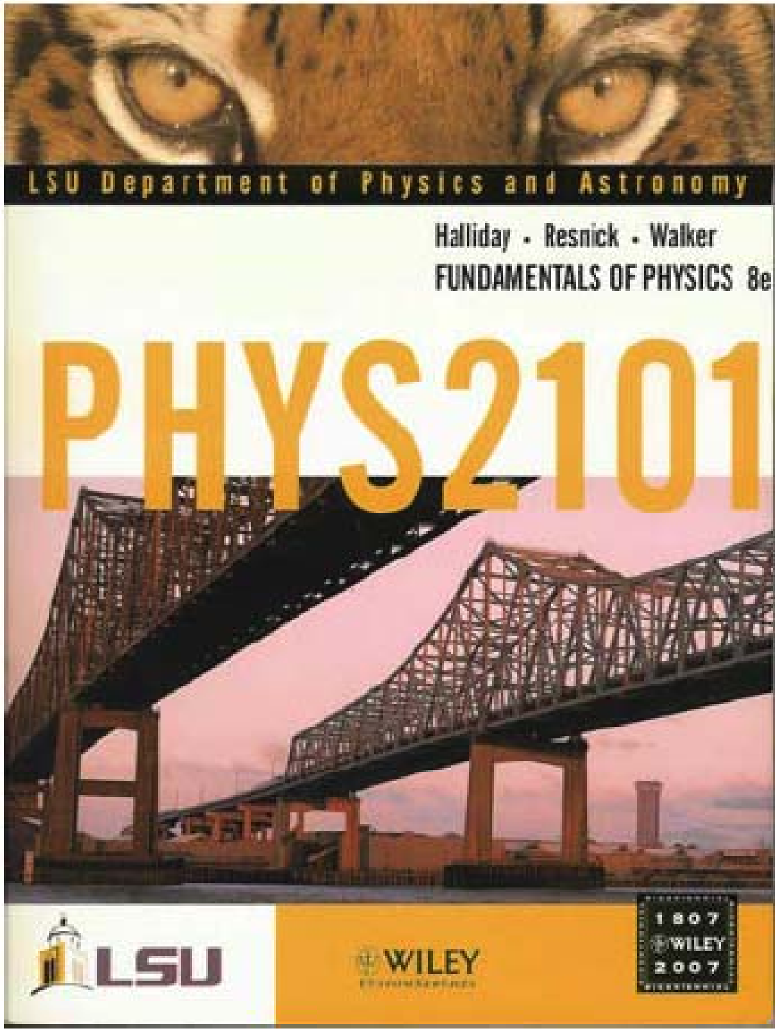


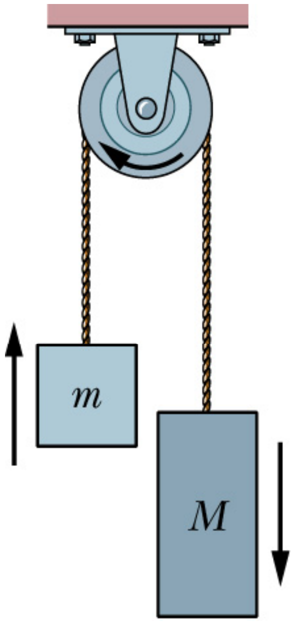
Physics 2101
Section 3
March 3rd : Ch. 10

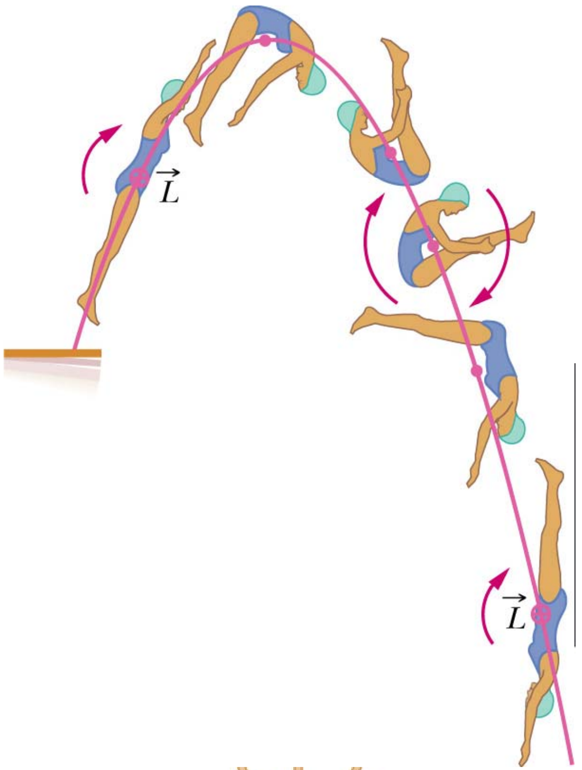
Announcements:

