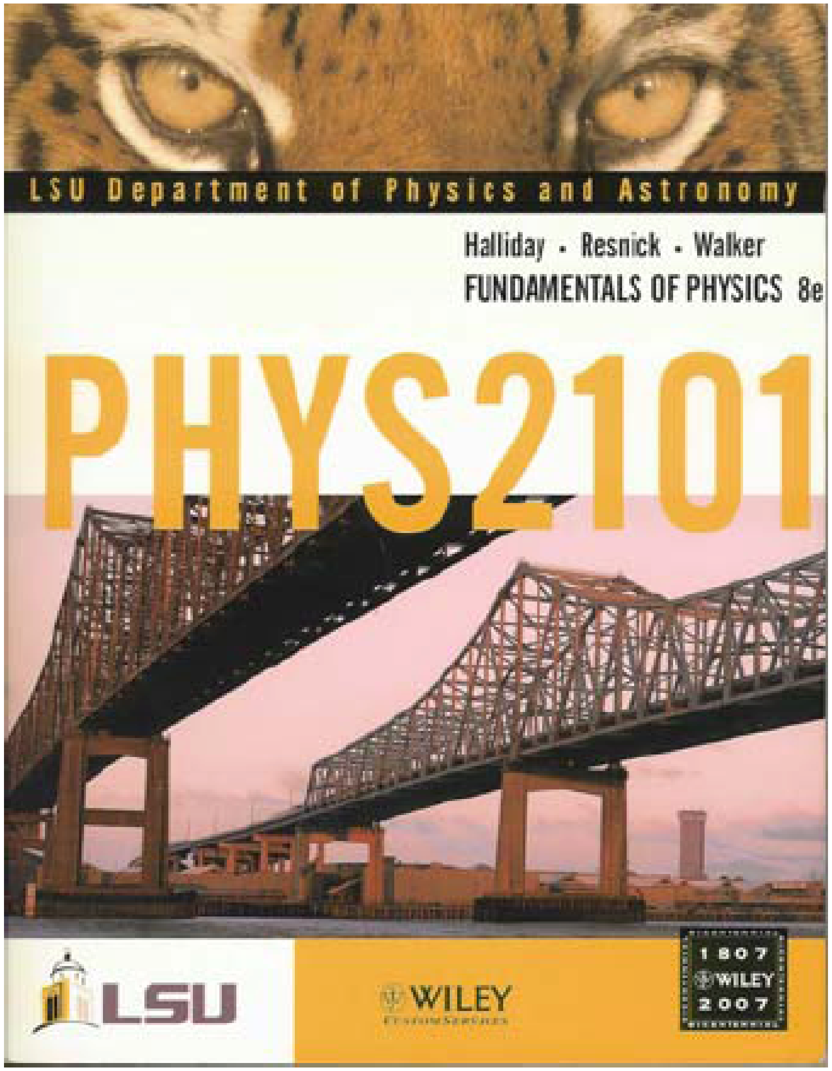


Physics 2101
Section 5
Feb. 19th : Ch. 9

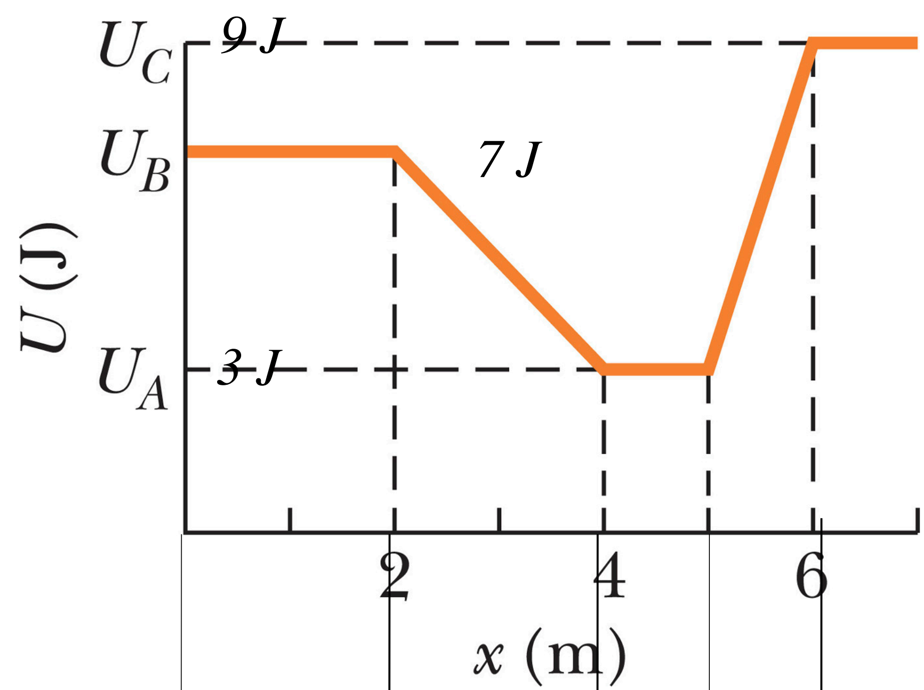
Announcements:

