = \theta R$$ displacement: translation→ rotation

$$v_{com} = \frac{ds}{dt} = \frac{d(\theta R)}{dt} = \omega R$$

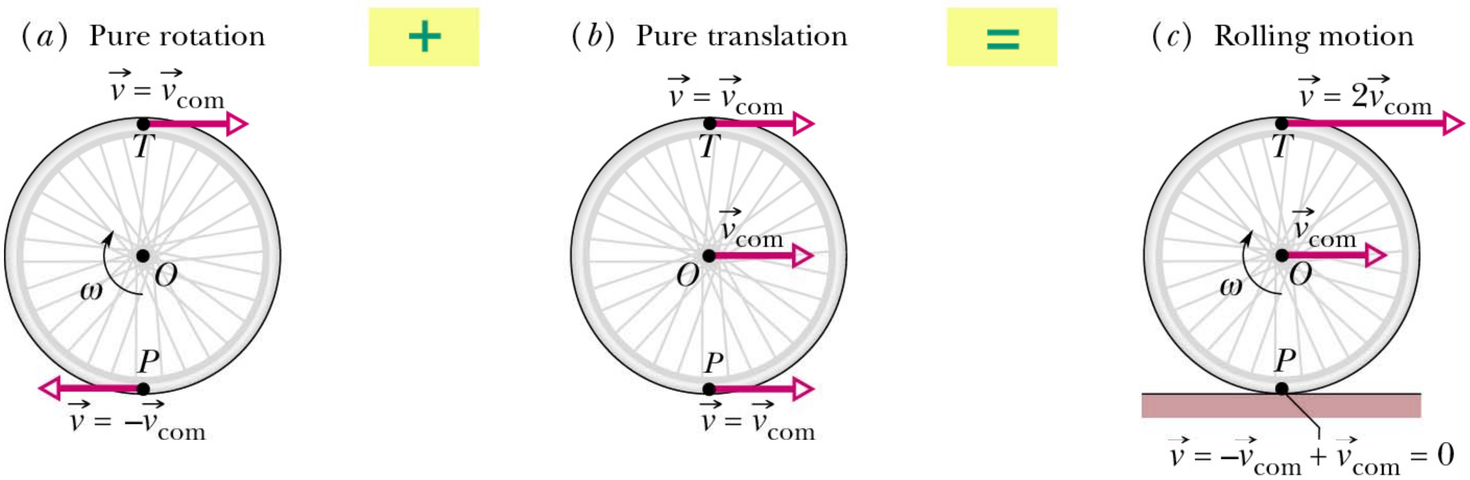
$$a_{com} = \frac{dv_{com}}{dt} = \frac{d(\omega R)}{dt} = \alpha R$$

Only if NO SLIDING [smooth rolling]

These relationships define "smooth rolling motion"

At point P (point of contact), wheel does not move

Understanding rolling with wheels II



All points on wheel move with same ω. All points on outer rim move with same linear speed v = v_{com}.

All points on wheel move to the right with same linear velocity v_{com} as center of wheel

