Physics 2101
Section 3
March 8rd : Ch. 10

Announcements:

- Next Quiz is March 12 on SHW#6
- I will be at the March APS meeting the week of 15-19th.
- Try to solve the SHW#5 as possible as you can.

Class Website:

http://www.phys.lsu.edu/classes/spring2010/phys2101-3/

http://www.phys.lsu.edu/~jzhang/teaching.html

Moment of inertia of a Pencil

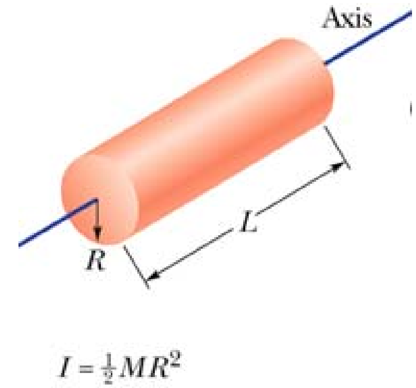
It depends on where the rotation axis is considered…

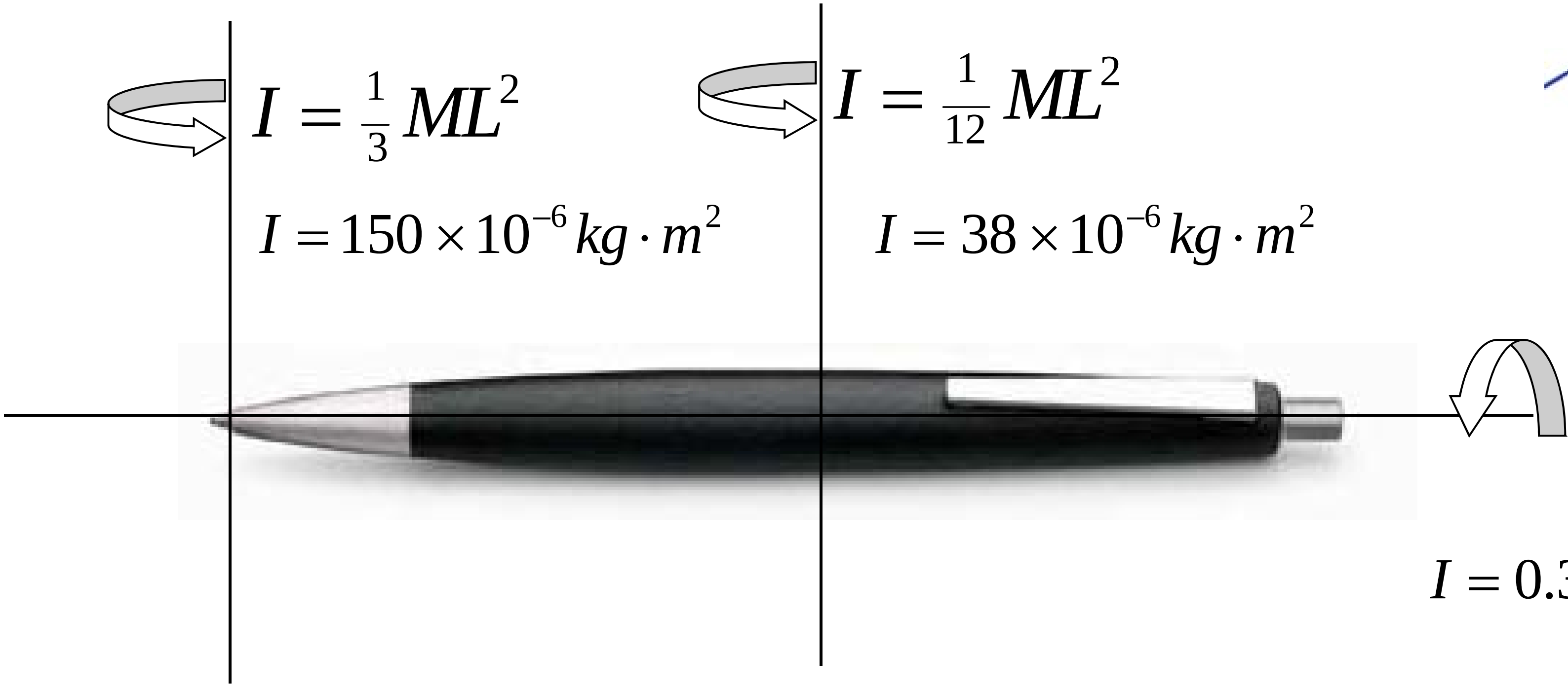
$$I = \frac{1}{3}ML^2$$

$$I = 150 \times 10^{-6}\, kg \cdot m^2$$

$$I = \frac{1}{12}ML^2$$

$$I = 38 \times 10^{-6}\, kg \cdot m^2$$





$$I = \frac{1}{2}MR^2$$

$$I = 0.3 \times 10^{-6}\, kg \cdot m^2$$

Consider a 20g pencil 15cm long and 1cm wide …

… somewhat like MASS, you can feel the difference in the rotational inertia

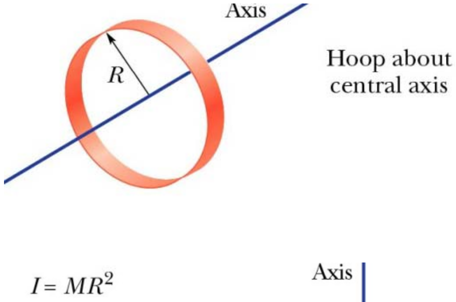