- Today Ch. 9
- Next Quiz will be Friday, Feb. 26 on SHW #5
- Next HW due Friday the 26th

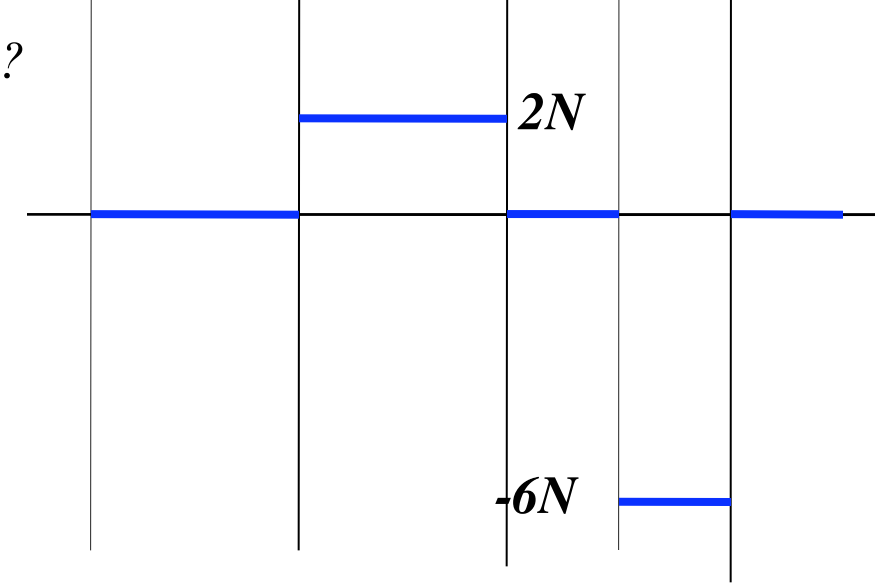
Test#2 (Ch. 7-9) will be at 6 PM, March 3 (6) Lockett)

Class Website:
http://www.phys.lsu.edu/classes/spring2010/phys2101-5/

Supplemental Homework #4: Look at the figure for potential vs x. The force acts on a 0.50 kg particles and U_A=3 J, U_B=7 J, U_C=9 J. (a) Calculate the magnitude and direction of the force in all five regions. (b) Draw the plot.



What happens at x=2, x=4, x=5, and x=6?

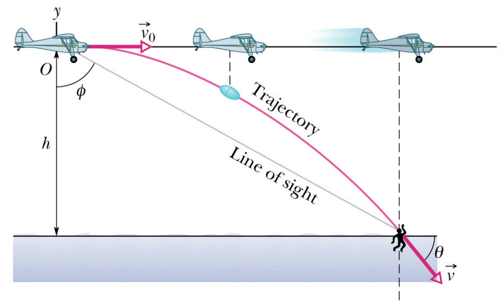


Chapter 9:

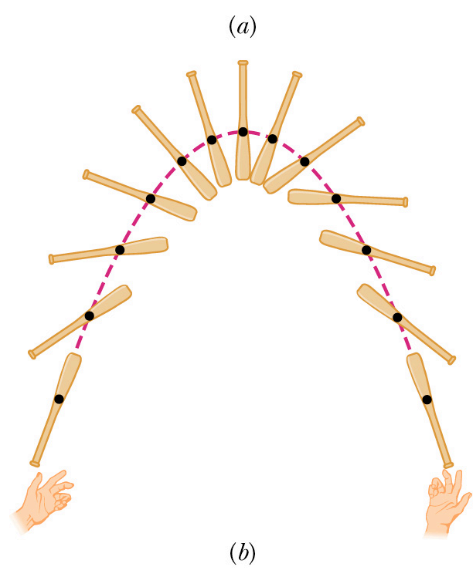
Center of Mass
+
Linear Momentum

The Center of Mass

Up to now, we've taken boxes, balls, pigs, penguins to be particles. We know how to apply Newton's laws to determine dynamics of a ***point*** particle.



Now we have a *real* mass (system of individual particles). How do we do this?



1) We find the <u>Center of Mass (COM) :</u>

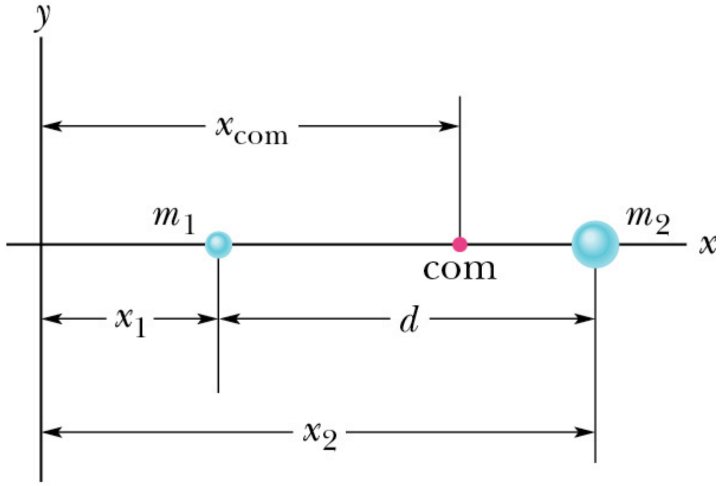
A point that moves as though all mass of a system were concentrated at that point *(doesn't have to be on the object)*

How to determine COM (*C*enter *O*f *M*ass) ?

