Physics 2101
Section 3
March 5rd : Ch. 10

Announcements:

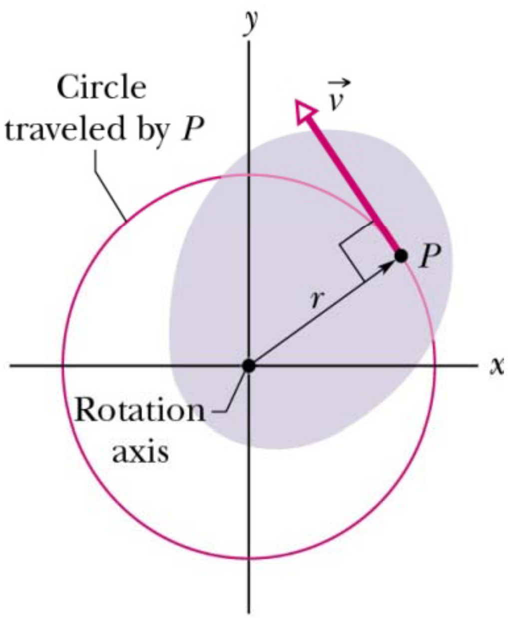
- Grades for Exam #2 posted next week.
- Today Ch. 10.7-10, 11.2-3
- Next Quiz is March 12 on SHW#6

Class Website:

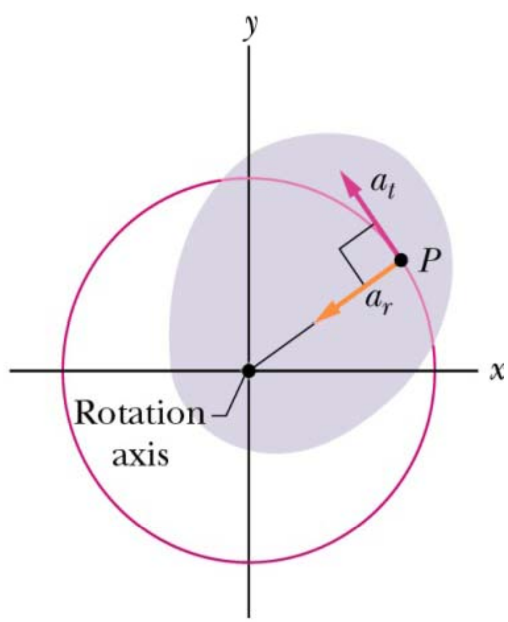
http://www.phys.lsu.edu/classes/spring2010/phys2101-3/

http://www.phys.lsu.edu/~jzhang/teaching.html

Summary: Translational and Rotational Variables



(a)



(b)

Rotational position and distance moved

$$s = |\theta| r \qquad \text{(only radian units)}$$

Vectors!

Rotational and translational speed

$$|\vec{v}| = \frac{|d\vec{r}|}{dt} = \frac{ds}{dt} = \frac{d|\theta|}{dt} r$$

$$v = |\omega| r$$

$$\vec{v} = \vec{\omega} \times \vec{r}$$

Rotational and translational acceleration

$$a_t = \frac{d|\omega|}{dt} r = |\alpha| r$$ **tangential acceleration**

$$\vec{a}_t = \vec{\alpha} \times \vec{r}$$

$$a_r = \frac{v^2}{r} = |\omega|^2 r$$ **radial/centripetal acceleration**

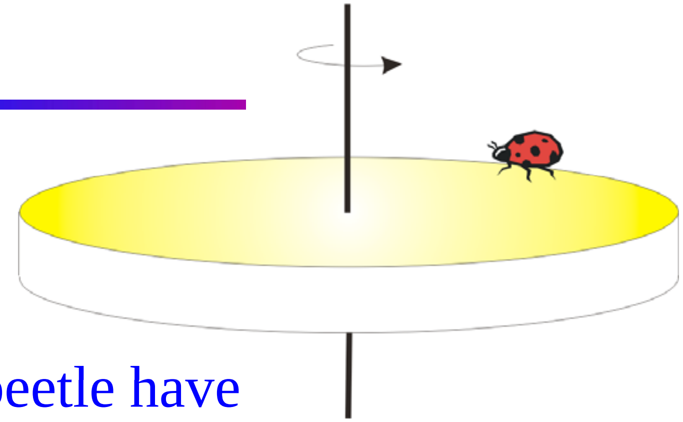
$$\vec{a}_r = \vec{\omega} \times (\vec{\omega} \times \vec{r})$$

$$|\vec{a}_{tot}|^2 = |\vec{a}_r|^2 + |\vec{a}_t|^2$$

$$= |\omega^2 r|^2 + |\alpha r|^2$$

$$\vec{a}_{tot} = \vec{a}_t + \vec{a}_r$$

Example: Beetle on a merry-go-round



A beetle rides the rim of a rotating merry-go-round.