Example #1

A bicycle wheel has a radius of 0.33m and a rim of mass 1.2 kg. The wheel has 50 spokes, each with a mass 10g.

What is the moment of inertial about axis of rotation?

What is moment of inertia about COM?

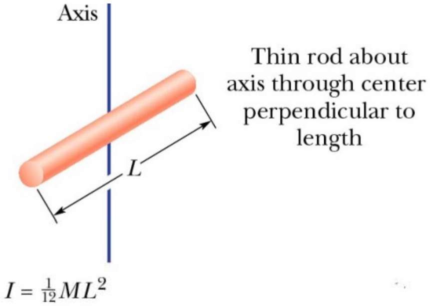
$$I_{tot,com} = I_{rim,center} + 50 I_{spoke}$$



→ What is I_{spoke} (parallel-axis)?

$$I_{spoke} = I_{rod.com} + Mh^2 = \tfrac{1}{12} ML^2 + M\left(\tfrac{1}{2} L\right)^2 = \tfrac{1}{3} ML^2$$

$$= \tfrac{1}{3}(0.01kg)(0.33m)^2 \cong 3.6 \times 10^{-4}\, kg \cdot m^2$$



→ Putting together

$$I_{tot,com} = I_{rim,center} + 50 I_{spoke}$$

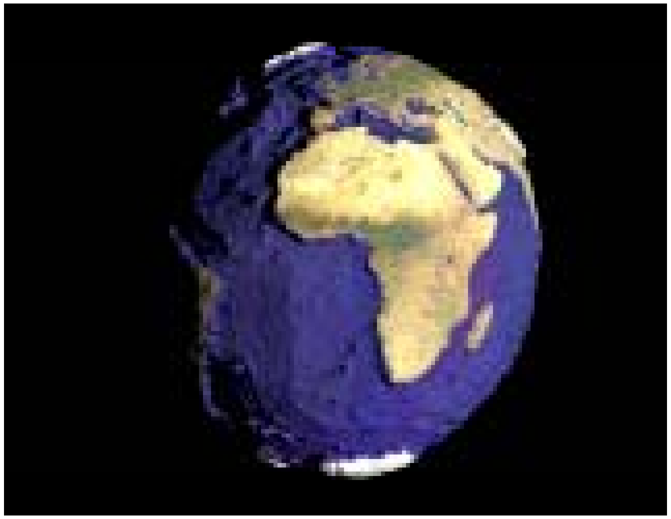
$$= M_{wheel} R^2 + 50 I_{spoke}$$

$$= (1.2kg)(0.33m)^2 + 50(3.6 \times 10^{-4}\, kg \cdot m^2) = 0.149 kg \cdot m^2$$

Example #2

How much work did Superman exert on earth in order to stop it?