Experimentally: *Find the point where it balances*
(as if all external forces are applied there)

Mathematically: *Find the "effective position"*

Simplest example: two particles



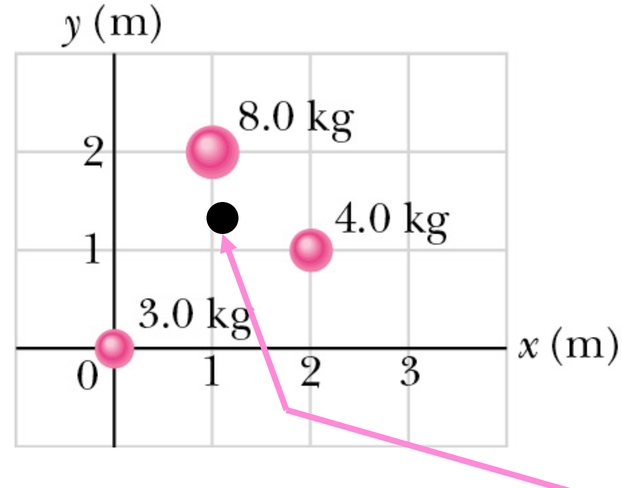
$$x_{com} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$$

$$y_{com} = \frac{m_1 y_1 + m_2 y_2}{m_1 + m_2}$$

$$= \frac{m_1(0) + m_2(0)}{m_1 + m_2} = 0$$

Note: Use symmetry!

Sample problem 9-3



What is the COM of the 3-particle system shown in the figure.

1) What is total mass

[redacted]

In General: How to determine COM?

N particles

with total mass M

$$M = \sum_{i=1}^{N} m_i$$

$$x_{com} = \frac{1}{M}\sum_{i=1}^{N} m_i x_i$$

$$y_{com} = \frac{1}{M}\sum_{i=1}^{N} m_i y_i$$

$$z_{com} = \frac{1}{M}\sum_{i=1}^{N} m_i z_i$$

In Vector notation:

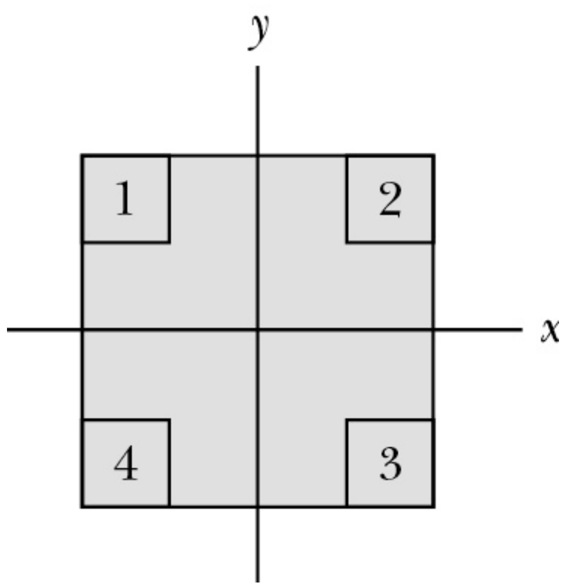
Position of ith particle: $\vec{r}_i = x_i\hat{i} + y_i\hat{j} + z_i\hat{k}$

Center of Mass : $\vec{r}_{com} = x_{com}\hat{i} + y_{com}\hat{j} + z_{com}\hat{k} = \frac{1}{M}\sum_{i=1}^{N} m_i\vec{r}_i$

Here N is large but still "countable"

Checkpoint

Checkpoint 1: ***The figure shows a uniform square plate from which four identical squares at the corners will be removed.*** *(answer all in terms of quadrants, axes, or points)*



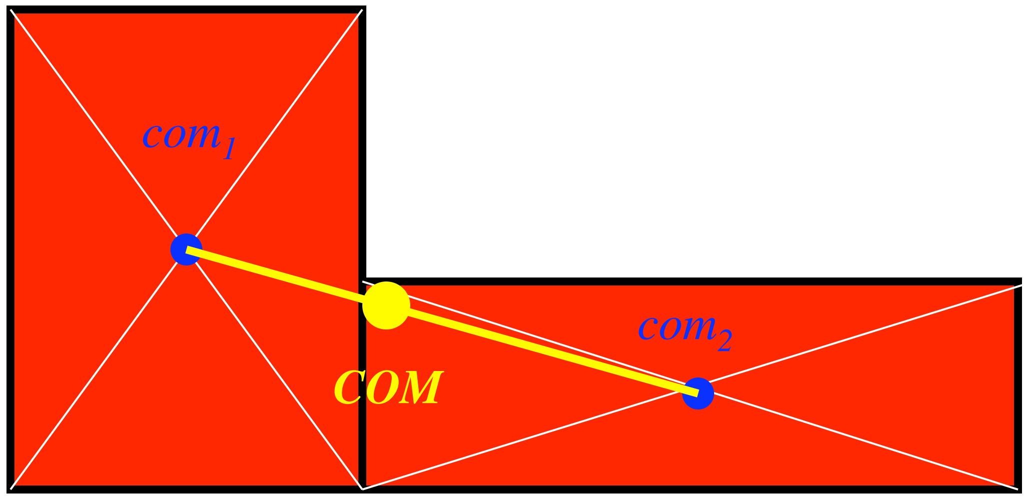
a) Where is the COM of the plate originally?