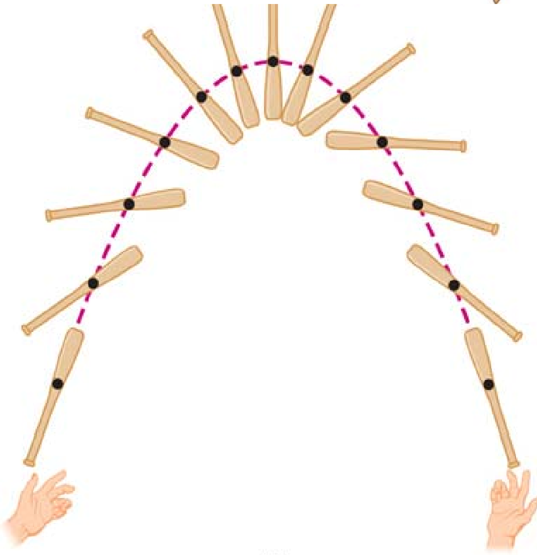
- Monday's Review Posted (in Plummer's section (4)
- Today start Ch. 10.
- Next Quiz will be next week

Test#2 (Ch. 7-9) will be at 6 PM, March 3, Lockett-6

Chap. 10: Rotation






(b)

Conservation of Angular Momentum

Describe their motion:

$$\theta(t) \to \omega(t) \to \alpha(t)$$

$$I \to \frac{I\omega^2}{2} \to Torque$$

Rotation

In this chapter we will study the rotational motion of rigid bodies about a fixed axis. To describe this type of motion we will introduce the following new concepts:

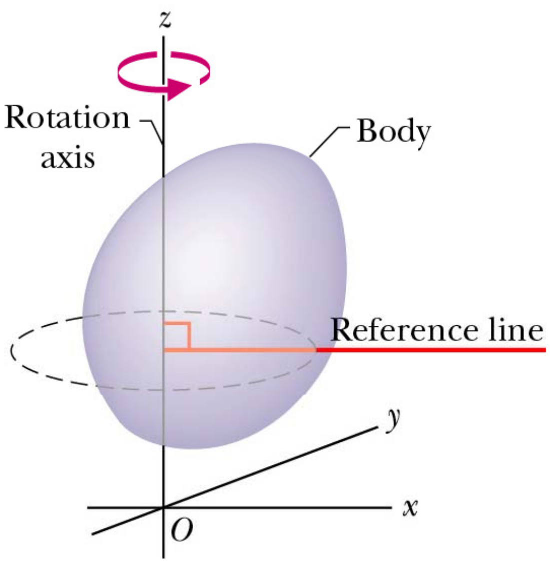
-Angular displacement $\theta = s/r$
- Average and instantaneous angular velocity (symbol: $\boldsymbol{\omega}$)
- Average and instantaneous angular acceleration (symbol: $\boldsymbol{\alpha}$)
- Rotational inertia, also known as moment of inertia (symbol **I**)
- Torque (symbol $\boldsymbol{\tau}$)

We will also calculate the kinetic energy associated with rotation, write Newton's second law for rotational motion, and introduce the work-kinetic energy theorem for rotational motion.

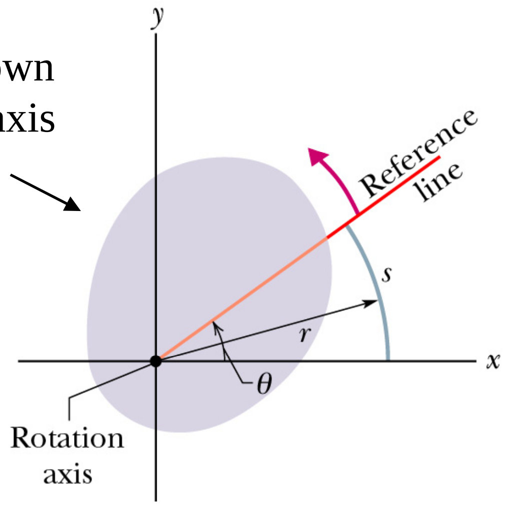
Rotation Variables

Here we assume (for now):