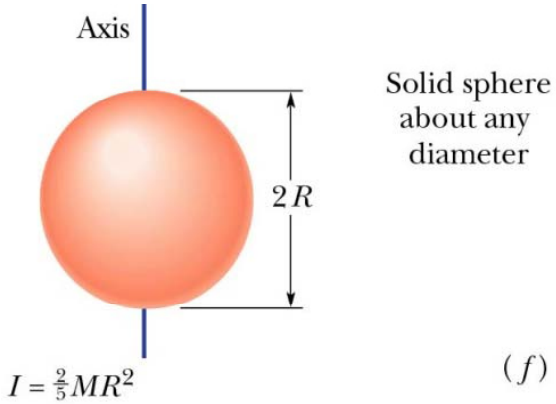
What is the kinetic energy of the earth's rotation about its axis?



Energy of rotational motion is found from:

$$KE_{rot} = \tfrac{1}{2} I\omega^2$$

What is earth's moment of inertia, I?



$$I_{earth} = I_{sphere} = \tfrac{2}{5} Mr^2$$

$$= \tfrac{2}{5}\left(6\times10^{24}\,kg\right)\left(6.4\times10^{6}\,m\right)^2 \cong 1\times10^{38}\,kg\cdot m^2$$

What is earth's angular velocity, ω?

$$From\ \ T = \frac{2\pi}{\omega} \ \ \Rightarrow \ \ \omega = \frac{2\pi\,\text{radians}}{\text{day}} = \frac{2\pi}{(3600\times24)} = 7.3\times10^{-5}\,rad/s$$

Now plug-'n-chug:

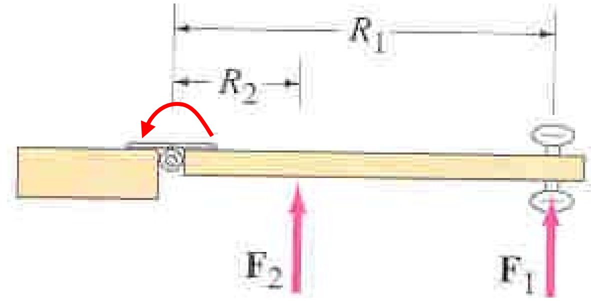
$$KE_{rot} = \tfrac{1}{2}\left(1\times10^{38}\,kg\cdot m^2\right)\left(7.3\times10^{-5}\,rad/s\right)^2 = \underline{2.6\times10^{29}\,J}$$

Rotational dynamics: Torque

What causes something to rotate

Ability of force $\vec{F}$ to rotate body

What does it depend on??



Depends on distance from pivot point

Depends on angle between force and radius

Moment of force about an axis = torque - "to twist"