b) Where is the COM after removal of square 1?

c) Where is the COM after removal of square 1 & 2?

d) Where is the COM after removal of square 1, 2, & 3?

e) Where is the COM after removal of all squares?



Checkpoint

CheckPoint #1: ***The figure shows a uniform square plate from which four identical squares at the corners will be removed.*** *(answer all in terms of quadrants, axes, or points)*

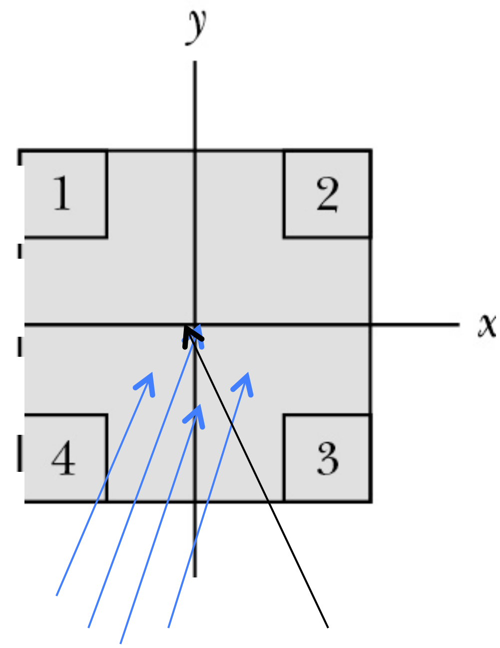
a) *Where is the COM of the plate originally?*

b) *Where is the COM after removal of square 1?*

c) *Where is the COM after removal of square 1 & 2?*

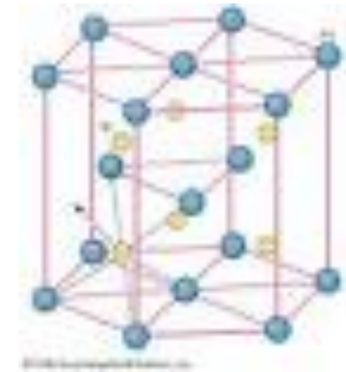
d) *Where is the COM after removal of square 1, 2 ,& 3?*

e) *Where is the COM after removal of all four squares?*



COM: Solid Bodies

What happens if N gets VERY large? For example a kilogram of material has 10^{26} atoms. Counting is impossible….



Treat as a continuum of material…

$$x_{com} = \frac{1}{M}\sum_{i=1}^{N} m_i x_i \longrightarrow x_{com} = \frac{1}{M}\int x\,dm \longrightarrow x_{com} = \frac{1}{V}\int x\,dV$$

$$y_{com} = \frac{1}{M}\sum_{i=1}^{N} m_i y_i \longrightarrow y_{com} = \frac{1}{M}\int y\,dm$$

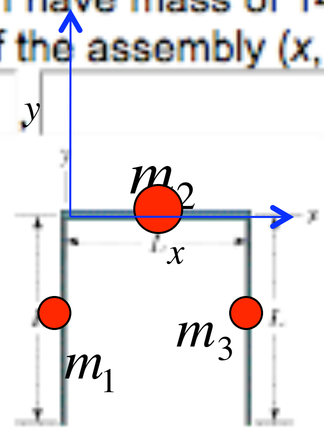
$$z_{com} = \frac{1}{M}\sum_{i=1}^{N} m_i z_i \longrightarrow z_{com} = \frac{1}{M}\int z\,dm$$

$$M = \sum_{i=1}^{N} m_i \longrightarrow \rho = \frac{dm}{dV} = \frac{M}{V}$$

Here "mass density" replaces mass $\rho=M/V$

If the figure below, three uniform thin rods, each of length L = 18 cm, form an inverted U. The vertical rods each have mass of 14 g; the horizontal rod has mass of 44 g. Where is the center of mass of the assembly (x, y)?