If the angular speed of the system is constant, does the beetle have a) radial acceleration and b) tangential acceleration?

$$If \quad \omega = \text{constant}, \quad \alpha = 0 \qquad |\vec{a}_{tot}|^2 = |\vec{a}_r|^2 + |\vec{a}_t|^2$$

$$= |\omega^2 r|^2 + |0|^2 \quad \Rightarrow \quad \vec{a}_{tot} = \omega^2 r(-\hat{r})$$

only radial

If the angular speed is decreasing at a constant rate, does the beetle have a) radial acceleration and b) tangential acceleration?

$$If \quad \alpha = negative \text{ but constant},$$

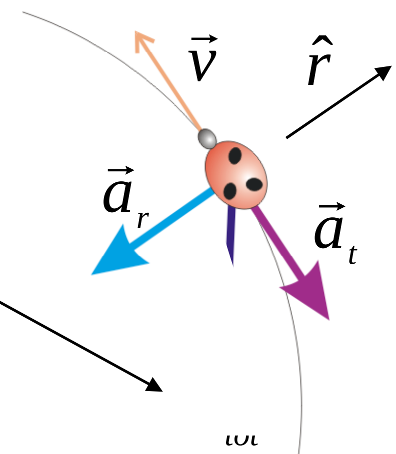
$$\omega = \omega_0 + \alpha t$$

$$|\vec{a}_{tot}|^2 = |\vec{a}_r|^2 + |\vec{a}_t|^2$$

$$= |\omega^2 r|^2 + |\alpha r|^2$$

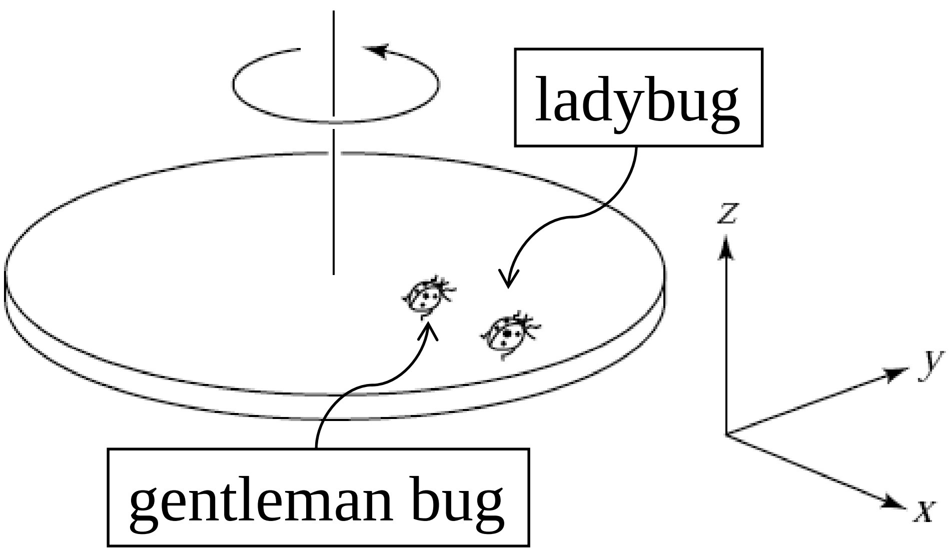
$$|a_{tot}| = \sqrt{((\omega_0 + \alpha t)^2 r)^2 + (\alpha r)^2}$$