$$\vec{\tau} = \vec{r} \times \vec{F}$$

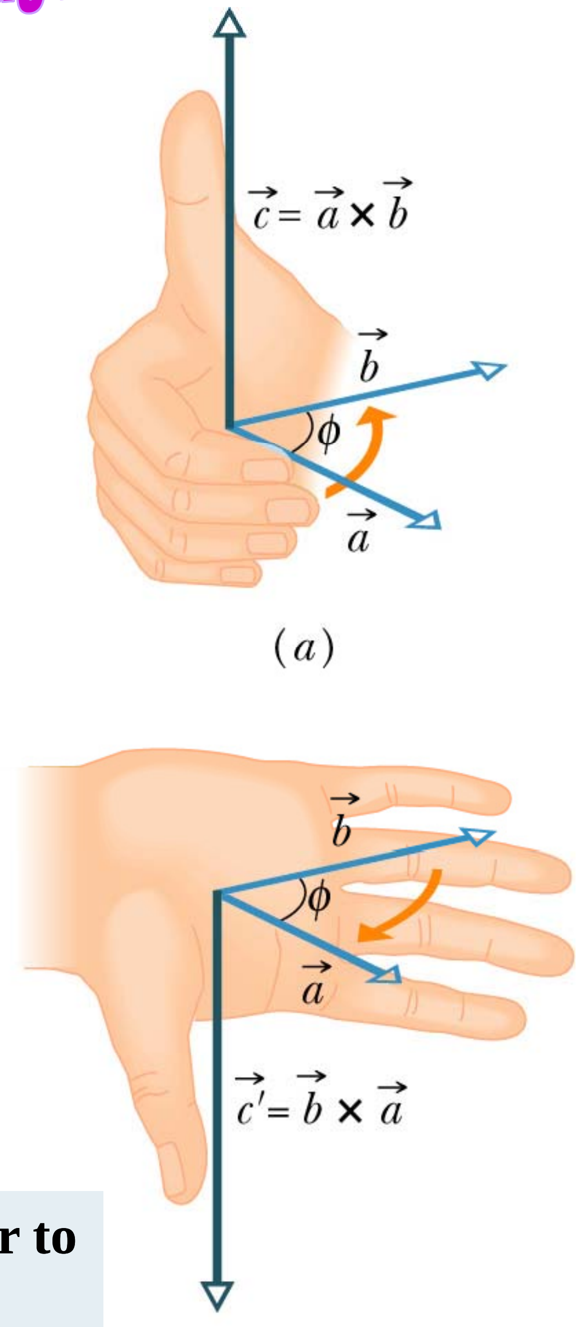
τ depends on: r, F and θ

$$|\vec{\tau}| = |\vec{r}||\vec{F}|\sin\vartheta$$

Units of torque: N·m (not Joule)

Sec. 3.7 Multiplying vectors:

See page A-10 in back of book



(b)

2) Cross Product (Vector × Vector = Vector)

$$\vec{C} = \vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

$$\left|\vec{C}\right| = |A||B| \sin\theta$$

If **A**⊥**B** then **A**×**B**=max

If **A**||**B** then **A**×**B**=0

Two vectors A and B define a plane. Vector C is perpendicular to plane according to right-hand rule.

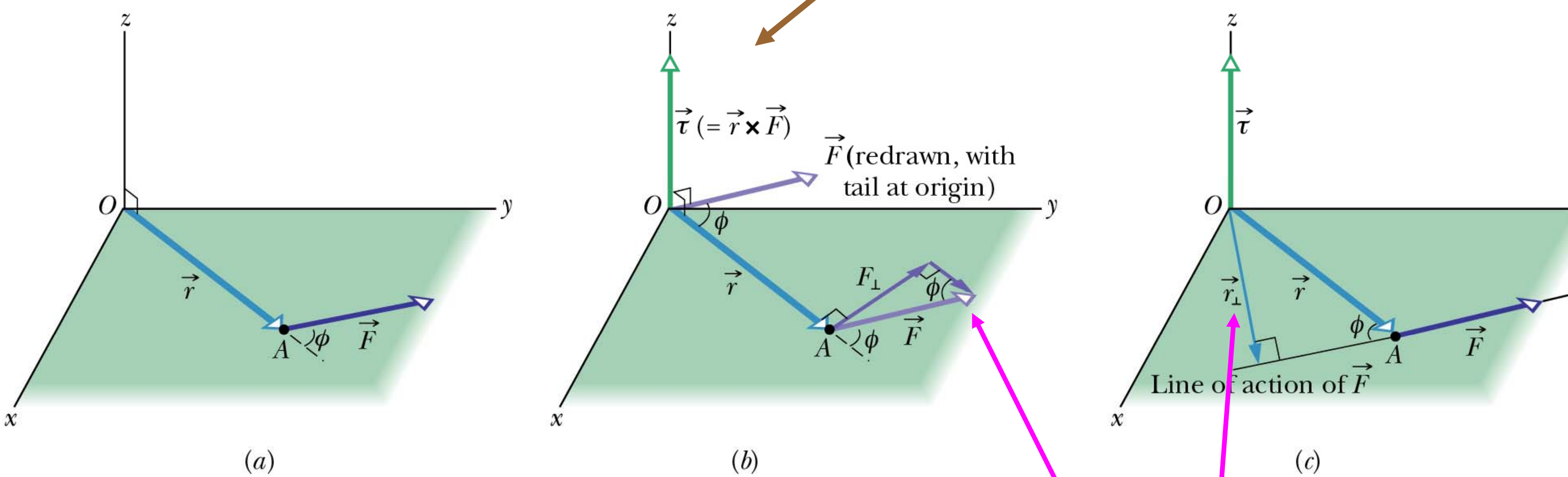
Rotational dynamics - *Torque*

vector = magnitude + direction

$$\vec{\tau} = \vec{r} \times \vec{F}$$

$$|\vec{\tau}| = |\vec{r}||\vec{F}|\sin\vartheta$$

vector $\vec{\tau}$ is always perpendicular to both vectors $\vec{r}$ and $\vec{F}$