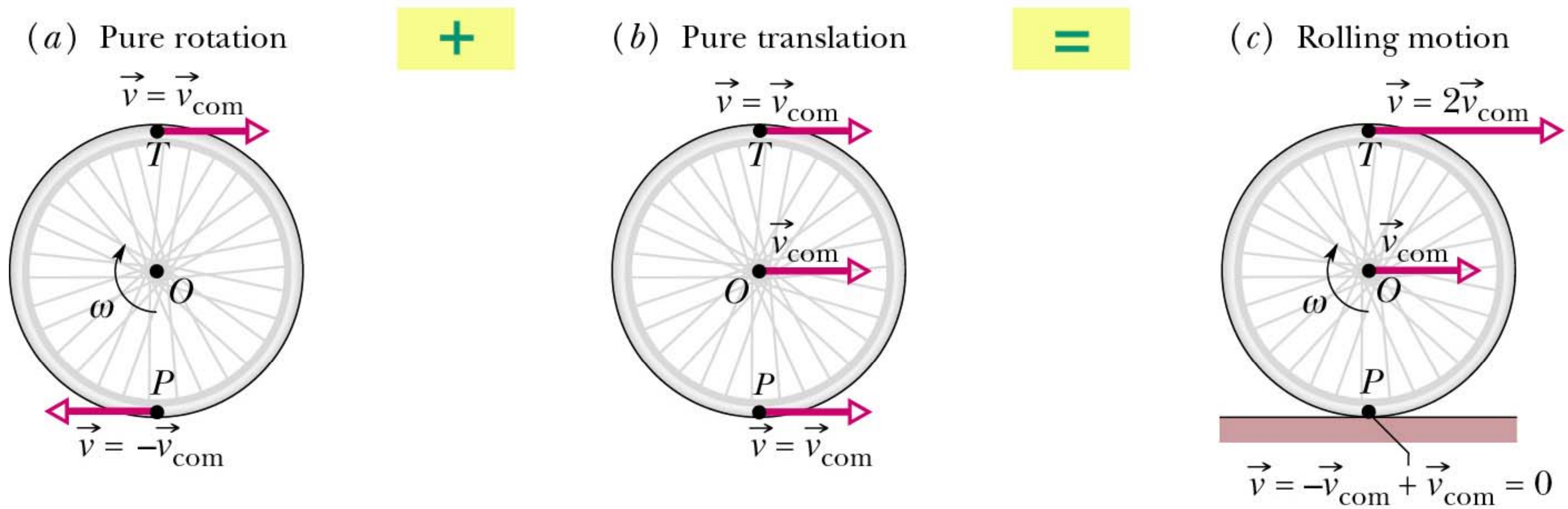
Combination of "pure rotation" and "pure translation"

$$\vec{v} = \vec{\omega} \times \vec{r}$$

Note at point P: vector sum of velocity = 0 (point of stationary contact)

at point T: vector sum of velocity = $2v_{com}$ (top moves twice as fast as com)

Kinetic energy of rolling



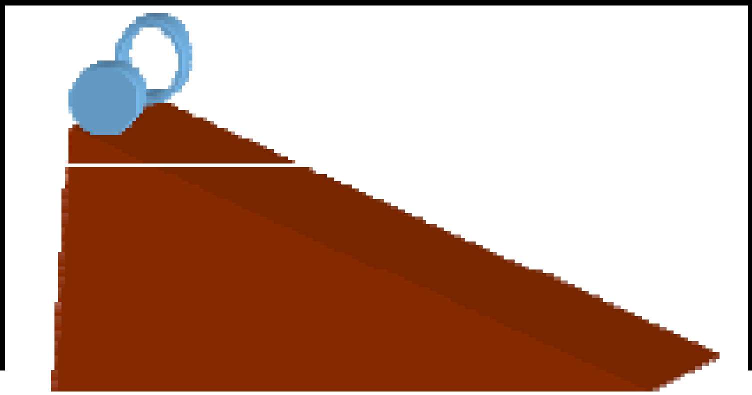
$$\frac{1}{2} I_{com}\omega^2 + \frac{1}{2} M v_{com}^2 = KE_{tot}$$

Note: rotation about COM and translation of COM combine for total KE

Remember: $v_{com} = \omega r$

Question

A ring and a solid disc, both with radius *r* and mass *m*, are released from rest at the top of a ramp. Which one gets to the bottom first?



1. Solid disc
2. Ring (hoop)
3. both reach bottom at same time

Question #2

Two solid disks of equal mass, but different radii, are released from rest at the top of a ramp. Which one arrives at the bottom first?