$$\vec{a}_{tot} = \vec{a}_t + \vec{a}_r$$



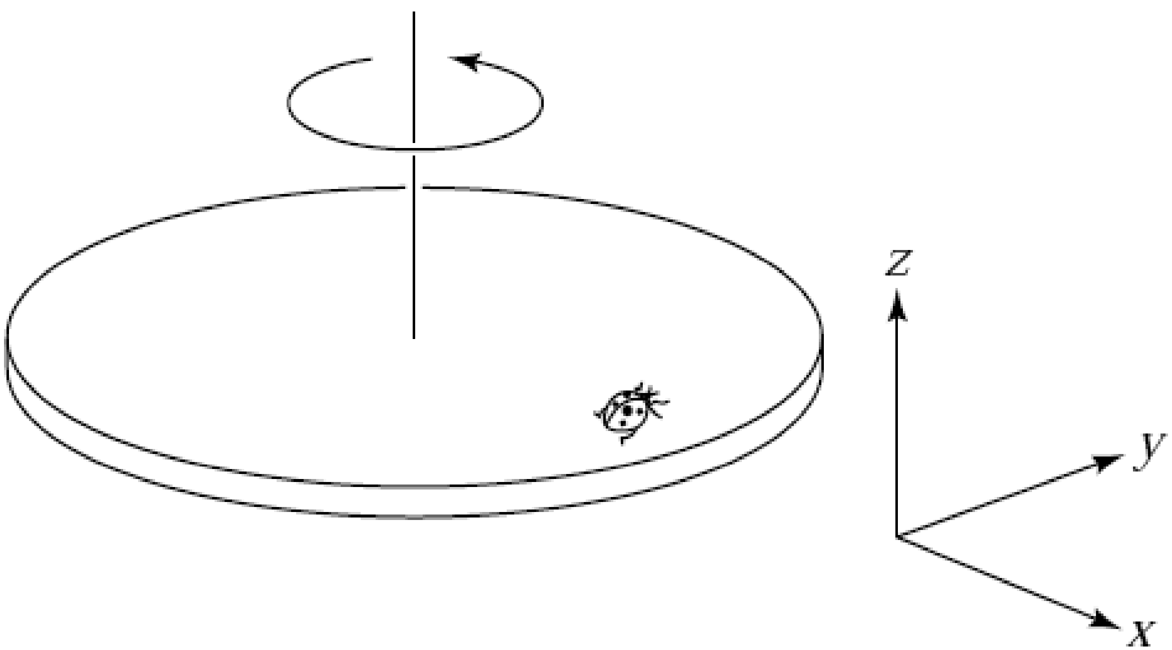
both radial and tangential

A ladybug sits at the outer edge of a merry-go-round, and a gentleman bug sits halfway between her and the axis of rotation. The merry-go-round makes a complete revolution once each second. The gentleman bug's angular speed is



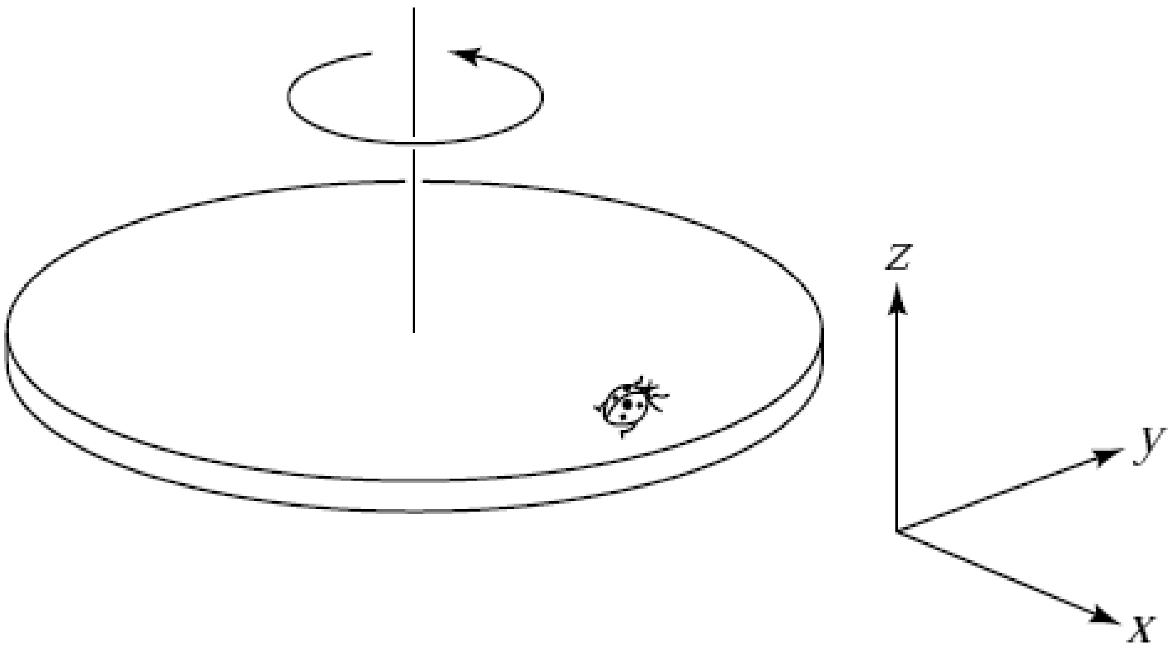
1. half the ladybug's.
2. the same as the ladybug's.
3. twice the ladybug's.
4. impossible to determine

A ladybug sits at the outer edge of a merry-go-round, that is turning and speeding up. At the instant shown in the figure, the tangential component of the ladybug's (Cartesian) acceleration is:



1. In the +x direction
2. In the - x direction
3. In the +y direction
4. In the - y direction
5. In the +z direction
6. In the - z direction
7. zero

A ladybug sits at the outer edge of a merry-go-round that is turning and is slowing down. The vector expressing her angular velocity is

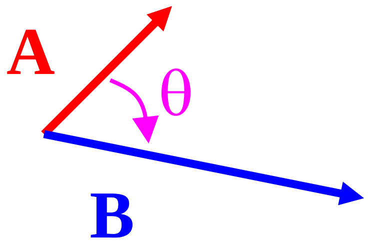


1. In the +x direction
2. In the - x direction
3. In the +y direction
4. In the - y direction
5. In the +z direction
6. In the - z direction
7. zero

Sec. 3.7 Multiplying vectors:

See page A-10 in back of book

1) **Dot Product or Scalar Product** (Vector · Vector = scalar)



$$\vec{A} \bullet \vec{B} = |A||B|\cos\theta$$
$$= A_x B_x + A_y B_y + A_z B_z$$

If A⊥B then A•B=0

If A||B then A • B=max

2) **Cross Product** (Vector × Vector = Vector)