(,) cm



From symmetry $\Rightarrow x_{cm} = \frac{L}{2} = 9cm$

$$y_{cm} = \frac{y_1 m_1 + y_2 m_2 + y_3 m_3}{m_1 + m_2 + m_3}$$

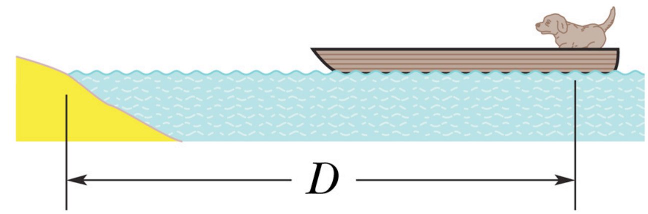
$$= \frac{(-\frac{L}{2}) \times 14 + (0) \times 44 + \left(-\frac{L}{2}\right) \times 14}{14 + 44 + 14} = -3.5cm$$

Rules of Center of Mass

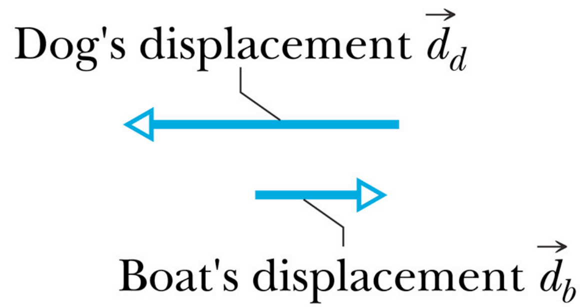
(1) Center of mass of a symmetric object always lies on an axis of symmetry.

(2) Center of mass of an object does NOT need to be on the object.

Problem #17: a dog of mass m_D stands on a boat of mass m_B at a distance D form the shore. The dow walx d m along the boat toward the shore and then stops. How far is the dog from the shore???



(a)



(b)

Step I: choose a system

$$\Rightarrow boat(m_b) + dog(m_d)$$

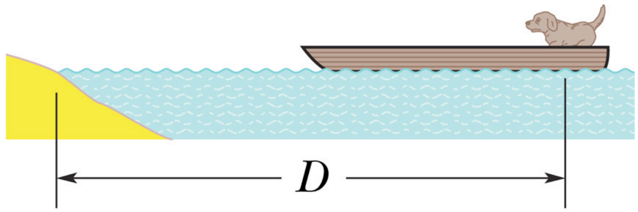
Step II: Find net force $F^x_{net} = 0 \Rightarrow$ COM remains unchanged

$$M\Delta x_{com} = 0 = m_B \Delta x_B + m_D \Delta x_D$$

the displacement of the boat is in the opposite direction of the dog, but

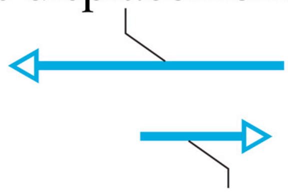
$$|\Delta x_B| = \frac{m_D}{m_B} |\Delta x_D|$$

Problem #17: a dog of mass m_D stands on a boat of mass m_B at a distance D form the shore. The dow walx d m along the boat toward the shore and then stops. How far is the dog from the shore???



(a)

$$|\Delta x_B| = \frac{m_D}{m_B}|\Delta x_D|$$



(b)

Important step: We look at the motion relative to the boat

The dog moved d relative to the boat

$$|\Delta x_B| + |\Delta x_D| = d$$

Now we substitute for Δx_B

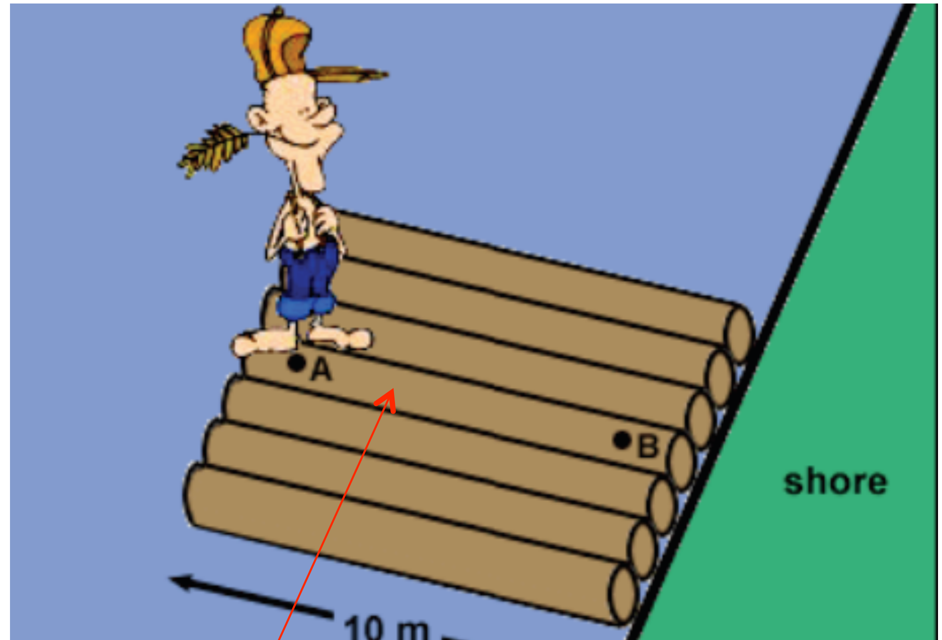
$$\frac{m_D}{m_B}|\Delta x_D| - |\Delta x_D| = d$$