- Rigid body - all parts are locked together (e.g. not a cloud)
- Axis of rotation - a line about which the body rotates (fixed axis)
- Reference line - rotates perpendicular to rotation axis



looking down Rotation axis



$$\theta(t) = \frac{s}{r} = \frac{\text{arc length}}{\text{radius}}$$

$$\Delta\theta = \theta_2 - \theta_1$$

$$\omega(t) = \frac{d\theta(t)}{dt}$$

$$\alpha(t) = \frac{d\omega(t)}{dt} = \frac{d^2\theta(t)}{dt^2}$$

Angular position (displacement) of line at time t: $\theta(t)$ $(\Delta\theta(t))$ (measured from $+\hat{x}$: ccw)

Angle is in units of radians (rad) : $1\ rev = 360^{\circ} = \dfrac{2\pi r}{r} = 2\pi\ rad$

Angular velocity of line at time t: $\omega(t)$ (positive if ccw)

All points move with same angular velocity Units of rad/sec

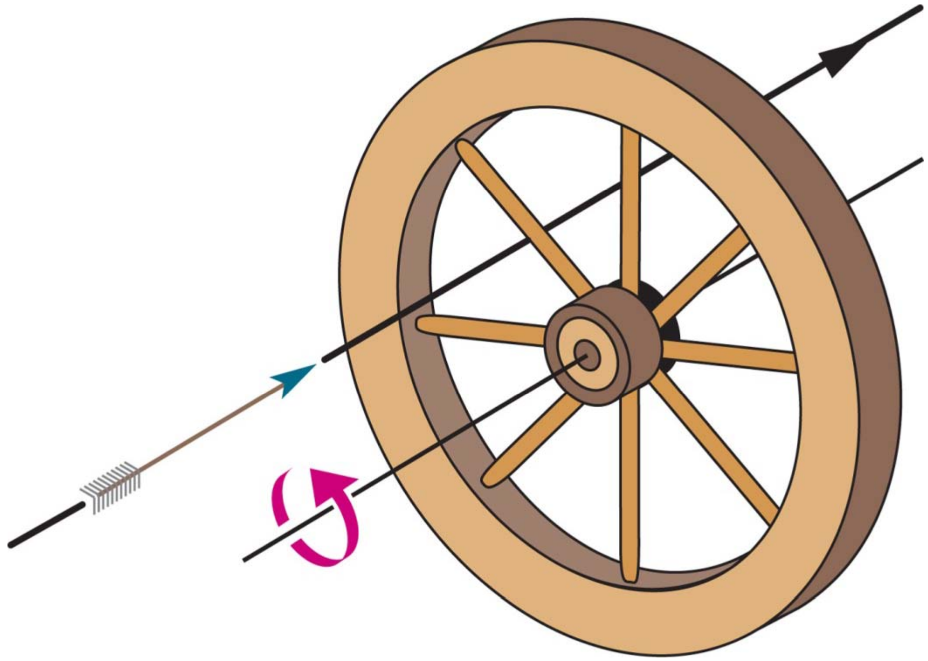
Angular acceleration of line at time t: $\alpha(t)$ (positive if ccw)