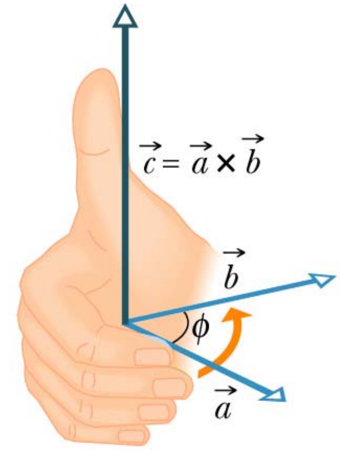
$$\vec{C} = \vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

$$|\vec{C}| = |A||B|\sin\theta$$

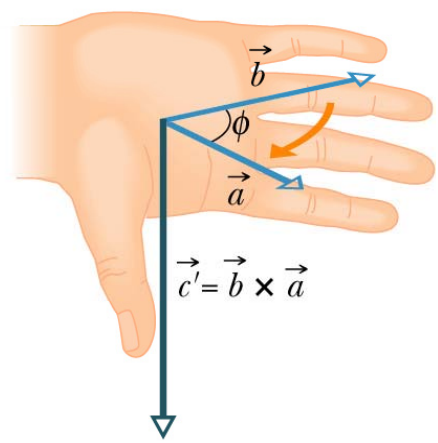
If **A**⊥**B** then **A**×**B**=max

If **A**||**B** then **A**×**B**=0

Two vectors A and B define a plane. Vector C is perpendicular to plane according to right-hand rule.



(a)



(b)

Kinetic Energy of Rotation

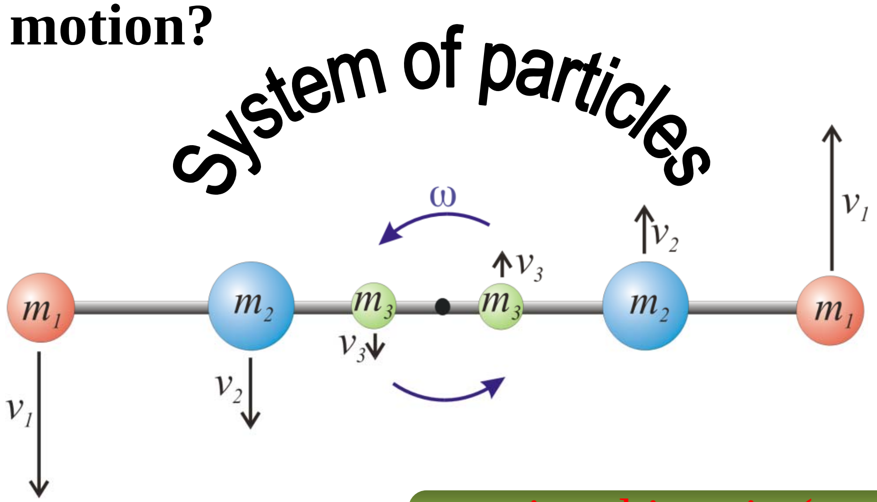
TRANSLATION Review:

Newton's 2nd Law ⇒ $\vec{F}_{net} \propto \vec{a}$ — proportionality is <u>inertia</u>, m ⇒ constant of an object

energy associated with state of translational motion — $KE_{trans} \propto v^2$ — "motion" of particles with same v

→ mass is translational inertia ←

What about rotation? What is energy associated with state of rotational motion?



$$KE_{system} = \sum \tfrac{1}{2} m_i v_i^2 \quad where \quad v_i = \omega r_i$$

trans→ rot

$$= \sum \tfrac{1}{2} m_i (\omega r_i)^2$$

$$= \tfrac{1}{2} \left[\sum \left(m_i r_i^2\right)\right] \omega^2$$

All particles have same ω

rotational inertia (moment of inertia) about some axis of rotation $\equiv I$

Energy of rotational motion $KE_{rot} = \tfrac{1}{2} I \omega^2$ $\left[KE_{trans} = \tfrac{1}{2} mv^2\right]$

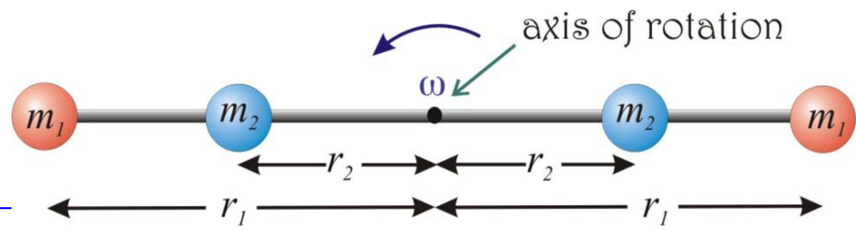
Moment of Inertia

For a discrete number of particles distributed about an axis of rotation

$$I \equiv \sum_{\text{all mass}} m_i r_i^2$$

units of kg·m²

Simple example:



$$I = \sum_{i=1}^{4} m_i r_i^2 = m_1 r_1^2 + m_2 r_2^2 + m_2 r_2^2 + m_1 r_1^2$$

$$= 2m_1 r_1^2 + 2m_2 r_2^2$$

what about other axis?

