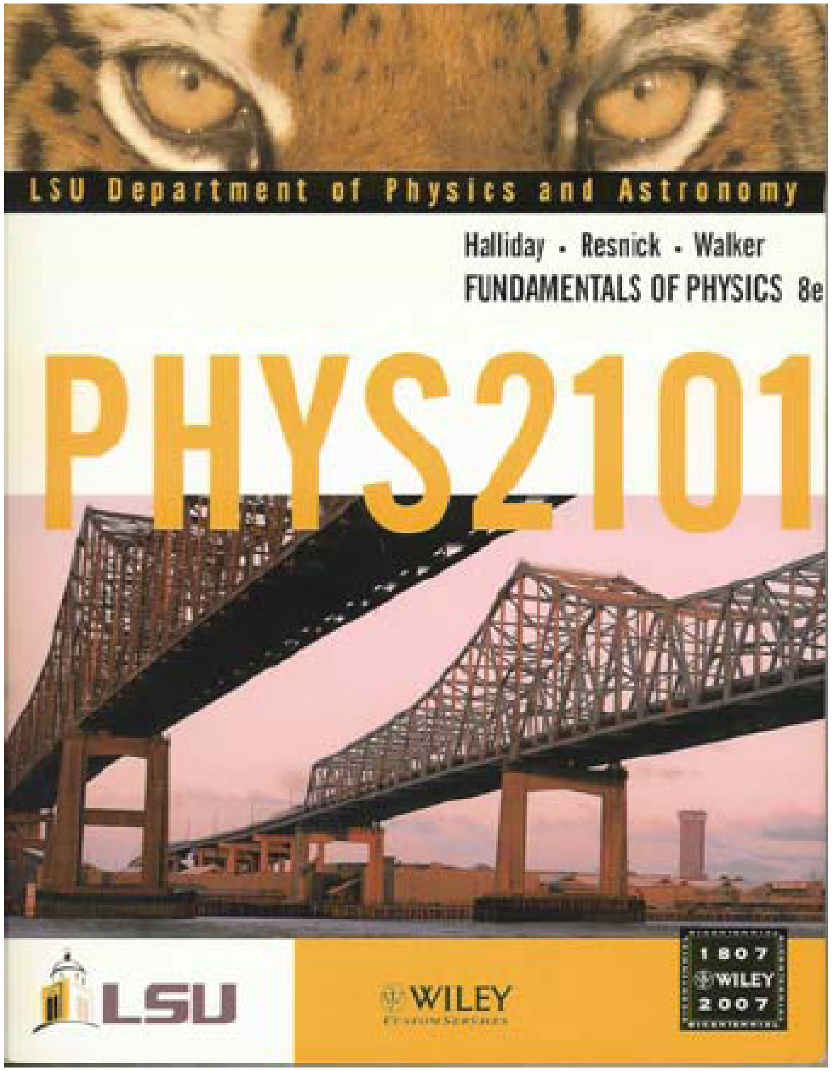



# Physics 2101
# Section 3
# Feb. 24$^{th}$ : Ch. 9

## Announcements:

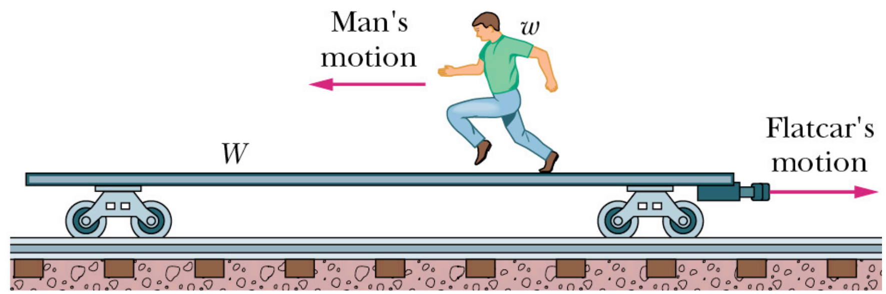
- Today finish Ch. 9
- Friday Review Ch. 7-9
- Next Quiz will be Friday, Feb. 26 on SHW #5
- Next HW due Friday the 26$^{th}$
- Study session next Monday at 6 pm

**Test#2 (Ch. 7-9) will be at 6 PM, March 3 (6) Lockett)**

**Class Website:**
http://www.phys.lsu.edu/classes/spring2010/phys2101-3/

# One more COM problem

***A railroad flatcar of mass M can roll without friction along a straight horizontal track. Initially, a man of mass m is standing on the car, which is moving to the right with speed $v_0$. What is the change in velocity of the car if the man runs to the left so that the speed relative to the car is $v_{rel}$?***

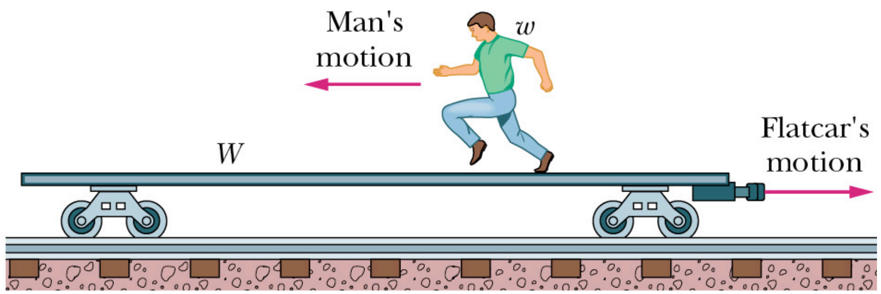



$$\Delta\vec{P} = 0 \quad \Rightarrow \quad \vec{P}_{before} = \vec{P}_{after}$$

[redacted]

# One more COM problem

***A railroad flatcar of mass M can roll without friction along a straight horizontal track. Initially, a man of mass m is standing on the car, which is moving to the right with speed $v_0$. What is the change in velocity of the car if the man runs to the left so that the speed relative to the car is $v_{rel}$?*** ***Lets do $v_0$=0 first.***



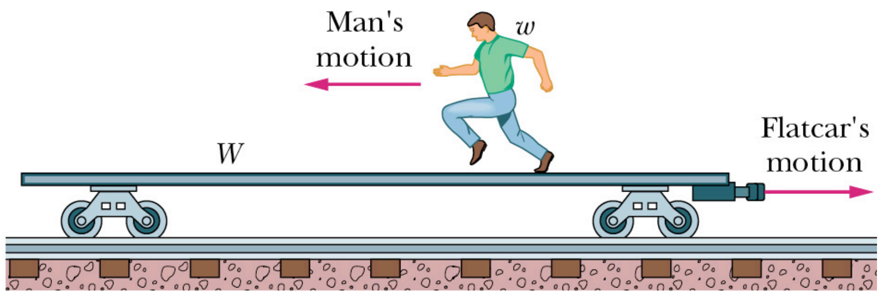

$$\Delta\vec{P} = 0 \quad \Rightarrow \quad \vec{P}_{before} = \vec{P}_{after}$$

$$\vec{P}_{before} = 0 \qquad \vec{P}_{after} = (M)v_{after}\hat{i} + (m)\left(v_{after}\hat{i} + v_{rel}(-\hat{i})\right) = 0$$

$$v_{after} = \frac{mv_{rel}}{M+m}$$

# One more COM problem

***A railroad flatcar of mass M can roll without friction along a straight horizontal track. Initially, a man of mass <u>m</u> is standing on the car, which is moving to the right with speed <u>$v_0$</u>. What is the change in velocity of the car if the man runs to the left so that the speed relative to the car is <u>$v_{rel}$</u>?***




$$\Delta\vec{P} = 0 \quad \Rightarrow \quad \vec{P}_{before} = \vec{P}_{after}$$

$$\vec{P}_{before} = (m + M)v_0\hat{i} \qquad \vec{P}_{after} = (M)v_{after}\hat{i} + (m)\left(v_{after}\hat{i} + v_{rel}(-\hat{i})\right)$$

$$\Delta\vec{v}_{car} = \left(v_{after} - v_0\right) = v_{after} - \frac{(M + m)v_{after} - mv_{rel}}{M + m}$$

$$\Delta\vec{v}_{car} = \frac{mv_{rel}}{M + m}$$

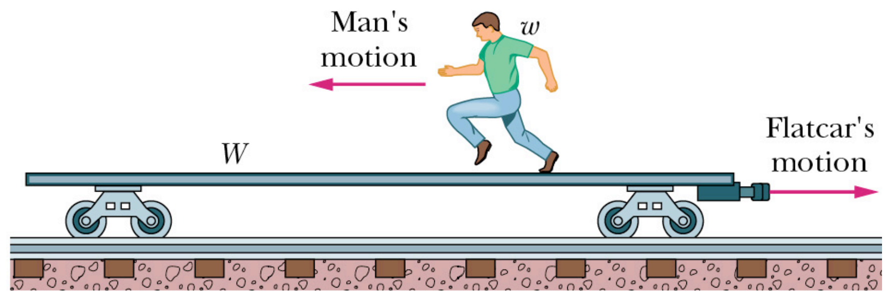
*- DOESN'T MATTER IF $v_0 = 0$*

*-Equivalent to Man = bullet & Flatcar = rifle*

*or Man = cannon ball & Flatcar = canon*

# One more COM problem: Center of Mass

***A railroad flatcar of mass M can roll without friction along a straight horizontal track. Initially, a man of mass m is standing on the car, which is moving to the right with speed $v_0$. What is the change in velocity of the car if the man runs to the left so that the speed relative to the car is $v_{rel}$?***