1. The smaller radius disk.
2. The larger radius disk.
3. Both arrive at the same time.

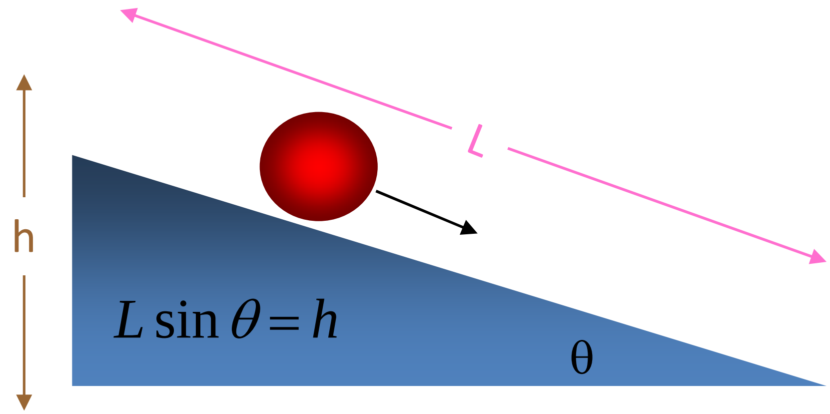
The equation for the speed of the a disk at the bottom of the ramp is

$$\sqrt{\frac{4}{3} g l \sin\theta}$$

Notice, it does not depend on the radius or the mass of the disk!!

Rolling down a ramp : Energy considerations

Object, with mass **m** and radius **r**, roles from top of incline plane to bottom. What is **v**, **a**, and **Δt** at bottom



$$\Delta E_{mech} = 0$$

AT BOTTOM $$\Delta KE_{tot} = -\Delta U$$

$$\left(KE_{rot+trans,COM}\right)_{final} - (0)_{init} = -\left[(0)_{final} - (mgh)_{init}\right]$$

$$\tfrac{1}{2}mv_{COM}^2 + \tfrac{1}{2}I_{com}\omega^2 = mgL\sin\theta$$

$$\tfrac{1}{2}mv_{COM}^2 + \tfrac{1}{2}I_{COM}\left(\frac{v}{r}\right)^2 = mgL\sin\theta$$

$$v_{COM}^2\left(m + \frac{I_{COM}}{r^2}\right) = 2mgL\sin\theta$$

$$\left|v_{COM}\right| = \sqrt{\frac{2gL\sin\theta}{\left(1 + \frac{I}{mr^2}\right)}}$$

AT BOTTOM

<u>Using 1-D kinematics</u>

$$v^2 = v_0^2 + 2aL$$

$$a = \frac{v^2}{2L} = \frac{g\sin\theta}{\left(1 + \frac{I}{mr^2}\right)}$$

AND

$$t = \sqrt{\frac{2L}{g\sin\theta}\left(1 + \frac{I}{mr^2}\right)}$$

Compare different objects

Assuming same work done (same change in U), objects with larger rotational inertial have larger KE_{rot} and during rolling, their KE_{trans} is smaller.

$$|v_{COM}| = \sqrt{\frac{2gL\sin\theta}{\left(1+\frac{I_{com}}{mr^2}\right)}}$$

$$KE_{tot} = KE_{trans} + KE_{rot} = KE_{trans}\left(1+\frac{I_{com}}{mr^2}\right)$$

Roll a hoop, disk, and solid sphere down a ramp - what wins?

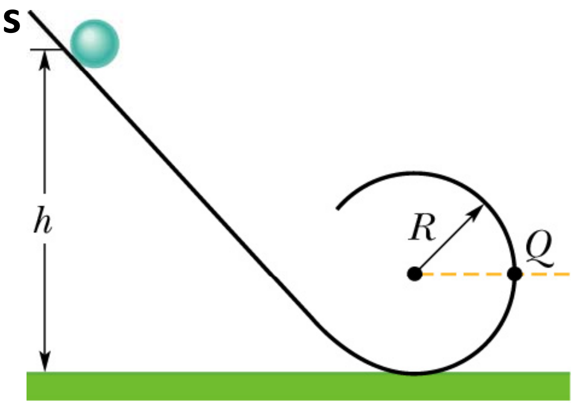
Moment of inertia large → small

Object	**Rotational Inertia, I_{com}**	**Fraction of Energy in Translation**	**Fraction of Energy in Rotation**	
Hoop	$1mr^2$	0.5	0.5	slowest
Disk	$\frac{1}{2}mr^2$	0.67	0.33	
Sphere	$\frac{2}{5}mr^2$	0.71	0.29	
sliding block (no friction)	0	1	0	fastest

$$\Delta t_{bottom} = \sqrt{\frac{2L}{g\sin\theta}\left(1+\frac{I}{mr^2}\right)}$$

Problem #1

A solid cylinder starts from rest at the upper end of the track as shown. What is the angular speed of the cylinder about its center when it is at the top of the loop?