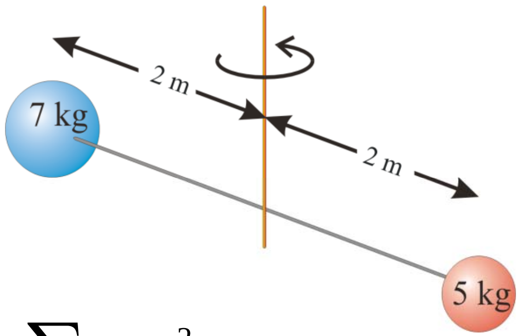
- Rotational inertia (moment of inertia) only valid about some axis of rotation.

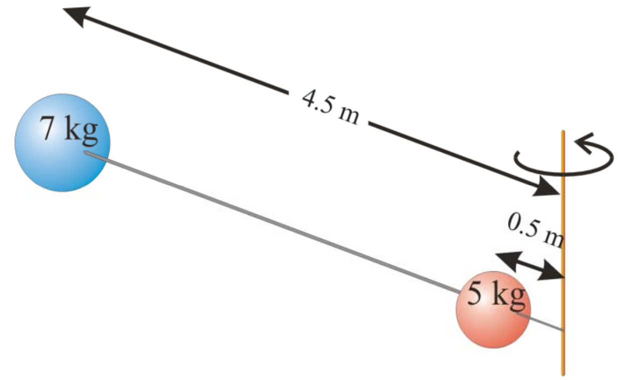
- For arbitrary shape, each different axis has a different moment of inertia.

- *I* relates how the mass of a rotating body is distributed about a given axis.

- *r* is perpendicular distance from mass to axis of rotation

Moment of inertia: *comparison*





$$I_1 = \sum m_i r_i^2$$
$$= (5 \cdot kg)(2 \cdot m)^2 + (7 \cdot kg)(2 \cdot m)^2$$
$$= 52 \cdot kgm^2$$

$$I_2 = (5 \cdot kg)(0.5 \cdot m)^2 + (7 \cdot kg)(4.5 \cdot m)^2$$
$$= (1.3 + 142) \cdot kgm^2 = 144 \cdot kgm^2$$

Note: 5 kg mass contributes <1% of total

- Mass close to axis of rotation contributes little to total moment of inertia.
- How does a larger moment of inertia “feel” ? $[F_{net} = \mathbf{m}a]$

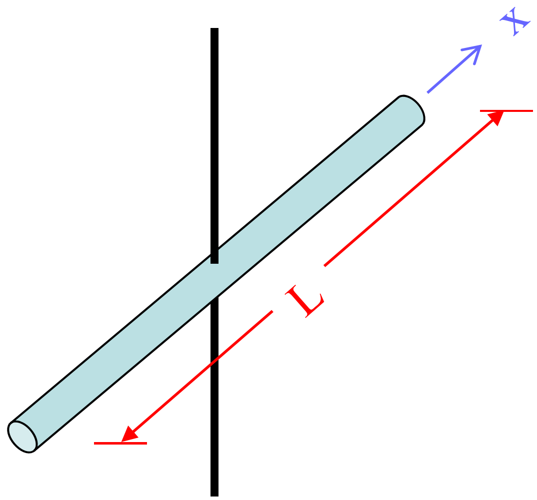
If rigid body = few particles $I = \sum m_i r_i^2$

If rigid body = too-many-to-count particles

Sum$\rightarrow$ Integral

Moment of inertia: *continuum mass*

$$I = \sum m_i r_i^2 \Rightarrow \int r^2 dm \qquad with\ \ \rho = \frac{m}{V} \qquad \Rightarrow \qquad I = \rho \int r^2 dV$$



Example: moment of inertia of thin rod with perpendicular rotation through center

$$I = \int r^2 dm = \rho \int r^2 dV \quad where \quad dV = area \cdot dx = A \cdot dx$$