$$|\Delta x_D| = \frac{d}{1 + m_D / m_B}$$

Look at position of Dog

$$x = D - \frac{d}{1 + m_D / m_B}$$

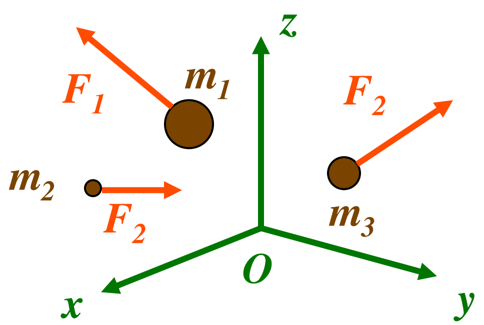
Supplemental Homework #5: A man, mass m_M stands at the edge of a raft of mass m_R that is L meters long. The edge of the raft is against the shore of the lake. The man walks toward the shore, the entire length of the raft. How far from the shore does the raft move?



This one is quite easy since it is symmetric.

[redacted]

Newton's 2nd Law for a System of Particles



Consider a system of n particles of masses $m_1, m_2, m_3, \ldots, m_n$ and position vectors $\vec{r}_1, \vec{r}_2, \vec{r}_3, \ldots, \vec{r}_n$, respectively.

The position vector of the center of mass is given by

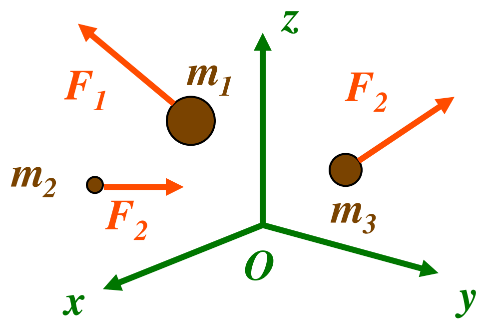
$M\vec{r}_{\text{com}} = m_1\vec{r}_1 + m_2\vec{r}_2 + m_3\vec{r}_3 + \ldots + m_n\vec{r}_n$. We take the time derivative of both sides $\rightarrow$

$$M\frac{d}{dt}\vec{r}_{\text{com}} = m_1\frac{d}{dt}\vec{r}_1 + m_2\frac{d}{dt}\vec{r}_2 + m_3\frac{d}{dt}\vec{r}_3 + \ldots + m_n\frac{d}{dt}\vec{r}_n \rightarrow$$

$M\vec{v}_{\text{com}} = m_1\vec{v}_1 + m_2\vec{v}_2 + m_3\vec{v}_3 + \ldots + m_n\vec{v}_n$. Here $\vec{v}_{\text{com}}$ is the velocity of the com and $\vec{v}_i$ is the velocity of the ith particle. We take the time derivative once more $\rightarrow$

$$M\frac{d}{dt}\vec{v}_{\text{com}} = m_1\frac{d}{dt}\vec{v}_1 + m_2\frac{d}{dt}\vec{v}_2 + m_3\frac{d}{dt}\vec{v}_3 + \ldots + m_n\frac{d}{dt}\vec{v}_n \rightarrow$$

$M\vec{a}_{\text{com}} = m_1\vec{a}_1 + m_2\vec{a}_2 + m_3\vec{a}_3 + \ldots + m_n\vec{a}_n$. Here $\vec{a}_{\text{com}}$ is the acceleration of the com and $\vec{a}_i$ is the acceleration of the ith particle.



$$M\vec{a}_{\text{com}} = m_1\vec{a}_1 + m_2\vec{a}_2 + m_3\vec{a}_3 + \ldots + m_n\vec{a}_n.$$

We apply Newton's second law for the ith particle:

$m_i\vec{a}_i = \vec{F}_i$. Here $\vec{F}_i$ is the net force on the ith particle,

$$M\vec{a}_{\text{com}} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \ldots + \vec{F}_n.$$

The force $\vec{F}_i$ can be decomposed into two components: applied and internal:

$\vec{F}_i = \vec{F}_i^{\text{app}} + \vec{F}_i^{\text{int}}$. The above equation takes the form:

$$M\vec{a}_{\text{com}} = \left(\vec{F}_1^{\text{app}} + \vec{F}_1^{\text{int}}\right) + \left(\vec{F}_2^{\text{app}} + \vec{F}_2^{\text{int}}\right) + \left(\vec{F}_3^{\text{app}} + \vec{F}_3^{\text{int}}\right) + \ldots + \left(\vec{F}_n^{\text{app}} + \vec{F}_n^{\text{int}}\right) \rightarrow$$

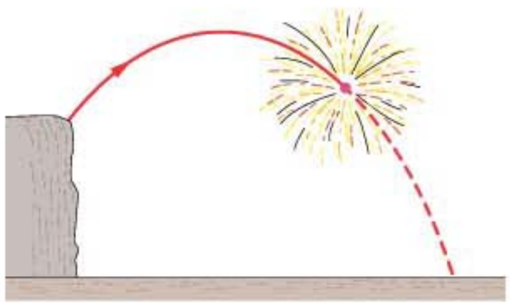
$$M\vec{a}_{\text{com}} = \left(\vec{F}_1^{\text{app}} + \vec{F}_2^{\text{app}} + \vec{F}_3^{\text{app}} + \ldots + \vec{F}_n^{\text{app}}\right) + \left(\vec{F}_1^{\text{int}} + \vec{F}_2^{\text{int}} + \vec{F}_3^{\text{int}} + \ldots + \vec{F}_n^{\text{int}}\right)$$

The sum in the second set of parentheses on the RHS vanishes by virtue of Newton's third law.