Using conservation of mechanical energy:

$$0 = \Delta E_{mech} = \Delta KE_{tot} + \Delta U_{grav}$$

$$0 = \left[\left(\tfrac{1}{2} m v_{com}^2 + \tfrac{1}{2} I_{com}\omega^2\right)_{final} - (0)_{init}\right] + \left[\left(mg(2R)\right)_{final} - \left(mg(h)\right)_{init}\right]$$

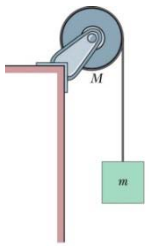
$$mg(h-2R) = \tfrac{1}{2} m(\omega r)^2 + \tfrac{1}{2}\left(\tfrac{1}{2} m r^2\right)\omega^2$$

Rearranging yields $\dfrac{2mg(h-2R)}{mr^2} = \omega^2\left[1 + \tfrac{1}{2}\right]$

$$\omega = \sqrt{\frac{4g(h-2R)}{3r^2}}$$

NOTE: Compare with before

Trans + Rot $\Rightarrow$



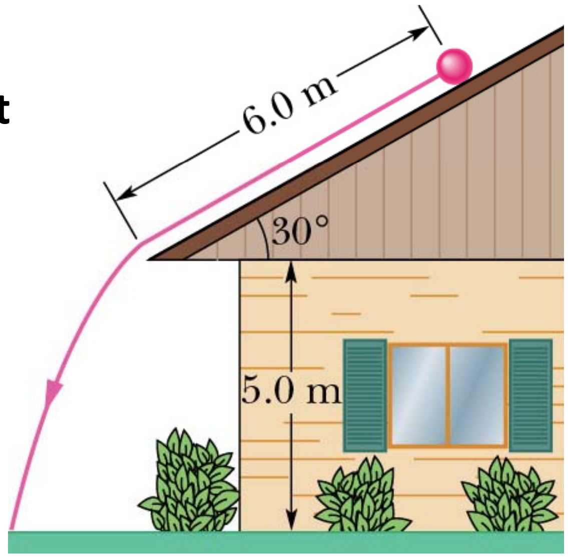
$$v = \sqrt{\tfrac{4}{3} g d_y}$$

m = M

$$v = \omega r = \sqrt{\tfrac{4}{3} g(h-2R)} = \sqrt{\tfrac{4}{3} g d_y}$$ $\Leftarrow$ Rolling

Problem #1

A solid cylinder of radius 10 cm and mass 12 kg starts from rest and rolls without slipping a distance of 6 m down a house roof that is inclined at 30º.



Where does it hit?

Using conservation of mechanical energy:

$$0 = \Delta E_{mech} = \Delta KE_{tot} + \Delta U_{grav}$$

$$0 = \left[\left(\tfrac{1}{2} mv_{com}^2 + \tfrac{1}{2} I_{com}\omega^2\right)_{final} - (0)_{init}\right] + \left[(0)_{final} - (mgh)_{init}\right]$$

$$mgL\sin\theta = \tfrac{1}{2} m(v)^2 + \tfrac{1}{2}\left(\tfrac{1}{2} mr^2\right)\left(\frac{v}{r}\right)^2$$

Rearranging yields $$\frac{2mgL\sin\theta}{m} = v^2\left[1 + \tfrac{1}{2}\right]$$

$$v_{bottom,COM} = \sqrt{\tfrac{4}{3} gl\sin\theta}$$

Then just use kinematics (v_{ox}, v_{oy}…)