If F_t is tangential component of force

$$|\vec{\tau}| = |\vec{r}|\left(|\vec{F}|\sin\vartheta\right) = rF_t$$

If $r_\perp$ is moment arm

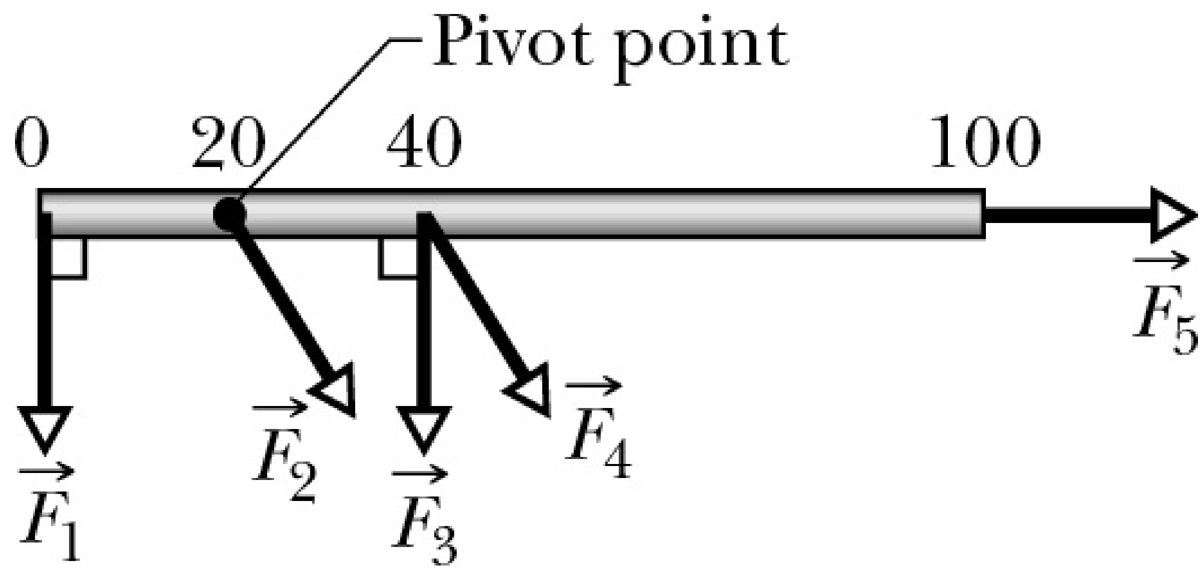
$$= \left(|\vec{r}|\sin\vartheta\right)|\vec{F}| = r_\perp F$$

Units of torque: N·m (not Joule)

Checkpoint

The figure shows an overhead view of a meter stick that can pivot about he dot at the position marked 20 (for 20 cm). All five horizontal forces on the stick have the same magnitude. Rank those forces according to the magnitude of the torque that they produce, greatest first.

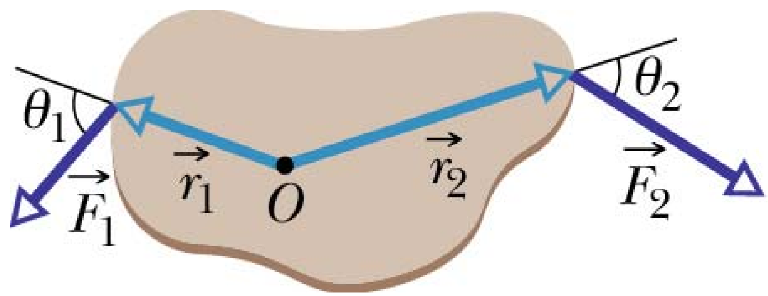
$$| \tau | = r F \sin\theta$$



$$|\tau_1| = |\tau_3| > |\tau_4| > |\tau_2| = |\tau_5| = 0$$

Problem 10-47

A body is pivoted at O and two forces act on it as shown. Find an expression for the net torque on the body about the pivot