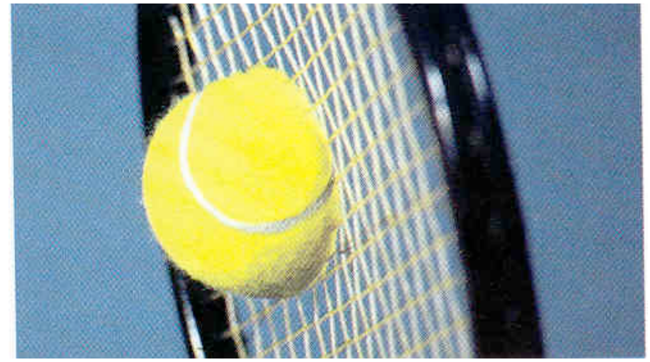

$$\Delta\vec{P} = 0 \quad \Rightarrow \quad \vec{P}_{before} = \vec{P}_{after}$$

Lets move along with the car at v=$v_{cm} = v_0$ : Then P(before)=0

$$\vec{P}_{before} = 0 \qquad \vec{P}_{after} = (M)v_{after}\hat{i} + (m)\left(v_{after}\hat{i} + v_{rel}(-\hat{i})\right) = 0$$

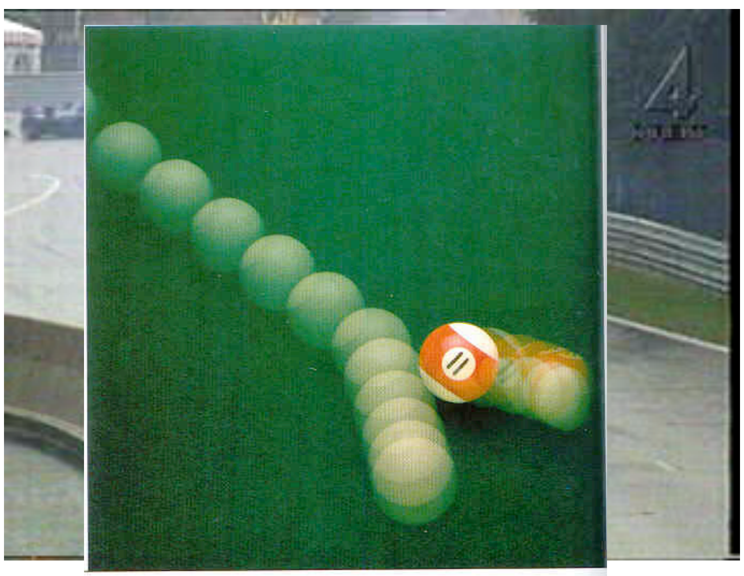
$$v_{after} = \frac{mv_{rel}}{M+m}$$

- *DOESN'T MATTER IF $v_0 = 0$ you can use a coordinate system moving with $v_{COM}$*



# Impulse & Collisions

*An isolated event in which two or more bodies exert relatively strong forces on each other for a relatively short time.*



***We study two types:***

***1) Inelastic (KE lost to $E_{therm}$)***

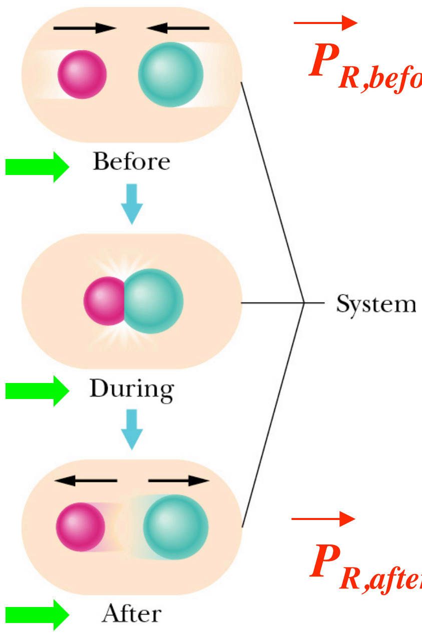
***2) Elastic (total KE=const)***

***[interaction through conservative forces]***

***However in both we assume closed and isolated systems***

# Forces during isolated, closed Collision

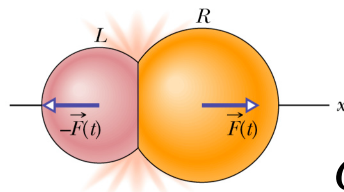
## Collision




$\vec{P}_{R,before}$

$\vec{P}_{R,after}$

**What changes momentum of each?**



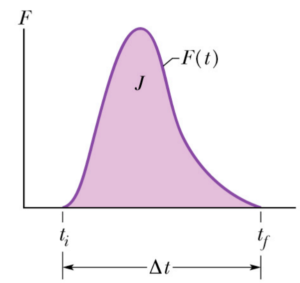

## Impulse

*Change in Momentum is equal to Impulse acting on it*

$$d\vec{p}_R = \vec{F}_{L\to R}(t)dt$$

$$\int_{\vec{p}_i}^{\vec{p}_f} d\vec{p}_R = \int_{t_i}^{t_f} \vec{F}_{L\to R}(t)dt$$

$$\vec{J} \equiv \int_{t_i}^{t_f} \vec{F}_{net}(t)dt$$




$$\vec{p}_f - \vec{p}_i = \Delta\vec{p} = \vec{J}$$

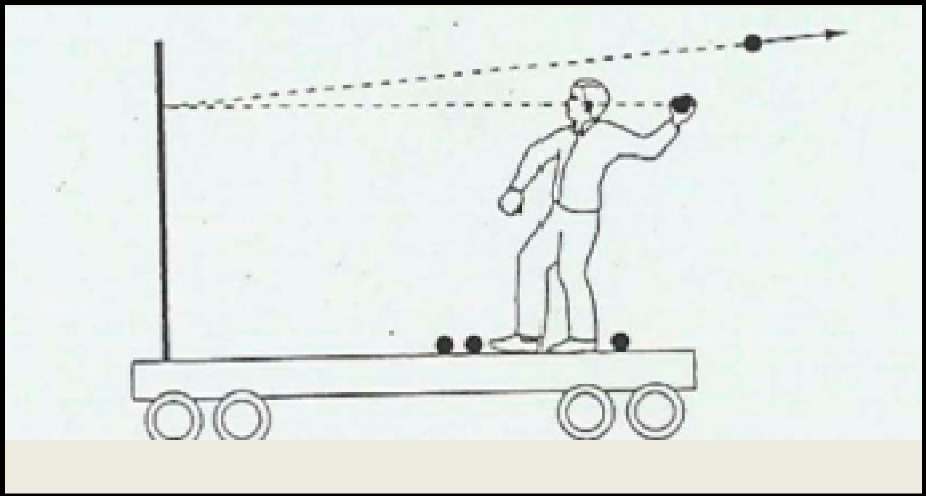
*Vector! Must satisfy for each direction!*

Impulse-momentum theorem

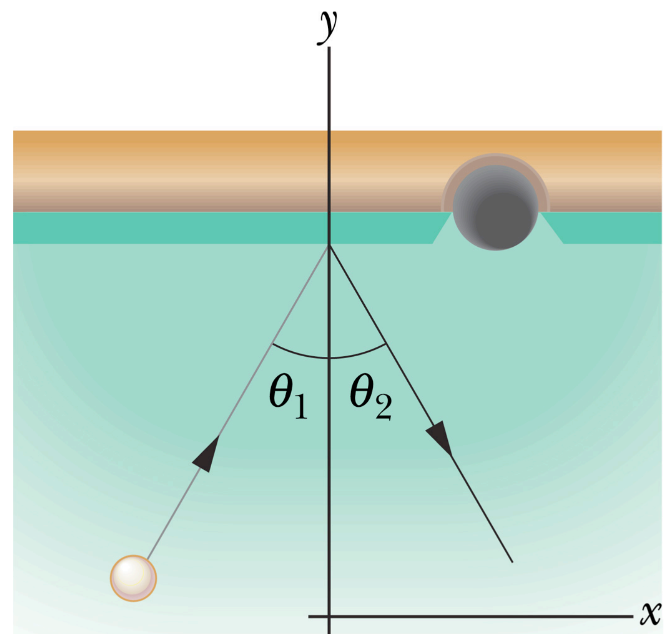
**Question**

Suppose you are on a cart, initially at rest on a track with no friction. You throw balls at a partition that is rigidly mounted to the cart. If the balls bounce straight back as shown in the figure, will the cart be put into motion?

1. Yes. The cart moves left.
2. Yes. The cart moves right.
3. No. The cart does not move.
4. Sometimes; it depends on the velocity of the ball and mass of the cart.



**Problem #20:** The figure shows the path of a cue ball of mass m as it bounces from a rail of a pool table. The ball's initial speed is $v_0$ and the angle from the normal is $\theta_1$. The bounce reverses the y component of the ball's velocity but does not alter the x component.
What are (a) the angle $\theta_2$ and change in the momentum in vector notation.

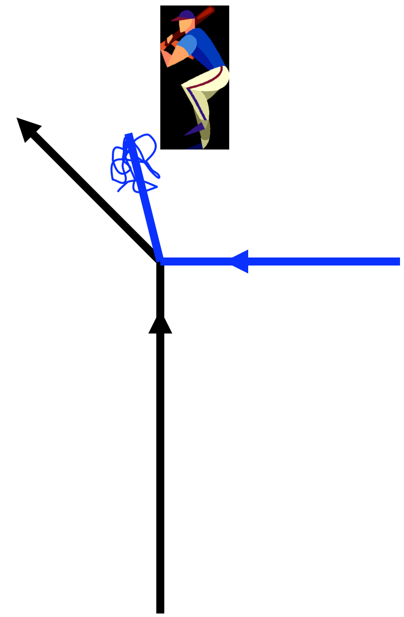



The x component of velocity is conserved

[redacted]

# Plot x vs. t and y vs. t

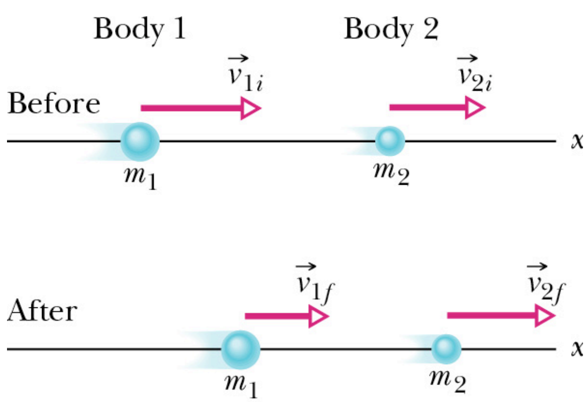
# Inelastic Collisions?