$$I = \rho A \int r^2 dx$$

$r^2 = x^2$ in this case

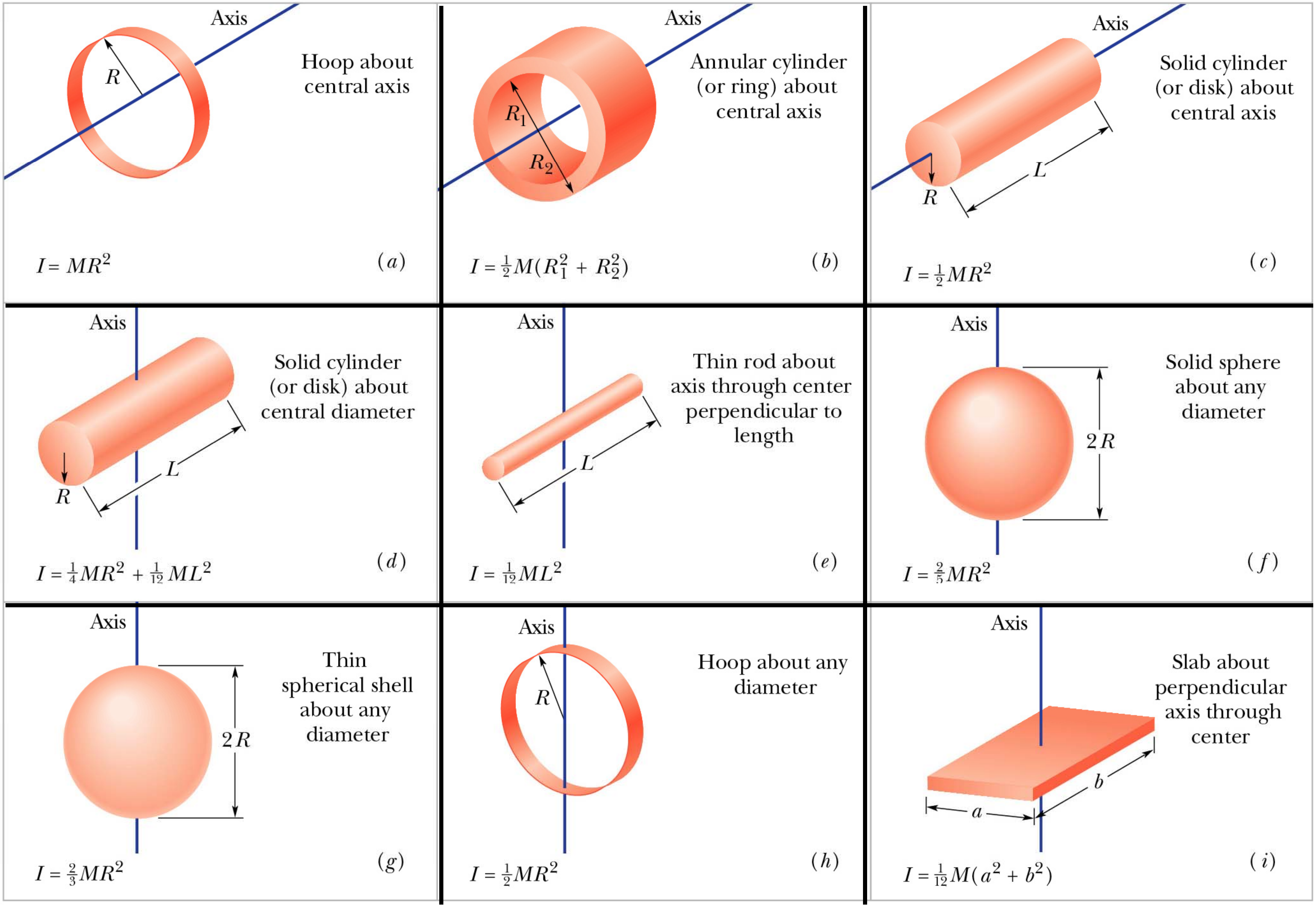
$-L/2 \rightarrow x \rightarrow +L/2$

$$I = \rho A \int_{-L/2}^{+L/2} x^2 dx \;= \rho A\left(\tfrac{1}{3}x^3\right)\Big|_{-L/2}^{+L/2} \;= \rho A\left(\tfrac{1}{3}\left(\tfrac{1}{8}L^3\right) - \tfrac{1}{3}\left(-\tfrac{1}{8}L^3\right)\right)$$

$$I = \left(\frac{m}{V}\right) A \left(\frac{L^3}{12}\right) = \frac{1}{12}\left(\frac{m \cdot A}{A \cdot L}\right)\left(L^3\right) \quad = \frac{1}{12} mL^2$$

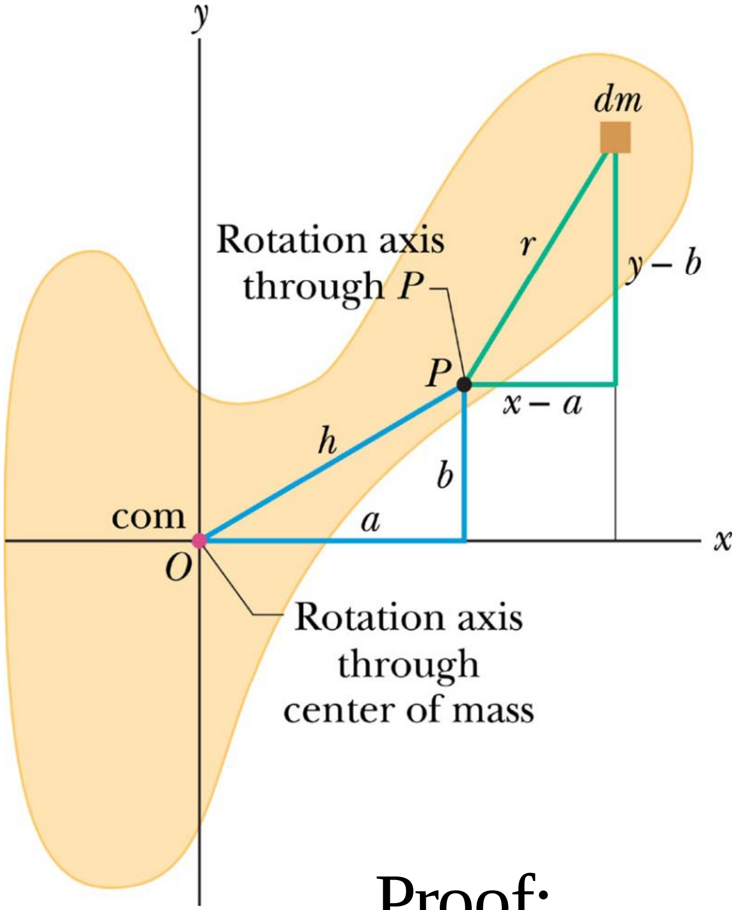
INDEPENDENT
of cross section area

Some Rotational Inertias



Each of these rotational inertias GO THROUGH the center of mass !