What is vector equation for net torque?

$$\vec{\tau}_{net} = \sum \vec{\tau}_i = \sum \vec{r}_i \times \vec{F}_i = \vec{r}_1 \times \vec{F}_1 + \vec{r}_2 \times \vec{F}_2$$

→ What is magnitude of torque?

$$\tau = |\vec{\tau}| = rF \sin\theta$$

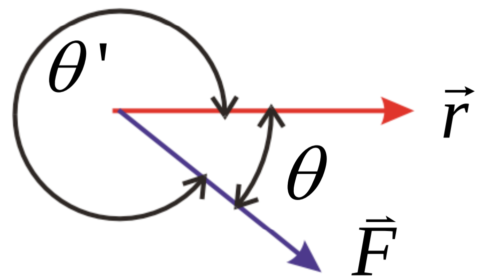
Thus expression is:

$$\vec{\tau}_{net} = [r_1 F_1 \sin\theta_1 - r_2 F_2 \sin\theta_2]\hat{k} \quad (out-of-page)$$

Note: When using the right-hand rule …

Use angle between F and r which is less than 180°

$$\tau = rF\sin\theta$$



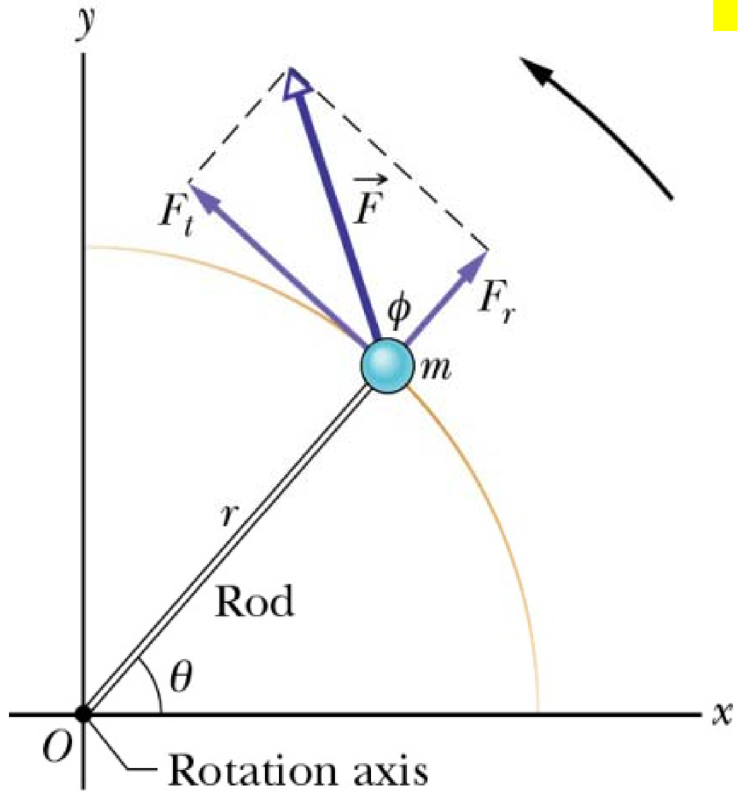
Which angle?

The one less than 180° → θ

Rotational dynamics - *Newton's 2nd Law*

Force about a point causes torque

$$\vec{\tau} = \vec{r} \times \vec{F}$$



Torque causes rotation

$$\vec{\tau}_{net} = \sum \vec{\tau}_i = I\vec{\alpha}$$

$$\left\{\vec{F}_{net} = \sum \vec{F}_i = m\vec{a}\right\}$$

- **Torque is <u>positive</u> when the force tends to produce a <u>counterclockwise</u> rotation about an axis is <u>negative</u> when the force tends to produce a <u>clockwise</u> rotation about an axis**