# Inelastic Collisions: 1-D

Inelastic collision : KE is not conserved (~ thermal energy )

However, if system is closed and isolated, the total linear momentum $P$ cannot change (whether the collision is elastic or inelastic !).

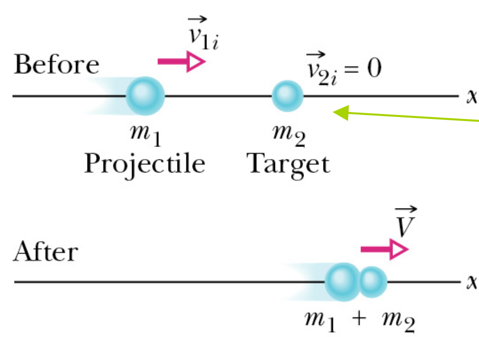



$$\vec{P}_{before} = \vec{P}_{after} \qquad \text{1 - D}$$

$$(m_1 v_{1i} + m_2 v_{2i}) = (m_1 v_{1f} + m_2 v_{2f})$$

*Only COLM: Conservation of Linear Momentum*

*Special Case:*
***Completely Inelastic Collision***
*(hit-'n-stick)*

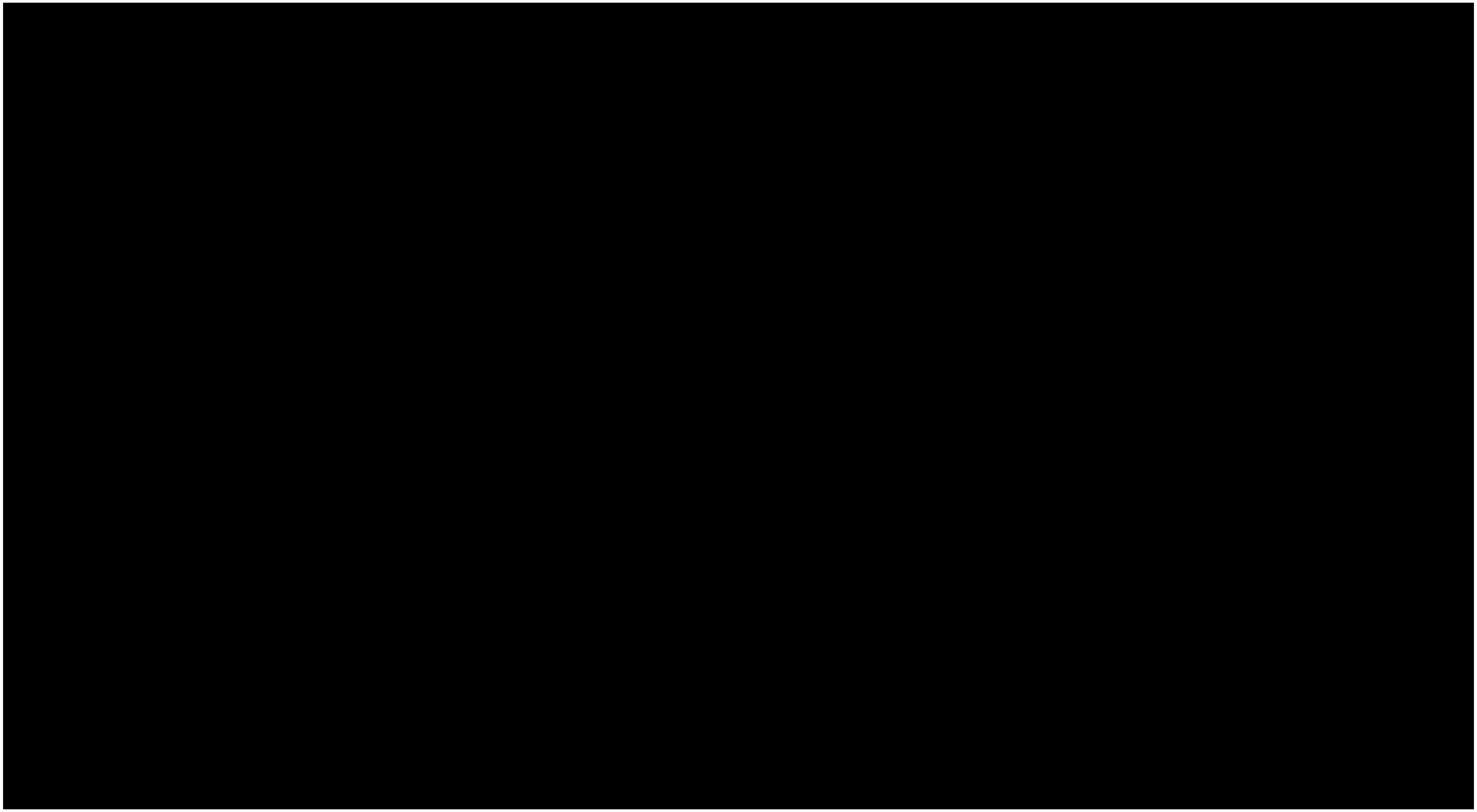



$$(m_1 v_{1i} + m_2 0) = (m_1 V + m_2 V)$$

target at rest

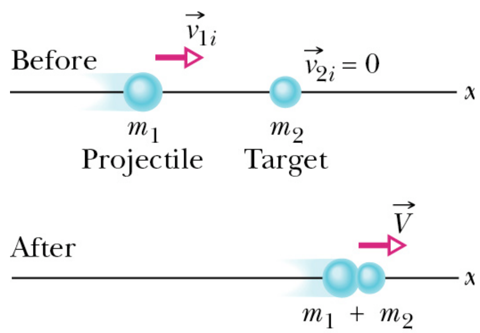
$$V = \frac{m_1 v_{1i}}{(m_1 + m_2)}$$

# Plot x vs. t for both masses



# Example #1

**Pure inelastic collision**
**Hit-'n-stick**




$$v_{com} = V = \frac{1m_1(1)}{(m_1 + 3m_1)}$$

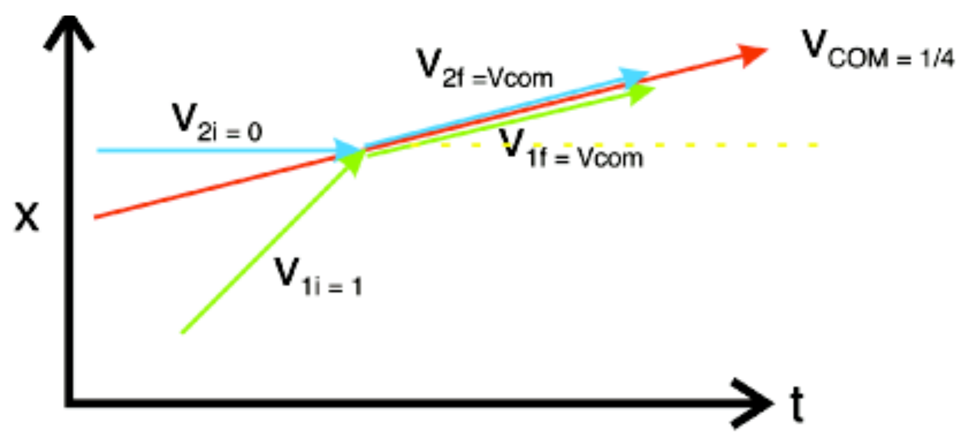
$$V = v_{com} = \frac{1}{4} \cdot m/s$$

$$\vec{P}_{before} = \vec{P}_{after} \qquad \text{1-D}$$

$$(m_1 v_{1i} + m_2 v_{2i}) = (m_1 v_{1f} + m_2 v_{2f})$$

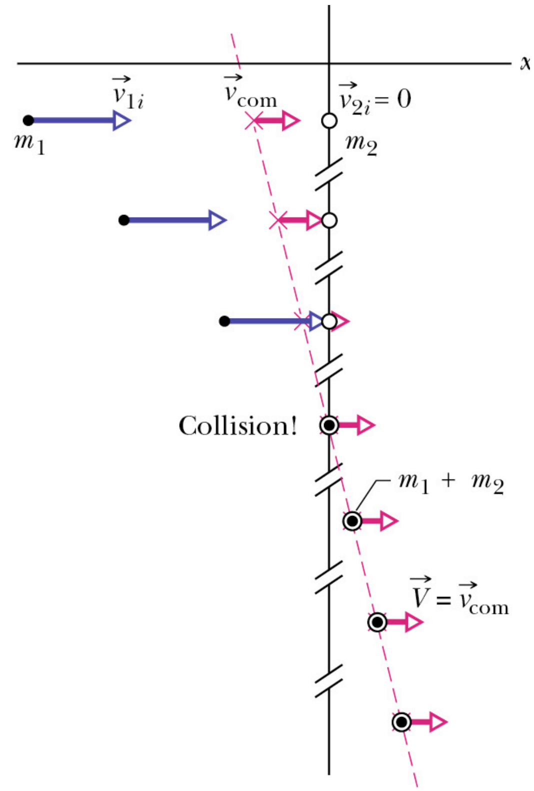
$$(m_1 v_{1i} + 0) = V_f (m_1 + m_2)$$

Special case: $v_{2i}=0$ & $m_2 = 3m_1$
take $v_{1i} = 1$ m/s



# Velocity of COM

*In a closed, isolated system the COM velocity ( $\vec{v}_{com}$) of the system is CONSTANT. Why?*