Parallel-Axis Theorem



$$I = I_{COM} + Mh^2$$

Proof:

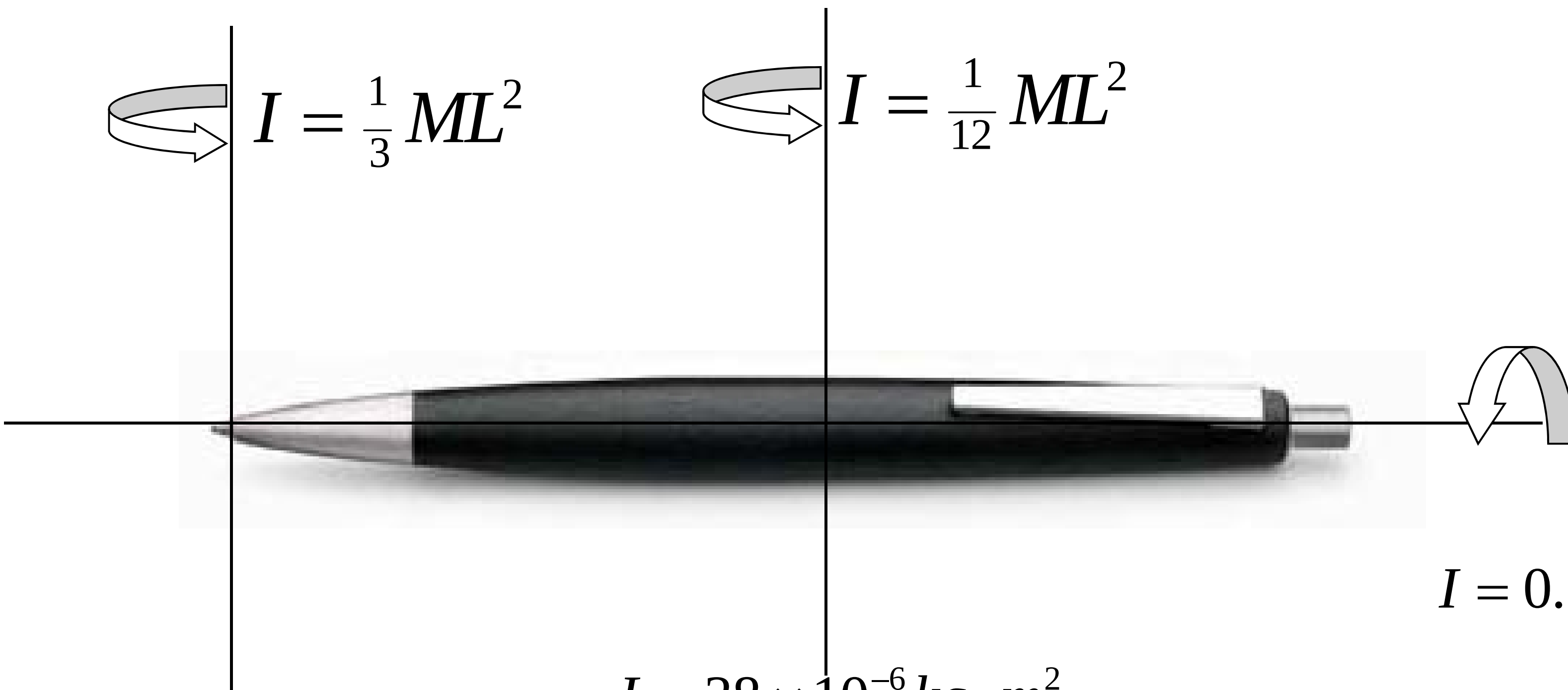
$$I = \int r^2\, dm = \int \{(x-a)^2 + (y-b)^2\}\, dm$$

$$I = \int (x^2 + y^2)\, dm - 2a\int x\, dm - 2b\int y\, dm + \int (a^2 + b^2)\, dm$$

$$I = I_{COM} + 0 + 0 + h^2 M$$

Moment of inertia of a Pencil

It depends on where the rotation axis is considered…



$$I = \frac{1}{3}ML^2$$

$$I = \frac{1}{12}ML^2$$

$$I = \frac{1}{2}MR^2$$

$$I = 0.3 \times 10^{-6}\, kg \cdot m^2$$

$$I = 150 \times 10^{-6}\, kg \cdot m^2$$

$$I = 38 \times 10^{-6}\, kg \cdot m^2$$

Consider a 20g pencil 15cm long and 1cm wide …

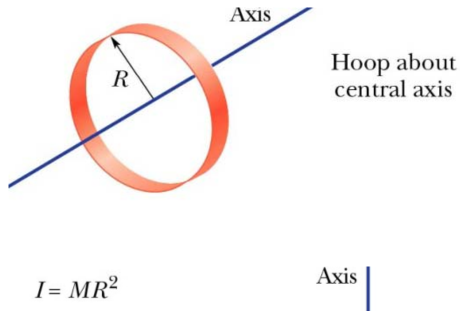
… somewhat like MASS, you can feel the difference in the rotational inertia

Example #1

A bicycle wheel has a radius of 0.33m and a rim of mass 1.2 kg. The wheel has 50 spokes, each with a mass 10g.
What is the moment of inertial about axis of rotation?