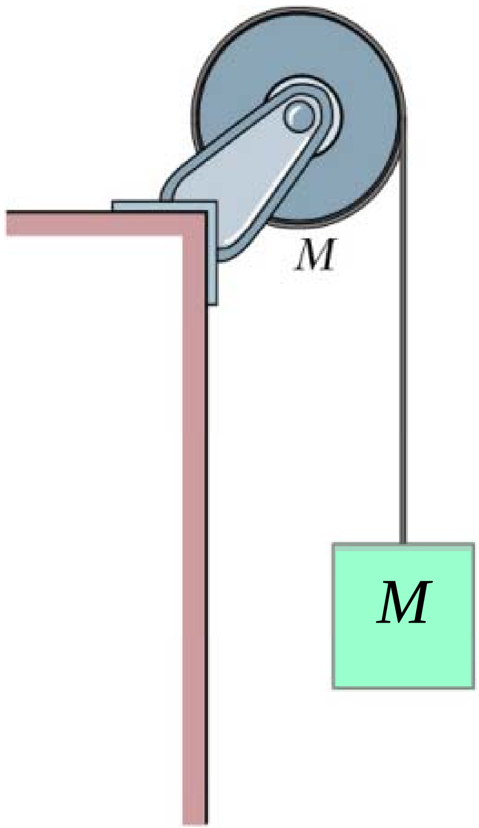
  (→ VECTOR! positive if pointing in +z, negative if pointing in -z)

- **Net torque and angular acceleration are parallel**

Example #1 **A uniform disk, with mass M and radius R, is mounted on a fixed horizontal axle. A block of mass M hangs from a massless cord that is wrapped around the rim of the disk.**

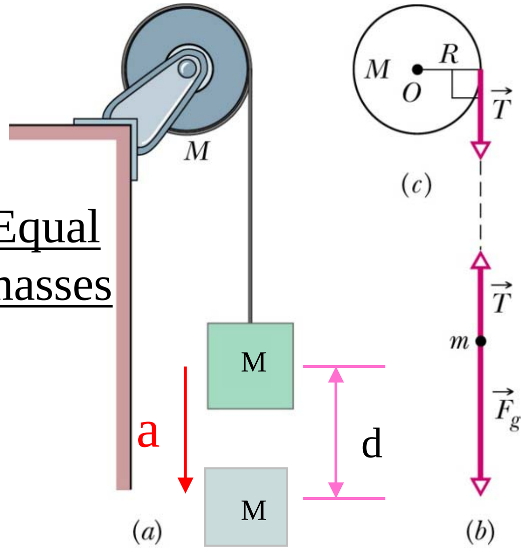
a) **Find the acceleration of the disk, and the tension in the cord.**

b) **b) Initially at rest, what is the speed of the block after dropping a distance d?**



Example #1

Equal masses



A wheel (mass **M**, radius **R,** and **I =** $\mathbf{\tfrac{1}{2}MR^2}$) is attached to block with equal mass **M** (i.e. $M_{disk}=M_{block}$). Initially at rest, what is the speed of the block after dropping a distance **d?**

Using conservation of Energy

$$\Delta KE_{tot} = -\Delta U$$

$$\left(\tfrac{1}{2}Mv^2 + \tfrac{1}{2}I\omega^2\right)_{final} - (0+0)_{init} = -\left[(0)_{final} - (Mgd)_{init}\right]$$

Substituting $v = \omega R \;\; \& \; I = \tfrac{1}{2}MR^2$

$$\tfrac{1}{2}Mv^2 + \tfrac{1}{2}\left(\tfrac{1}{2}MR^2\right)\left(\frac{v}{R}\right)^2 = Mgd$$