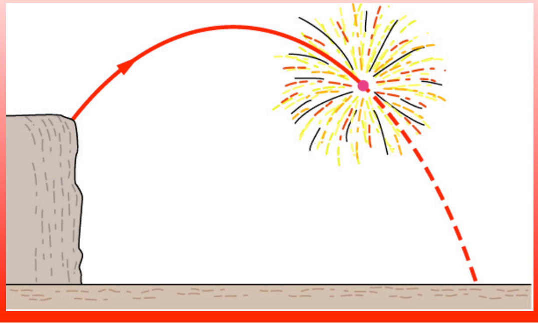



$$\vec{F}_{net} = \frac{d\vec{P}_{tot}}{dt} = 0 \Longrightarrow \vec{P}_{tot} = M\vec{v}_{com} = (m_1 + m_2)\vec{v}_{com}$$

$$\vec{P}_{tot} = \vec{p}_{1i} + \vec{p}_{2i}$$

$$\vec{v}_{com} = \frac{\vec{P}_{tot}}{(m_1 + m_2)} = \frac{\vec{p}_{1i} + \vec{p}_{2i}}{(m_1 + m_2)} = \frac{\vec{p}_{1f} + \vec{p}_{2f}}{(m_1 + m_2)}$$

***Constant !!***

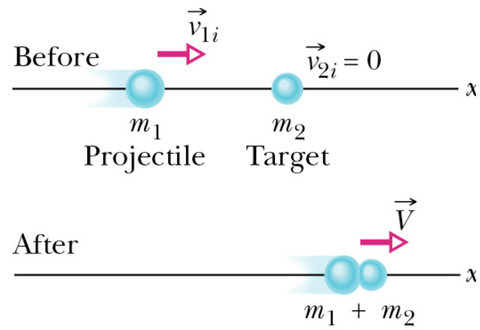


*Exploding things from Chapt. 9:*

# Summary: Velocity & Position of COM

*(closed & isolated system)*

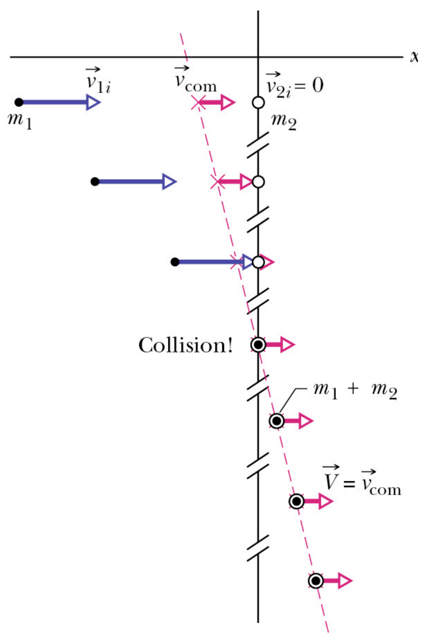
*Special Case:*
*Completely*
*Inelastic*
*Collision*
*(hit-'n-stick)*

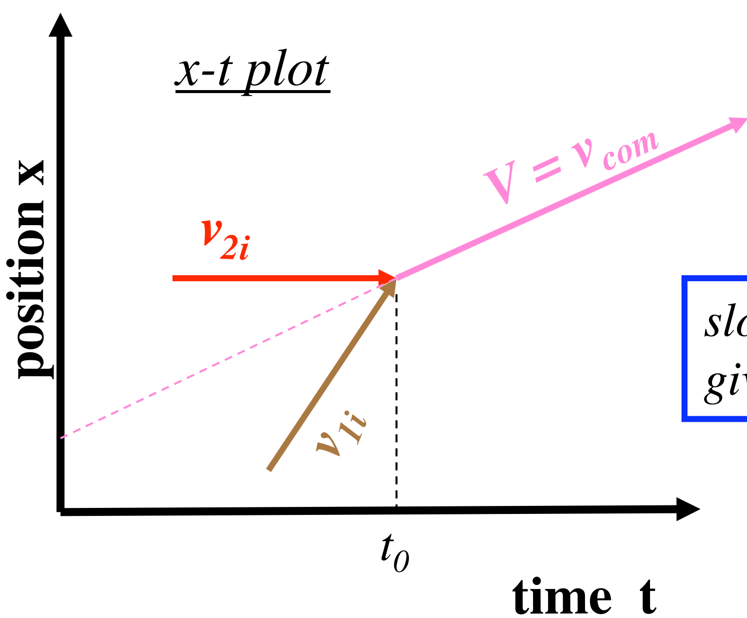



$$(m_1 v_{1i} + m_2 v_{2i}) = (m_1 V + m_2 V)$$

target at rest

$$V = \frac{m_1 v_{1i}}{(m_1 + m_2)} = v_{com}$$




*slope in x-t plot gives velocity*

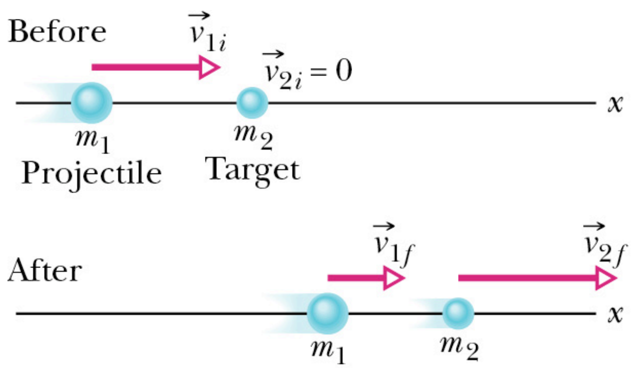
# *Elastic* Collisions: 1-D

Elastic collision : TOTAL KE is conserved (~ Conservative forces )

AND if system is closed and isolated, the total linear momentum $\vec{P}$ cannot change (whether the collision is elastic or inelastic !).

*For example:*

$$v_{2i} = 0$$




During Collision: transfer KE and momentum between objects through conservative internal forces

$$KE_{before} = KE_{after}$$

$$\left(\tfrac{1}{2} m_1 v_{1i}^2\right) = \left(\tfrac{1}{2} m_1 v_{1f}^2 + \tfrac{1}{2} m_2 v_{2f}^2\right)$$

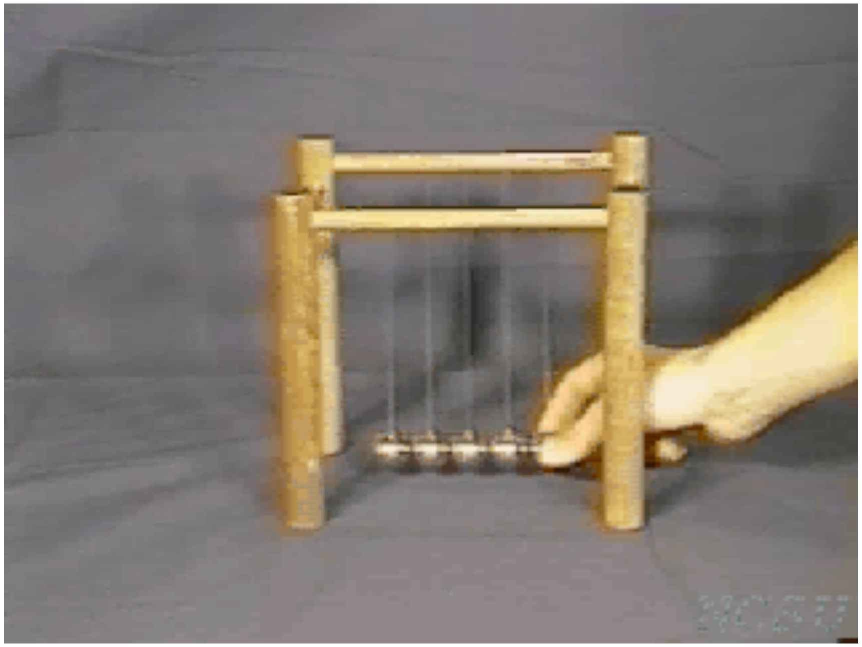
*"COKE"*

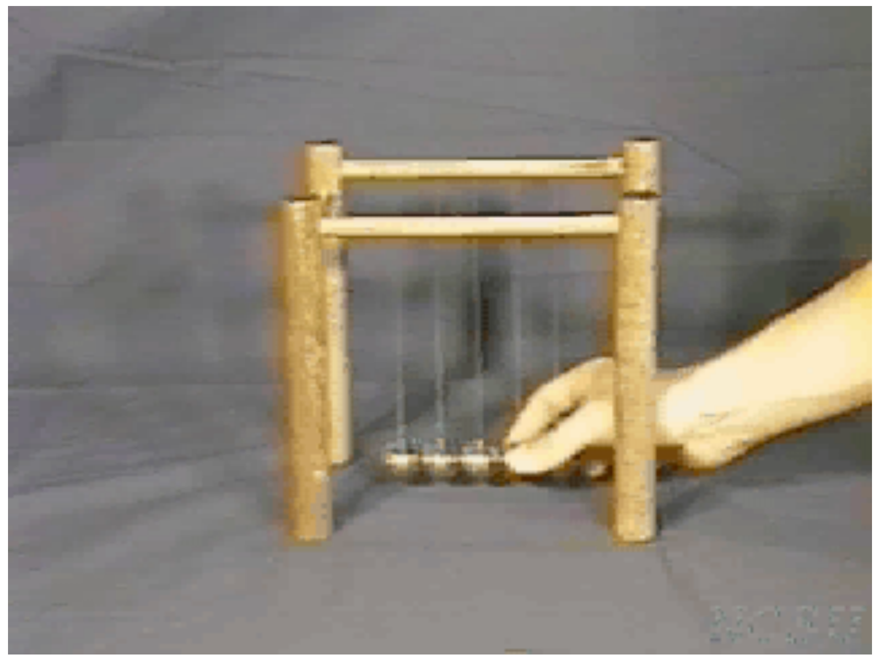
$$\vec{P}_{before} = \vec{P}_{after} \qquad \text{1 - D}$$