What is moment of inertia about COM?

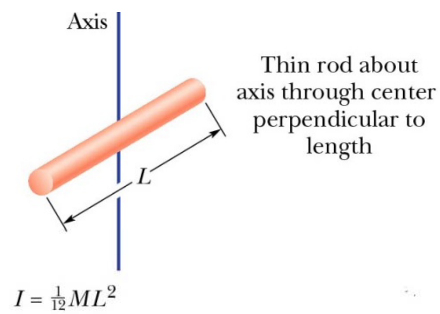
$$I_{tot,com} = I_{rim,center} + 50 I_{spoke}$$



→ What is I_{spoke} (parallel-axis)?

$$I_{spoke} = I_{rod.com} + Mh^2 = \tfrac{1}{12} ML^2 + M\left(\tfrac{1}{2} L\right)^2 = \tfrac{1}{3} ML^2$$

$$= \tfrac{1}{3}(0.01kg)(0.33m)^2 \cong 3.6 \times 10^{-4}\, kg \cdot m^2$$



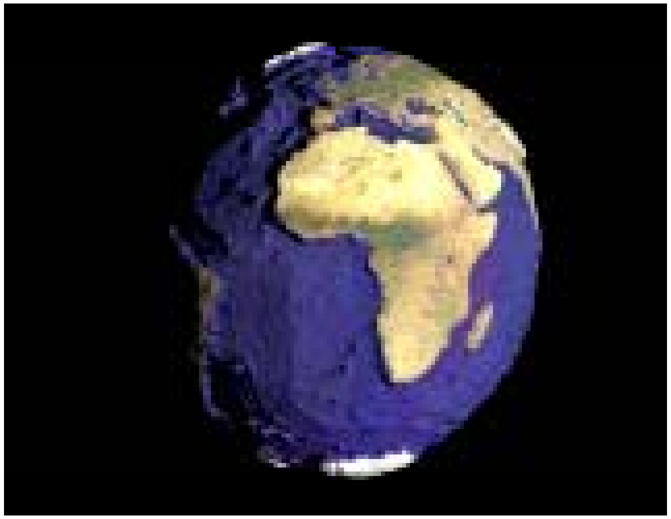
→ Putting together

$$I_{tot,com} = I_{rim,center} + 50 I_{spoke}$$

$$= M_{wheel} R^2 + 50 I_{spoke}$$

$$= (1.2kg)(0.33m)^2 + 50(3.6 \times 10^{-4}\, kg \cdot m^2) = 0.149 kg \cdot m^2$$

Example #2

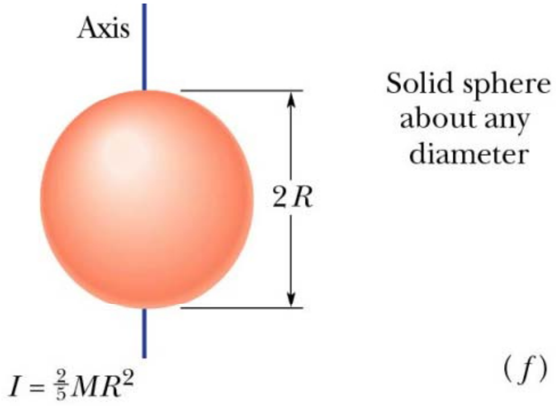


How much work did Superman exert on earth in order to stop it?

What is the kinetic energy of the earth's rotation about its axis?

Energy of rotational motion is found from:

$$KE_{rot} = \tfrac{1}{2} I\omega^2$$



What is earth's moment of inertia, I?

$$I_{earth} = I_{sphere} = \tfrac{2}{5} Mr^2$$

$$= \tfrac{2}{5}\left(6\times10^{24}\,kg\right)\left(6.4\times10^{6}\,m\right)^2 \cong 1\times10^{38}\,kg\cdot m^2$$

What is earth's angular velocity, ω?

$$From\;\; T = 2\pi/\omega \;\;\Rightarrow\;\; \omega = 2\pi\,\text{radians}/\text{day} = 2\pi/(3600\times24) = 7.3\times10^{-5}\,rad/s$$

Now plug-'n-chug:

$$KE_{rot} = \tfrac{1}{2}\left(1\times10^{38}\,kg\cdot m^2\right)\left(7.3\times10^{-5}\,rad/s\right)^2 = \underline{2.6\times10^{29}\,J}$$