$$v^2 + \left(\tfrac{1}{2}\right)(v)^2 = 2gd$$

$$v^2\left(1+\tfrac{1}{2}\right) = \tfrac{3}{2}v^2 = 2gd$$

$$|v| = \sqrt{\tfrac{4}{3}gd}$$

Using force + torque

$$\hat{y}: \; T - Mg = -Ma$$

$$\Rightarrow T = M(g-a)$$

$$\hat{z}: \; \vec{\tau}_{net} = \vec{r}\times\vec{F}$$

$$= (RT)(-\hat{z})$$

$$\vec{\tau} = I\vec{\alpha}$$

$$(RT)(-\hat{z}) = (I\alpha)(-\hat{z})$$

$$a_t = \alpha R \;\; \&$$

$$I = \tfrac{1}{2}MR^2 \;\Rightarrow\; R\left[M(g-a)\right] = \left(\tfrac{1}{2}MR^2\right)\left(\frac{a}{R}\right)$$

$$a\left(\tfrac{1}{2}M + M\right) = Mg$$

$$a = \frac{Mg}{\tfrac{3}{2}M} = \frac{2}{3}g$$

Now use 1-D kinematics

$$v^2 = v_0^2 + 2ad$$

$$v^2 = 0 + 2d\left(\tfrac{2}{3}g\right)$$

$$|v| = \sqrt{\tfrac{4}{3}gd}$$

Newton's 2nd law for rotation

$$\vec{\tau}_{net} = \sum \vec{\tau}_i = I\vec{\alpha}$$

$$\left\{ \vec{F}_{net} = \sum \vec{F}_i = m\vec{a} \right\}$$

Work and Rotational Kinetic Energy

NET Work done ON system

$$W = \Delta KE_{rot} = \tfrac{1}{2} I\left(\omega_f^2 - \omega_i^2\right)$$

$$\left\{ W = \Delta KE_{trans} = \tfrac{1}{2} m\left(v_f^2 - v_i^2\right) \right\}$$

Rotational work, fixed axis rotation

$$W = \int_{\theta_1}^{\theta_2} \tau_{net}\, d\theta$$

$$\left\{ W = \int_{x_1}^{x_2} F\,dx \quad 1-D\ motion \right\}$$

(if torque is const)

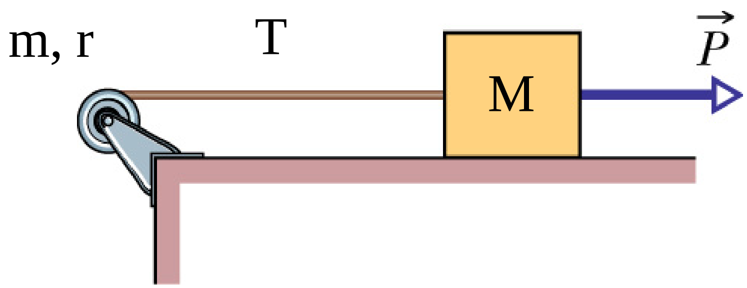
$$= \tau\left(\theta_f - \theta_i\right)$$

Power, fixed axis rotation

$$P = \frac{dW}{dt} = \frac{d}{dt}(\tau\theta) = \tau\omega$$

$$\left\{ P = \frac{dW}{dt} = \frac{d}{dt}\left(\vec{F} \bullet \vec{x}\right) = \vec{F} \bullet \vec{v} \right\}$$

Block and Pulley Accelerating



Probelme 10-54: A disk wheel of radius *r* (m) and mass *m* (kg) is mounted on a frictionless horizontal axis. The rotational inertia of the wheel is *I* kg/m². A massless cord wrapped around the wheel is attached to a M kg block that slides on a horizontal frictionless surface. If a horizontal force of magnitude P is applied, what is the magnitude of the angular acceleration of the wheel? Assume that the string does not slip on the wheel.

Force about a point causes torque

$$\tau = F\ell$$

Torque causes rotation

$$\vec{\tau}_{net} = \sum \vec{\tau}_i = I\vec{\alpha}$$

Trans - rotational relationship

$$a_T = r\alpha$$

$$P - T = Ma_T$$

$$Tr = \tau = I\alpha = \frac{Ia_T}{r} \quad \text{or} \quad T = \frac{Ia_T}{r^2}$$

For a disc $I = \frac{mr^2}{2}$

$$P - \frac{Ia_T}{r^2} = Ma_T$$

$$a_T = \frac{P}{M + \frac{I}{r^2}}$$

$$a_T = \frac{P}{M + \frac{m}{2}}$$

Check M =0, m=0