$$\left(m_1 v_{1i}\right) = \left(m_1 v_{1f} + m_2 v_{2f}\right)$$

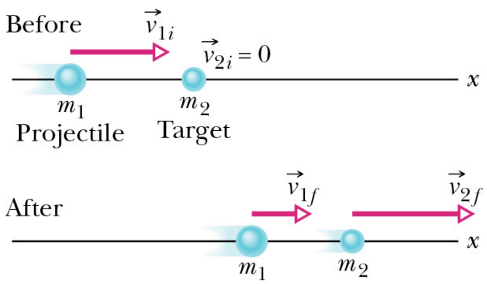
*"COLM"*

# Examples of Elastic Collisions





# 1-D Elastic: Solve Eqn's with $v_{2i} = 0$




$$v_{f1} = \frac{m_1 - m_2}{m_1 + m_2} v_{o1} \qquad \text{(Eqn. 9.67)}$$

$$v_{f2} = \frac{2m_1}{m_1 + m_2} v_{o1} \qquad \text{(Eqn. 9.68)}$$

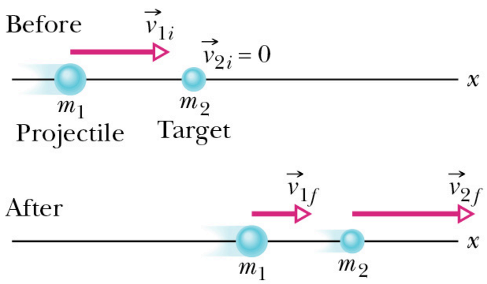
*5 variables and 2 Equations*

*2 mass & 3 velocity*

*Special Cases:*

*1) Equal masses* If $m_1 = m_2$ then $v_{f1} = 0$ and $v_{f2} = v_{o1}$

# 1-D Elastic: Solve Eqn's with $v_{2i} = 0$




$$v_{f1} = \frac{m_1 - m_2}{m_1 + m_2} v_{o1} \qquad \text{(Eqn. 9.67)}$$

$$v_{f2} = \frac{2m_1}{m_1 + m_2} v_{o1} \qquad \text{(Eqn. 9.68)}$$

*5 variables and 2 Equations*

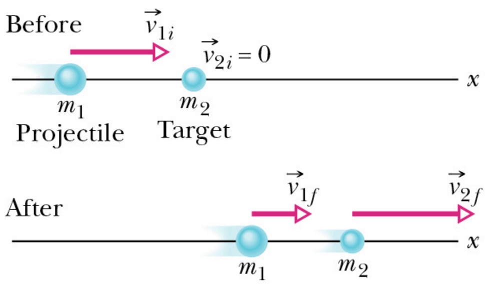
*2 mass & 3 velocity*

*Special Cases:*

*2) Massive target*

*If* $m_1 << m_2$ *then* $v_{f1} \approx -v_{o1}$ *and* $v_{f2} \approx \frac{2m_1}{m_2} v_{o1}$

# 1-D Elastic: Solve Eqn's with $v_{2i} = 0$




$$v_{f1} = \frac{m_1 - m_2}{m_1 + m_2} v_{o1} \qquad \text{(Eqn. 9.67)}$$

$$v_{f2} = \frac{2m_1}{m_1 + m_2} v_{o1} \qquad \text{(Eqn. 9.68)}$$

*5 variables and 2 Equations*

*2 mass & 3 velocity*

*Special Cases:*

*3) Massive Projectile* If $m_1 >> m_2$ then $v_{f1} \approx v_{o1}$ and $v_{f2} \approx 2v_{o1}$

# 1-D Elastic Collision: 2-particles, moving target


**No need to memorize**

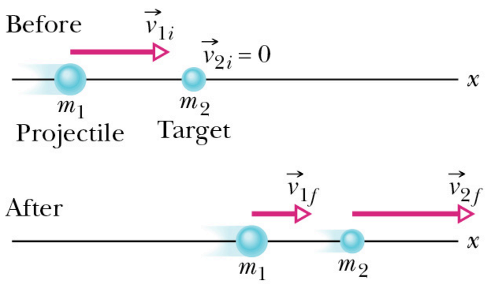
$$v_{f1} = \frac{m_1 - m_2}{m_1 + m_2} v_{o1} + \frac{2m_2}{m_1 + m_2} v_{o2}$$

$$v_{f2} = \frac{2m_1}{m_1 + m_2} v_{o1} + \frac{m_2 - m_1}{m_1 + m_2} v_{o2}$$

*Note: if $v_{o2}=0$, the equations above (9.75 & 9.76) reduce to Eqn's 9.67 & 9.68*

*6 variables and 2 Equations*
*2 mass & 4 velocity*

# Example: 1-D Elastic $\quad v_{2i} = 0$



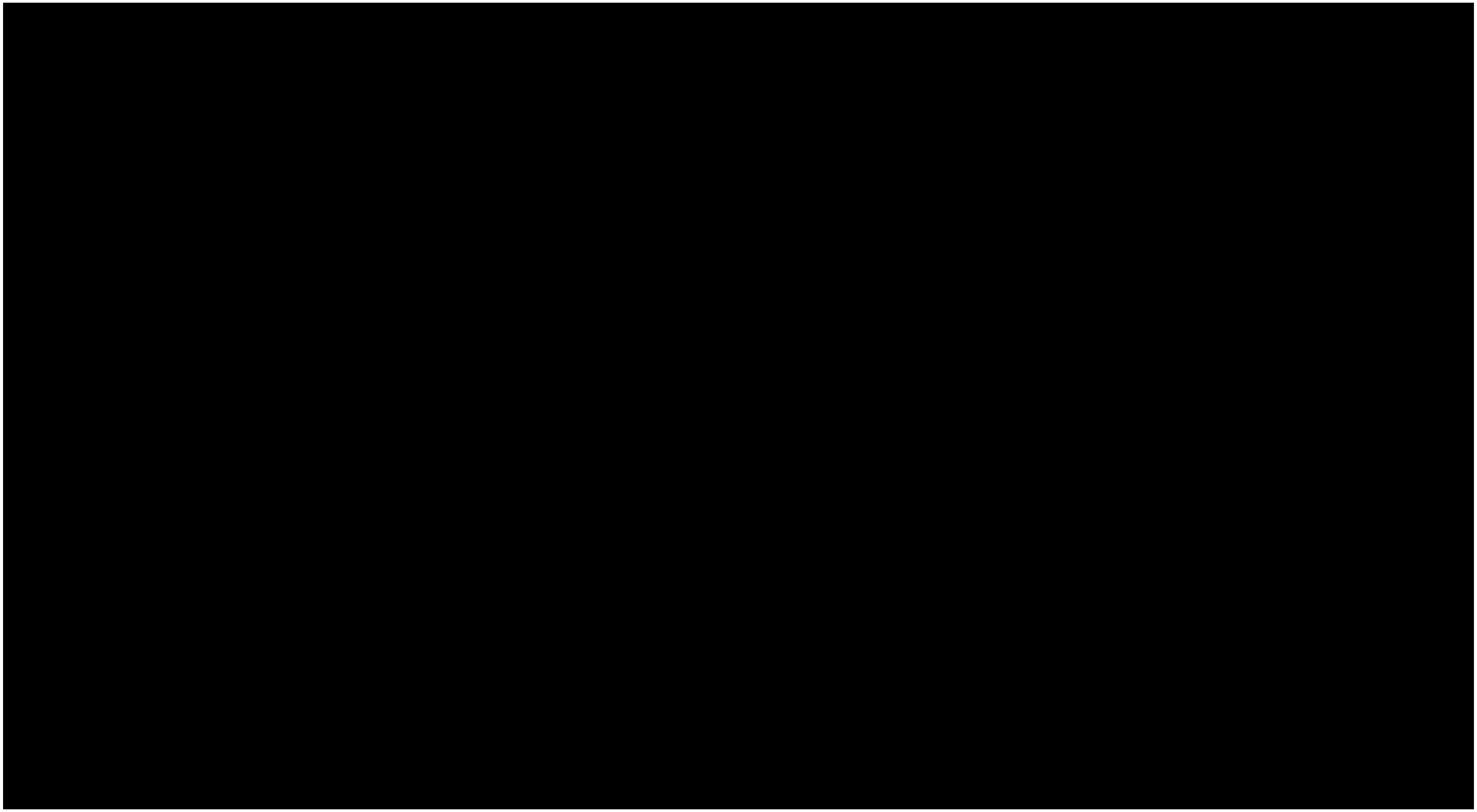

$$v_{f1} = \frac{m_1 - m_2}{m_1 + m_2} v_{o1} \qquad \text{(Eqn. 9.67)}$$

$$v_{f2} = \frac{2m_1}{m_1 + m_2} v_{o1} \qquad \text{(Eqn. 9.68)}$$

*Special Cases: Equal masses* $\quad$ *If* $m_1 = m_2$ *then* $v_{f1} = 0$ *and* $v_{f2} = v_{o1}$