The equation of motion for the center of mass becomes $M\vec{a}_{\text{com}} = \vec{F}_{\text{net}}$.

In terms of components we have:

$$F_{\text{net},x} = Ma_{\text{com},x} \qquad F_{\text{net},y} = Ma_{\text{com},y} \qquad F_{\text{net},z} = Ma_{\text{com},z}$$

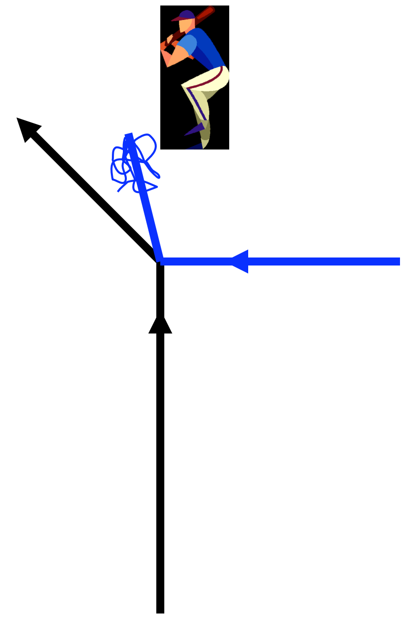


$$\boxed{M\vec{a}_{com} = \vec{F}_{net}}$$

$$\boxed{\begin{aligned} F_{net,x} &= Ma_{com,x} \\ F_{net,y} &= Ma_{com,y} \\ F_{net,z} &= Ma_{com,z} \end{aligned}}$$

A dramatic example is given in the figure. In a fireworks display a rocket is launched and moves under the influence of gravity on a parabolic path (projectile motion). At a certain point the rocket explodes into fragments. If the explosion had not occurred, the rocket would have continued to move on the parabolic trajectory (dashed line). The forces of the explosion, even though large, are all internal and as such cancel out. The only external force is that of gravity and this remains the same before and after the explosion. This means that the center of mass of the fragments follows the same parabolic trajectory that the rocket would have followed had it not exploded.

Collisions in 2D – elastic or inelastic?



Sample problem 9.3: The tree particles shown in the figure experience an *external* forces due to bodies outside of the system. F_1=6.0 N, F_2= 12 N, F_3=14 N.
What is the acceleration of the center of mass of the system? What direction?