All points move with same angular acceleration Units of rad/sec^2

Analogies Between Translational and Rotational Motion

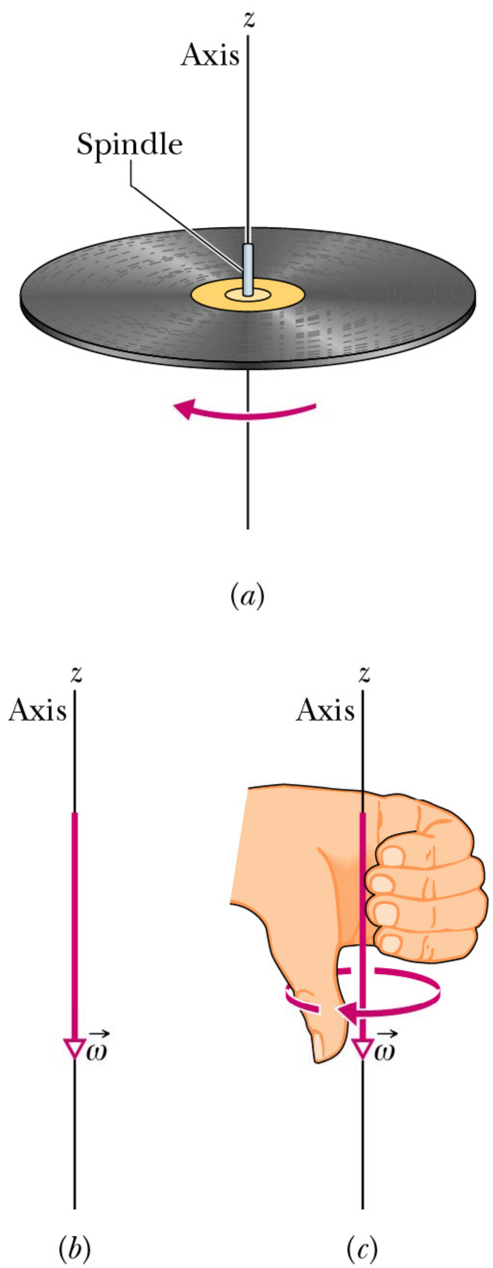
Translational Motion		Rotational Motion
x	$\leftrightarrow$	θ
v	$\leftrightarrow$	ω
a	$\leftrightarrow$	α
$v = v_0 + at$	$\leftrightarrow$	$\omega = \omega_0 + \alpha t$
$x = x_o + v_0 t + \frac{at^2}{2}$	$\leftrightarrow$	$\theta = \theta_0 + \omega_0 t + \frac{\alpha t^2}{2}$
$v^2 - v_0^2 = 2a\left(x - x_o\right)$	$\leftrightarrow$	$\omega^2 - \omega_0^2 = 2\alpha\left(\theta - \theta_0\right)$
$K = \frac{mv^2}{2}$	$\leftrightarrow$	$K = \frac{I\omega^2}{2}$
m	$\leftrightarrow$	I
$F = ma$	$\leftrightarrow$	$\tau = I\alpha$
F	$\leftrightarrow$	τ
$P = Fv$	$\leftrightarrow$	$P = \tau\omega$

Problem 10-8: The wheel in the picture has a radius of 30cm and is rotating at 2.5rev/sec. I want to shoot a 20 cm long arrow parallel to the axle without hitting an spokes. (a) What is the minimum speed? (b) Does it matter where between the axle and rim of the wheel you aim? If so what is the best position.



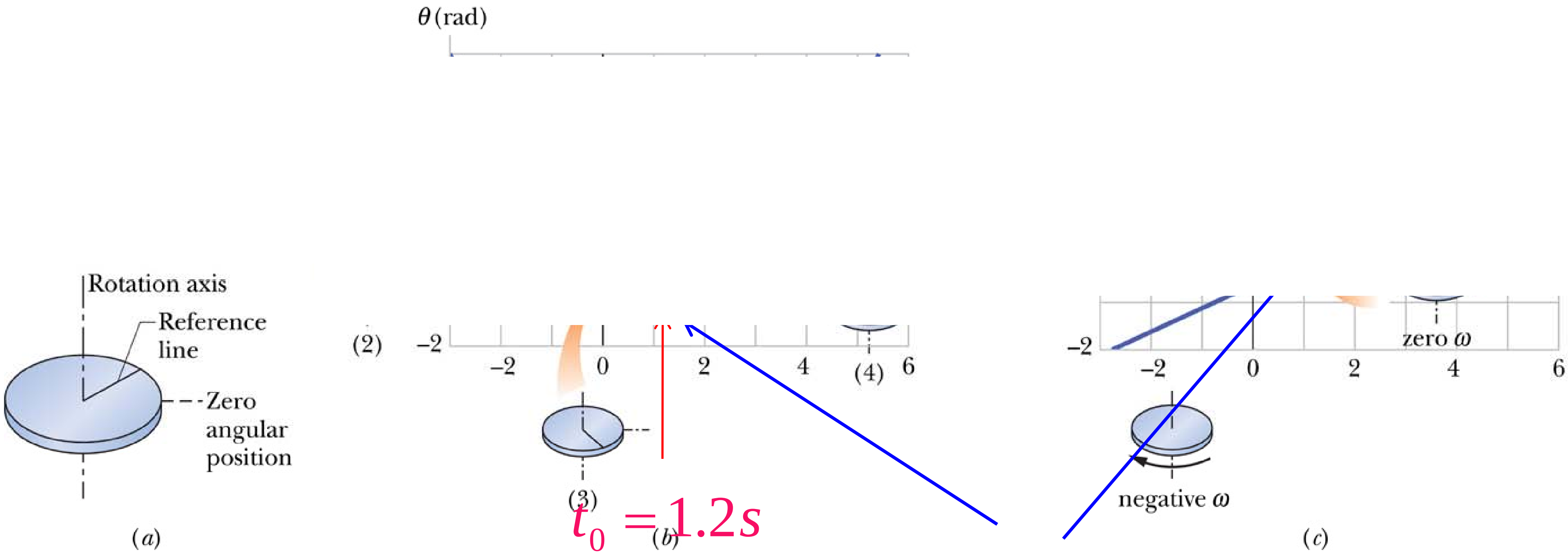
Are angular quantities Vectors?