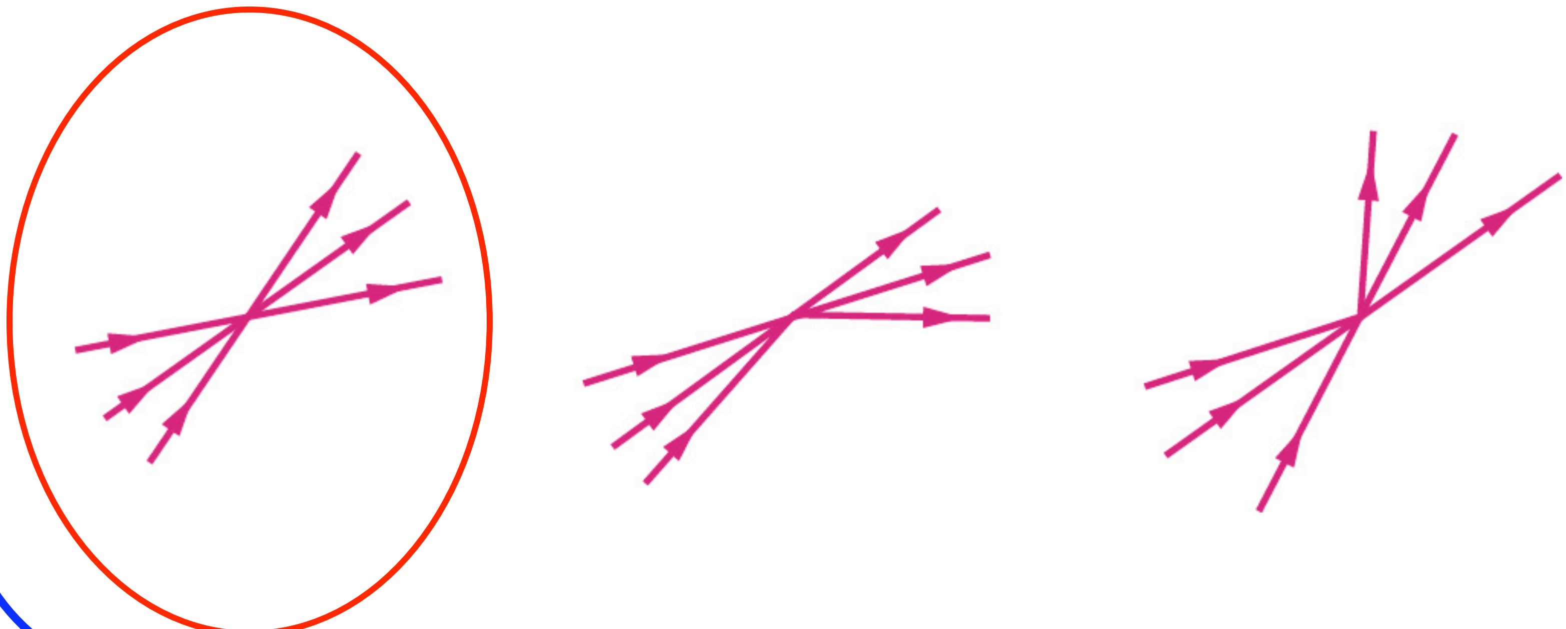
**How do you "draw" this collision?**

# Plot x vs. t for both masses

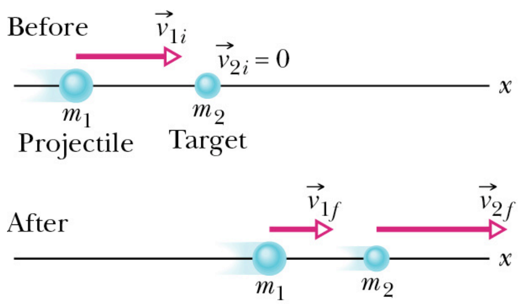


# Question 9-10*

Two bodies that form a closed, isolated system undergo an elastic collision in 1-D. Which of the three choices best represents the position-versus-time ($x$-$t$ plot) of those bodies and their center of mass velocity ($v_{com}$)?

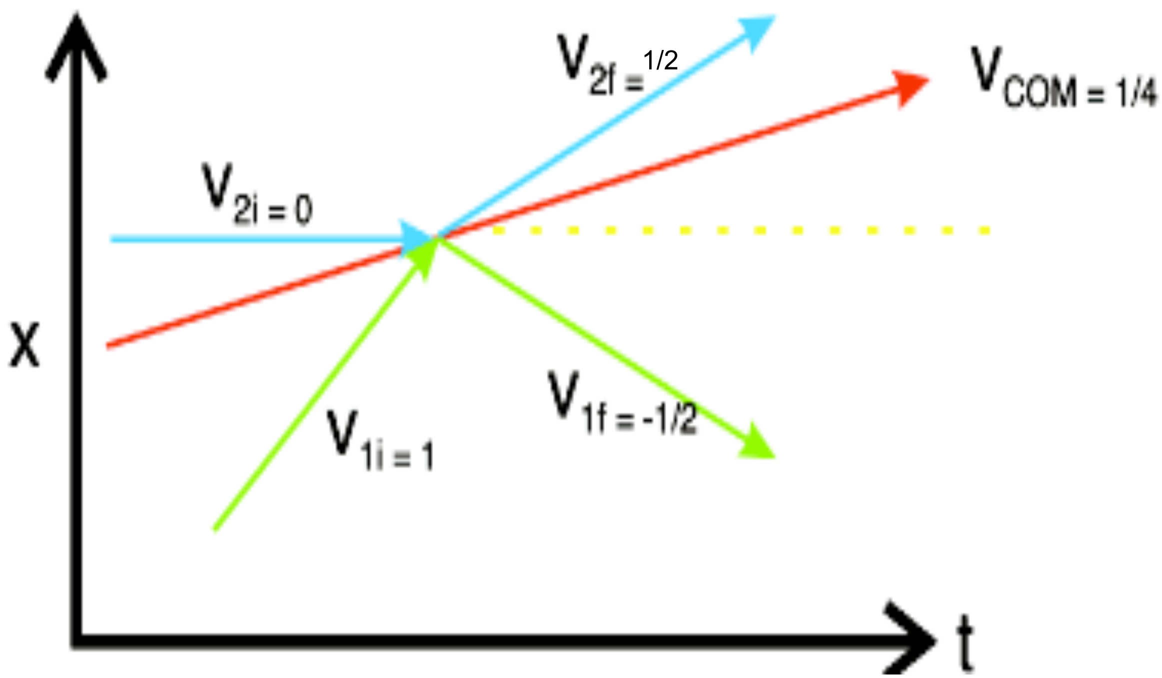


# Example #2 1-D elastic collision with $m_2 = 3m_1$ and $v_{2i} = 0$




$$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2} v_{1i} = \frac{m_1 - 3m_1}{m_1 + 3m_1} v_{1i} = -\frac{1}{2} v_{1i}$$

$$v_{2f} = \frac{2m_1}{m_1 + m_2} v_{1i} = \frac{2m_1}{m_1 + 3m_1} v_{1i} = \frac{1}{2} v_{1i}$$



# Review: Impulse & Collision s

*An isolated event in which two or more bodies exert relatively strong forces on each other for a relatively short time.*

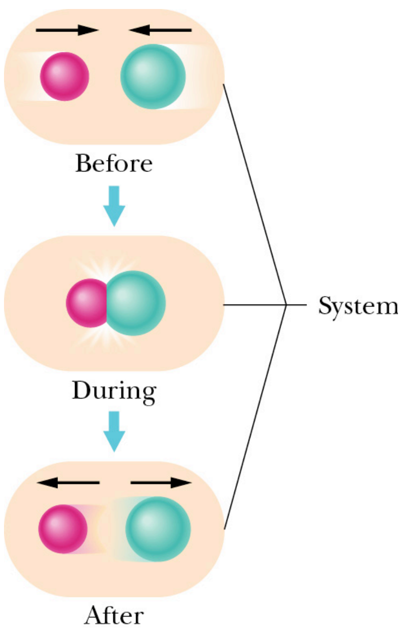
***We study two types:***

***1) Inelastic (KE lost to $E_{therm}$)***

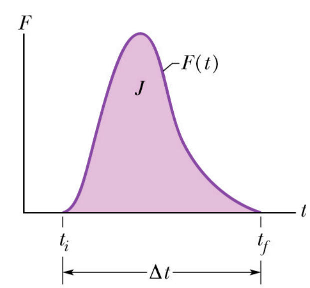
***2) Elastic (total KE=const)***

***[interaction through conservative forces]***

***However in both we assume closed and isolated systems***




$$\vec{J} \equiv \int_{t_i}^{t_f} \vec{F}_{net}(t)dt$$



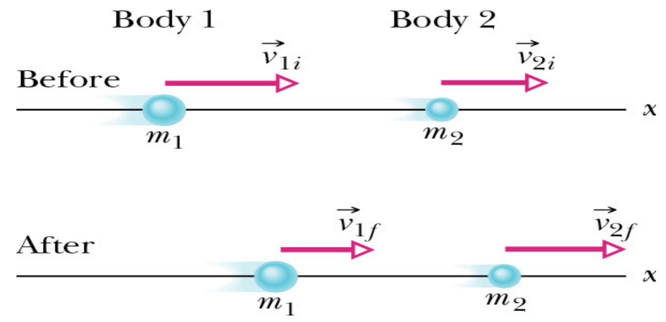

$$\vec{p}_f - \vec{p}_i = \Delta\vec{p} = \vec{J}$$

*Vector! Must satisfy for each direction!*

<u>Impulse-momentum theorem</u>

# *Review: 1-D* Collisions

If system is closed and isolated, the total linear momentum $\vec{P}$ cannot change




Inelastic collision : KE is not conserved (~ thermal energy )

$$\vec{P}_{before} = \vec{P}_{after} \qquad 1\text{-D} \qquad 2 \text{ particles}$$

$$\left(m_1 v_{1i} + m_2 v_{2i}\right) = \left(m_1 v_{1f} + m_2 v_{2f}\right)$$

Elastic collision : TOTAL KE is conserved (~ Conservative forces )

$$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2} v_{1i} + \frac{2m_2}{m_1 + m_2} v_{2i} \qquad \text{(Eqn. 9-75)}$$

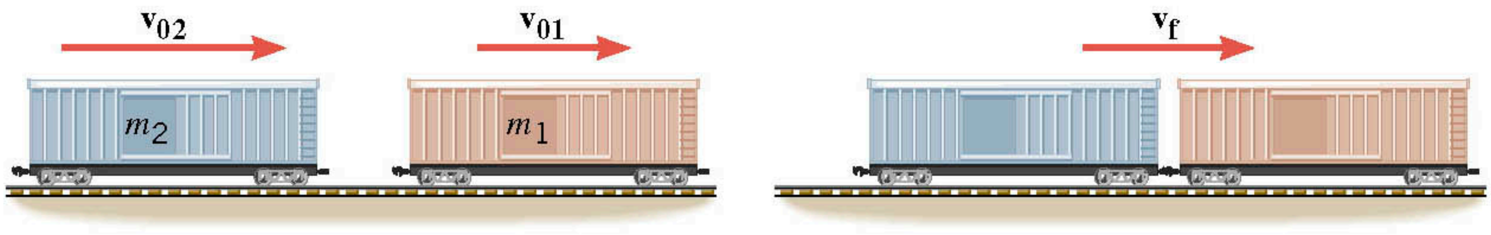
$$v_{2f} = \frac{2m_1}{m_1 + m_2} v_{1i} + \frac{m_2 - m_1}{m_1 + m_2} v_{2i} \qquad \text{(Eqn. 9-76)}$$

$$\vec{P}_{before} = \vec{P}_{after} \qquad 1\text{-D}$$

$$KE_{before} = KE_{after} \qquad 2 \text{ particles}$$

$$\left(\tfrac{1}{2} m_1 v_{1i}^2 + \tfrac{1}{2} m_2 v_{2i}^2\right) = \left(\tfrac{1}{2} m_1 v_{1f}^2 + \tfrac{1}{2} m_2 v_{2f}^2\right)$$

# Collisions – elastic or inelastic?




(*a*) Before (*b*) After

Assemble a train. Car 2 overtakes and locks onto Car 1 What kind of collision is this?

1) Momentum is ***always*** conserved

Momentum conservation: $P_{initial} = P_{final}$

$$m_1 v_{o1} + m_2 v_{o2} = (m_1 + m_2) v_f$$

Is energy conserved?

2) Unless it states that it is an elastic collision, ***you do not know***

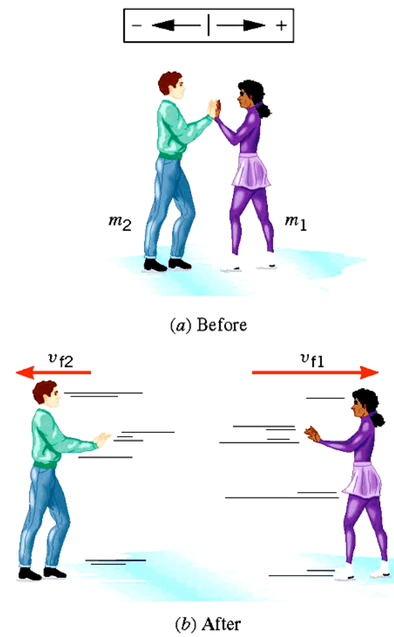
Does the $KE_{initial} = KE_{final}$?

$$KE_o = \tfrac{1}{2} m_1 v_{o1}^2 + \tfrac{1}{2} m_2 v_{o2}^2 \qquad KE_f = \tfrac{1}{2}(m_1 + m_2) v_f^2$$

Usually you will find that they are NOT equal !!

# Skaters Push Off

*Two skaters of $m_1$=54 kg and $m_2$=88 kg push off and the woman moves off with $v_{f1}$=2.5 m/s. What is the velocity of the man?*



(a) Before

(b) After

In this (and all) "collisions" momentum is conserved:

$$P_{initial} = P_{final}$$

$$0 = m_1 v_{1f} + m_2 v_{2f} \qquad v_{2f} = -\frac{m_1 v_{1f}}{m_2}$$

$$v_{2f} = -\frac{(54kg)(2.5\, m/s)}{88kg} = -1.5\, m/s$$

Is this "collision" elastic or inelastic?

*It <u>has</u> to be inelastic !!*

$$KE_{initial} = 0 \qquad KE_{final} = \tfrac{1}{2} m_1 v_{1f}^2 + \tfrac{1}{2} m_2 v_{2f}^2$$

$$= \tfrac{1}{2}(54)(2.5)^2 + \tfrac{1}{2}(88)(-1.5)^2$$

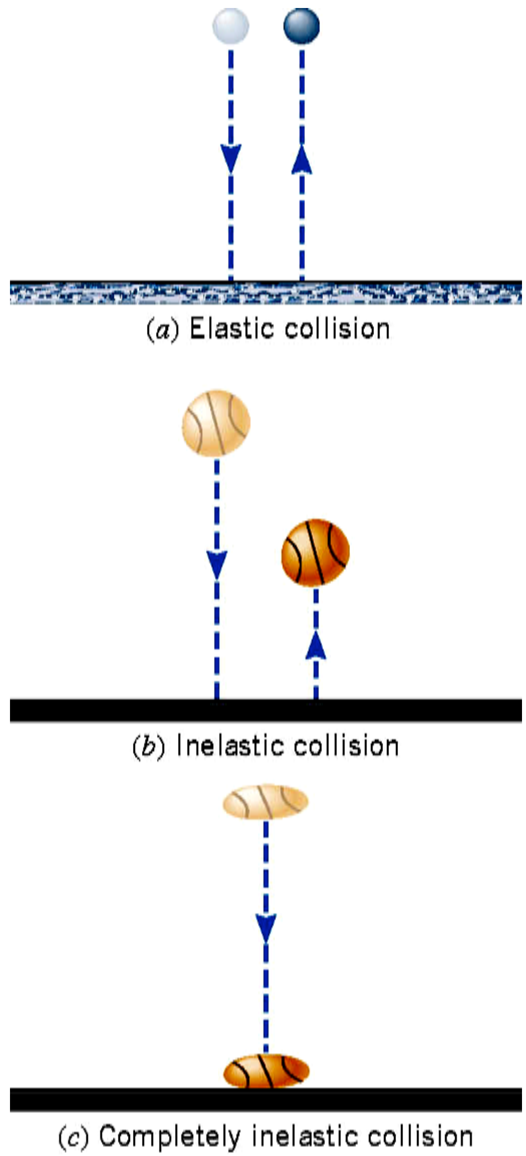
$$= 268J$$

*Energy is not conserved here!*

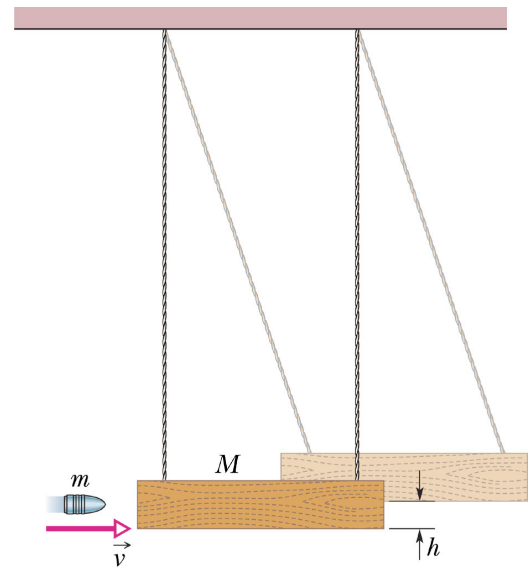
# Collisions can often have multiple parts

*Always, momentum is conserved in the collision*

You may, however, have energy conserved before or after the collision

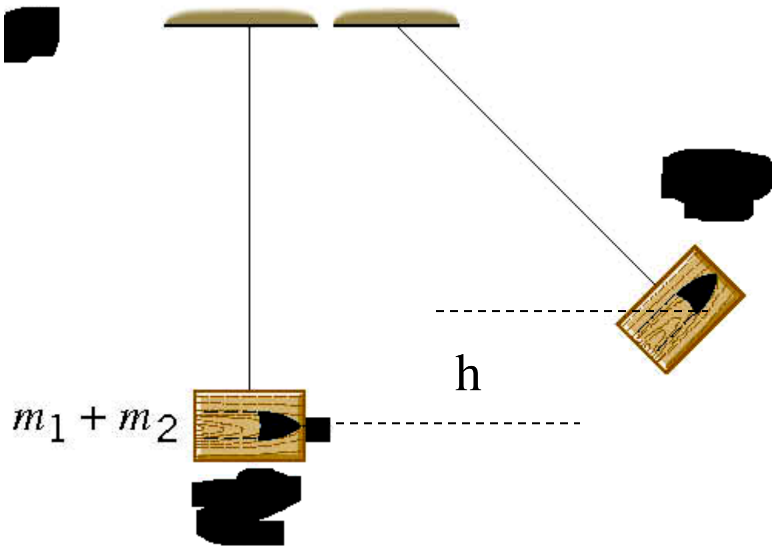


**Sample problem 9:** A bullet of mass m and initial velocity $v_0$ collides with and sticks onto a large wooden block of mass M. Find the velocity of M+m immediately after the collision. How high does the combined block + bullet go before coming to rest temporarily.




[redacted]

# Collisions – ballistic pendulum




A bullet with with an initial velocity of 896 m/s and a mass of 0.01 kg strikes a 2.5 kg block hung from the ceiling. How high do the the block/bullet combination go ?