Some! Once coordinate system is established for rotational motion, rotational quantities of velocity and acceleration are given by right-hand rule (angular displacement is NOT a vector)



Note: the movement is NOT along the direction of the vector. Instead the body is moving <u>around</u> the direction of the vector

Sample Problem 10-1: The disc in the figure below is rotating about an axis described by: $\theta(t) = -1.00 - 0.600t + 0.25t^2$



$t_0 = 1.2s$

$\omega = 0 rad/s$

Rewrite the equation

$$\theta(t) = -1.36 + 0.25(t - 1.2)^2$$

$$\theta(-2) = 1.2 rad$$

$$\theta(0) = -1.0 rad$$

$$\theta(1.2) = -1.36 rad$$

$$\omega(t) = \frac{d\theta(t)}{dt} = +0.50(t - 1.2) rad/s$$

$$\alpha(t) = \frac{d\omega(t)}{dt} = +0.50 rad/s^2$$

Rotation with Constant Rotational Acceleration

1-D translational motion

$$v = v_0 + at$$

$$x = x_0 + v_0 t + \tfrac{1}{2} a t^2$$

Rotational motion

$$\omega = \omega_0 + \alpha t$$

$$\theta = \theta_0 + \omega_0 t + \tfrac{1}{2} \alpha t^2$$

Problem: A disk, initially rotating at 120 rad/s, is slowed down with a constant acceleration of magnitude 4 rad/s^2.

a) How much time does the disk take to stop?

$$\omega = \omega_0 + \alpha t \quad \Rightarrow \quad t = \frac{(0 - 120 rad/s)}{-4 rad/s^2} = 30s$$

b) Through what angle does the disk rotate during that time?

$$\theta = \theta_0 + \omega_0 t + \tfrac{1}{2} \alpha t^2 \quad \Rightarrow \quad \Delta\theta = (120 rad/s)(30s) + \tfrac{1}{2}(-4 rad/s^2)(30s)^2$$

$$= 1800 rad = 286.5 rev$$

Equations for constant Angular Acceleration

Equations of Motion for Constant Linear Acceleration and for Constant Angular Acceleration

Equation Number	Linear Equation	Missing Variable		Angular Equation	Equation Number
(2-11)	$v = v_0 + at$	$x - x_0$	$\theta - \theta_0$	$\omega = \omega_0 + \alpha t$	(10-12)
(2-15)	$x - x_0 = v_0 t + \frac{1}{2}at^2$	v	ω	$\theta - \theta_0 = \omega_0 t + \frac{1}{2}\alpha t^2$	(10-13)
(2-16)	$v^2 = v_0^2 + 2a(x - x_0)$	t	t	$\omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0)$	(10-14)
(2-17)	$x - x_0 = \frac{1}{2}(v_0 + v)t$	a	α	$\theta - \theta_0 = \frac{1}{2}(\omega_0 + \omega)t$	(10-15)
(2-18)	$x - x_0 = vt - \frac{1}{2}at^2$	v_0	ω_0	$\theta - \theta_0 = \omega t - \frac{1}{2}\alpha t^2$	(10-16)

Remember these? Solve angular problems the same way.

Checkpoint 2: In four situations, a rotating body has angular position give by the following. To which situations do the angular equations for constant acceleration apply?