1) Only momentum conservation during collision

2) Conservation of Energy as it swings up under conservative force

Momentum conservation:

$$P_o = m_1 v_{o1} = P_f = (0.01kg)(896\, {}^{m}\!/\!_{s}) = 8.96\, {}^{kg\cdot m}\!/\!_{s}$$

$$P_f = (m_1 + m_2)v_f = 8.96\, {}^{kg\cdot m}\!/\!_{s}$$

$$v_f = \frac{8.96\, {}^{kg\cdot m}\!/\!_{s}}{2.51kg} = 3.57\, {}^{m}\!/\!_{s}$$

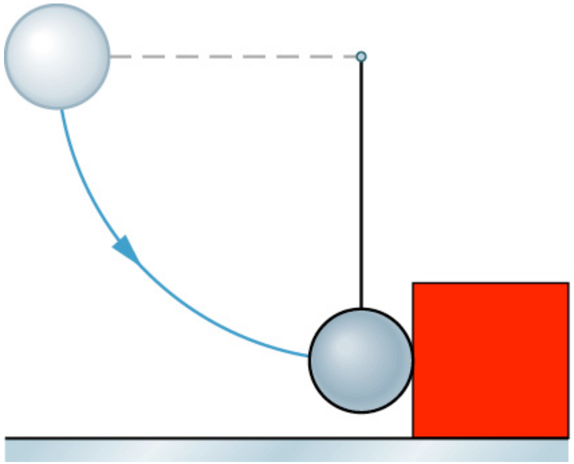
Energy conservation:

$$KE_o = PE_f = \tfrac{1}{2}(m_1 + m_2)v_f^2 = (m_1 + m_2)gh$$

$$h = \frac{\frac{1}{2}v_f^2}{g} = 0.650m$$

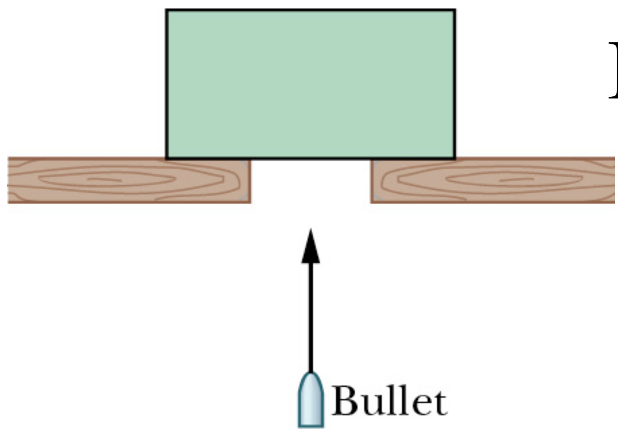
# Supplemental HW #5

*A ball of mass **m** is fastened to a cord of length **L**. The ball is released when cord is horizontal. At bottom of path, the ball elastically strikes block of mass **M** initially at rest on frictionless floor.*



a) *What is the speed of the ball right after the collision?*

[redacted]

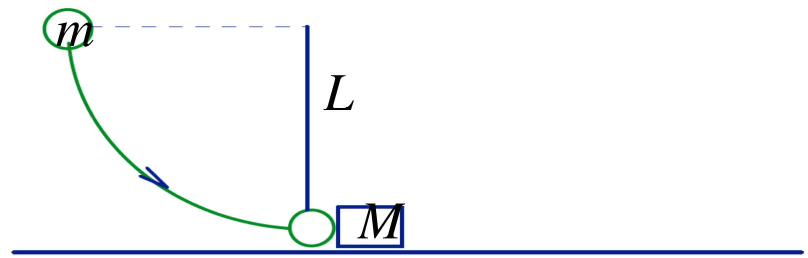



# Bullet Into Block

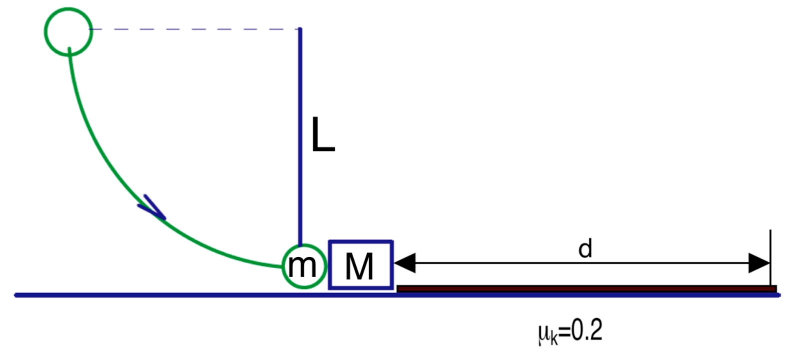
A 10 g bullet moving directly upward at 1000 /ms strikes and lodges in the center of a 5.0 kg block initially at rest. To what maximum height does the block/bullet combination then rise above its initial position?

[redacted]

Supplemental HW#5: A ball of mass m is fastened to a cord that is L m long and fixed at the end. The ball is released when the cord is horizontal. At the bottom of its path, the ball strikes a block of Mass M initially at rest. The collision is elastic find the speed of the ball and the block just after the collision.



***SHW#5:*** *A ball of mass m is fastened to a cord that is L m long and fixed at the far end. The ball is then released when the cord is horizontal. At the bottom of its path, the ball strikes a block of mass M initially at rest on a surface with kinetic friction constant* $\mu_k$*. The collision is elastic. Find the speed of the ball, the speed of the block (both just after the collision), and the distance d that the block travels.*




$$v_{f2} = \frac{2m}{m+M}\sqrt{2gL} \Longrightarrow E_{Mech} = \frac{Mv_{2F}^2}{2} = \frac{M}{2}\left(\frac{2m}{m+M}\sqrt{2gL}\right)^2$$

When it comes to rest

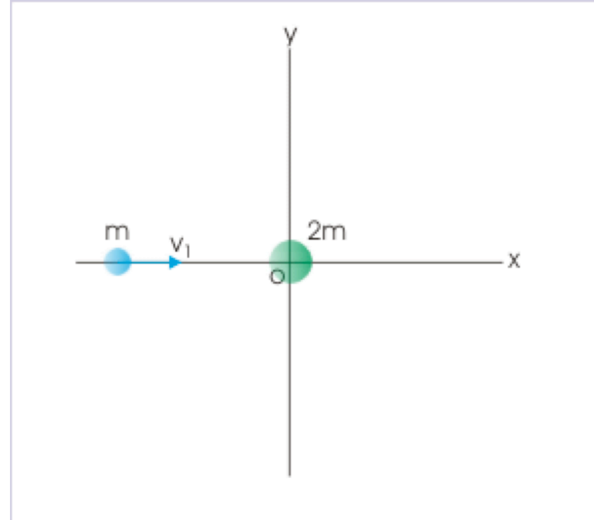
$$E_{Mech} = \mu_k Mgd$$

$$\mu_k Mgd = \frac{M}{2}\left(\frac{2m}{m+M}\sqrt{2gL}\right)^2$$

*Check—no gravity, no friction*

$$d = \frac{1}{2\mu_k g}\left(\frac{2m}{m+M}\sqrt{2gL}\right)^2$$

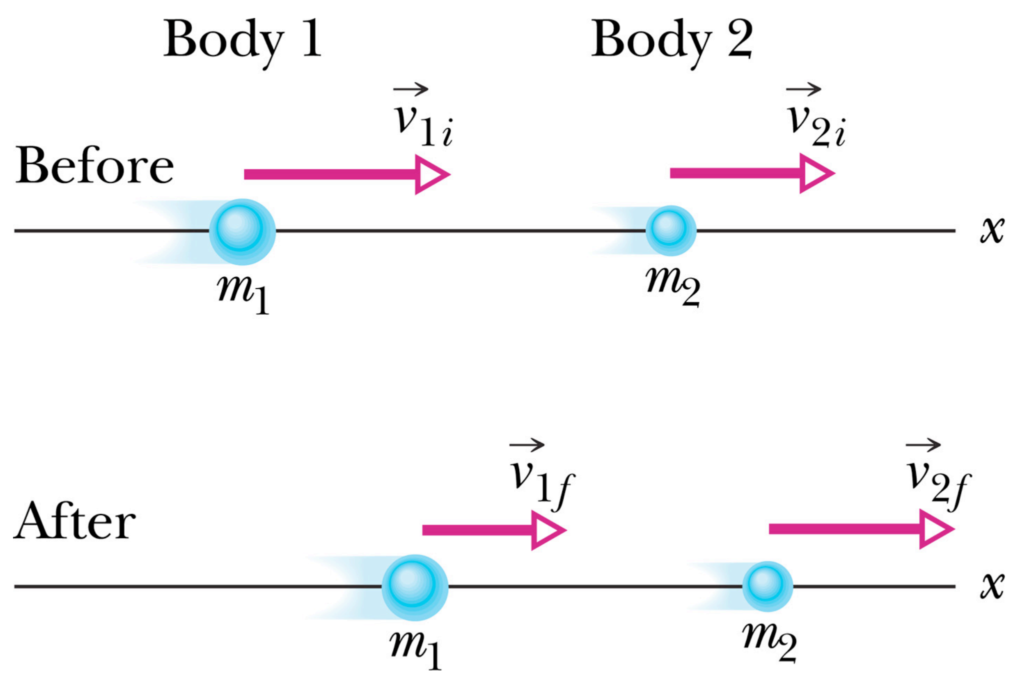
*SHW#4. A ball of mass "m", which is moving with a speed "v1" in x-direction, strikes another ball of mass "2m", placed at the origin of horizontal planar coordinate system. The lighter ball comes to rest after the collision, whereas the heavier ball breaks in two equal parts. One part moves along y-axis with a speed "v2". Find the direction of the motion of other part.*
*[Answer: θ=tan−1(v2/v1) below horizontal]*




[redacted]

# Enough for Today
# Continue on Monday

# Momentum and Kinetic Energy in Collisions



# Inelastic Collisions in one Dimension