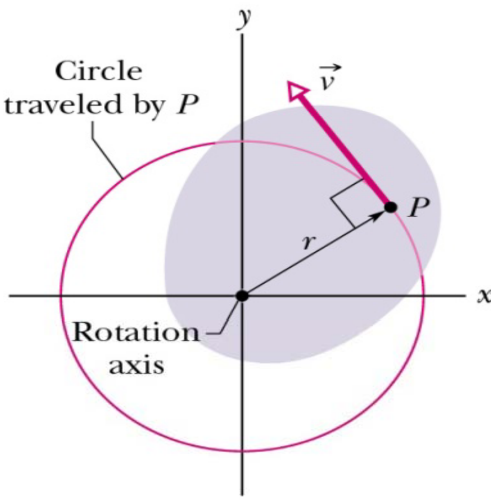
$$(a):\ \theta(t) = 3t - 4$$

$$(b):\ \theta(t) = -5t^3 + 4t^2 + 6$$

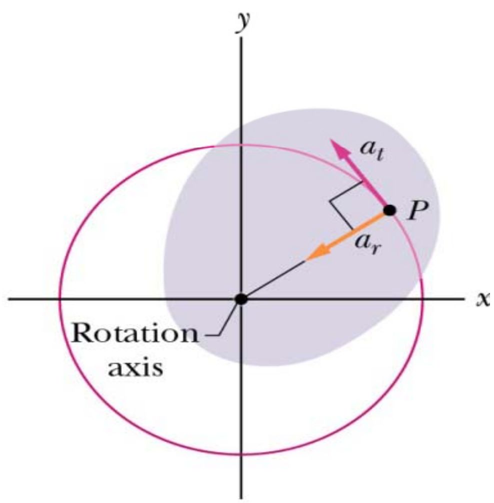
$$(c):\ \theta(t) = \frac{2}{t^2} - \frac{4}{t}$$

$$(d):\ \theta(t) = 5t^2 - 3$$

Relating Translational and Rotational Variables



(a)



(b)

Rotational position and distance moved

$$s = |\theta| r \qquad \text{(only radian units)}$$

Rotational and translational speed

$$|\vec{v}| = \frac{|d\vec{r}|}{dt} = \frac{ds}{dt} = \frac{d|\theta|}{dt} r$$

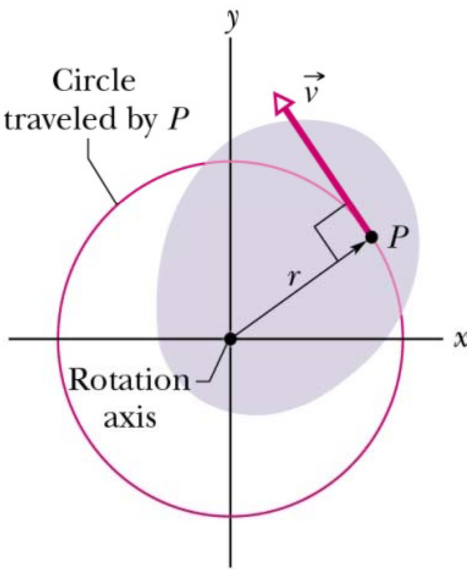
$$v = |\omega| r$$

Relating period and rotational speed [distance = rate × time]

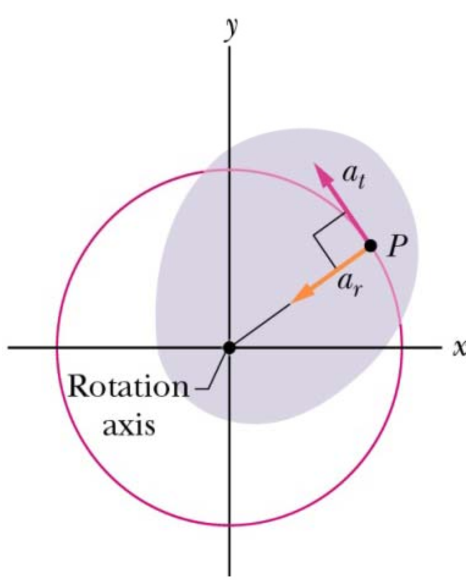
$$T = \frac{2\pi r}{v} = \frac{2\pi}{\omega} \qquad \text{units of time (s)}$$

$$\omega T = 2\pi$$

Relating Translational and Rotational Variables



(a)



(b)

“acceleration” is a little tricky

<u>Rotational and translational acceleration</u>

a) *from* $v = |\omega| r$

$$\frac{dv}{dt} = \frac{d|\omega|}{dt} r$$ from change in angular speed

$$a_t = \frac{d|\omega|}{dt} r = |\alpha| r$$ **tangential acceleration**

b) from before we know there’s also a “radial” component

$$a_r = \frac{v^2}{r} = |\omega|^2 r$$ **radial acceleration**

c) must combine two distinct rotational accelerations

$$|\vec{a}_{tot}|^2 = |\vec{a}_r|^2 + |\vec{a}_t|^2$$
$$= |\omega^2 r|^2 + |\alpha r|^2$$

Problem 10-30: Wheel A of radius r_A=10 cm is coupled by belt B to wheel C of radius r_C=25 cm. The angular speed of wheel A is increased from rest at a constant rate of 1.6 rad/s^2. Find the time needed for wheel C to reach an angular speed of 100 rev/min.