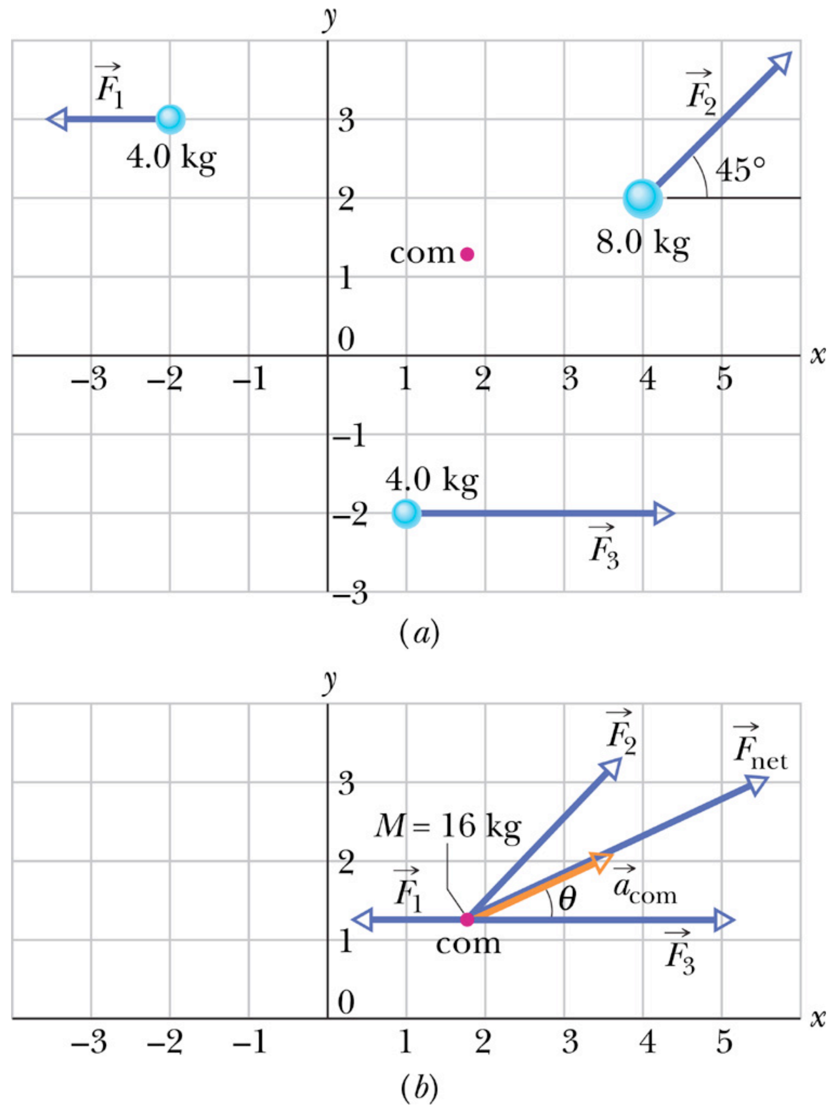
$$\vec{F}_{net} = M\vec{a}_{com}$$

$$\vec{a}_{com} = \frac{\vec{F}_1 + \vec{F}_2 + \vec{F}_3}{M}.$$

$$M = 4 + 8 + 4 = 16kg$$

$$a_{com,x} = \frac{F_{1x} + F_{2x} + F_{3x}}{M} = 1.03m/s^2$$

$$a_{com,y} = \frac{F_{1y} + F_{2y} + F_{3y}}{M} = 0.53m/s^2$$

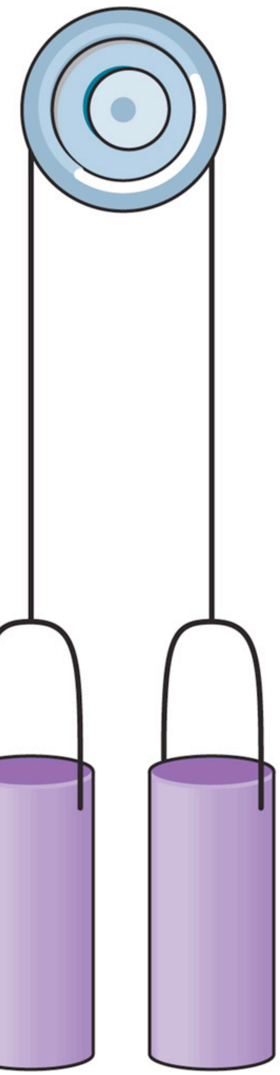


Checkpoint

Two skaters on frictionless ice hold opposite ends of a pole of negligible mass. An axis runs along the pole, and the origin of the axis is at the COM. On skater, Fred, weighs twice as much as the other skater, Ethel. Where do the skaters meet if (a) Fred pulls hand over hand along the pole so as to to draw himself to Ethel, (b) Ethel pulls hand over hand to draw herslef to Fred, and (c) both skaters pull hand over hand?

- Center of Mass

Problem #105: Tow identical containers of sugar are connected by a cord that passes over a frictionless pulley. Each container has mass m_0 and they are separated by d_0.
(a) Where is the cm originally?
(b) We now transfer Δm from 1 to 2. After the transfer describe the acceleration (magnitude and direction) of the CM.



(a) $$x_{cm} = \frac{m_0 x_1 + m_0\left(x_1 + d\right)}{m_0 + m_0} = \frac{d}{2}$$

(b) The acceleration of the cm is down-y direction

$$a_{com,y} = \frac{F_2 - F_1}{\left(m_0 + \Delta m\right) + \left(m_0 - \Delta m\right)}$$

(b) Putting in the appropriate values

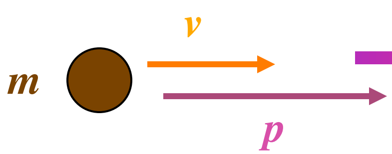
$$a_{com,y} = \frac{\left(m_0 + \Delta m\right)g - \left(m_0 - \Delta m\right)g}{2m_0}$$

$$a_{com,y} = \frac{\Delta m g}{m_0}$$

Check: Δm=0

Check: Δm=m_0

Linear Momentum of a Particle



$$\boxed{\vec{p} = m\vec{v}}$$

Linear momentum $\vec{p}$ of a particle of mass m and velocity $\vec{v}$ is defined as $\vec{p} = m\vec{v}$.

The SI unit for linear momentum is the kg.m/s.

Below we will prove the following statement: The time rate of change of the linear momentum of a particle is equal to the magnitude of net force acting on the particle and has the direction of the force.

In equation form: $\vec{F}_{\text{net}} = \dfrac{d\vec{p}}{dt}$. We will prove this equation using Newton's second law:

$$\vec{p} = m\vec{v} \rightarrow \frac{d\vec{p}}{dt} = \frac{d}{dt}\left(m\vec{v}\right) = m\frac{d\vec{v}}{dt} = m\vec{a} = \vec{F}_{\text{net}}$$

This equation is stating that the linear momentum of a particle can be changed only by an external force. If the net external force is zero, the linear momentum cannot change:

$$\boxed{\vec{F}_{\text{net}} = \frac{d\vec{p}}{dt}}$$

Linear Momentum of a Particle

Define Linear Momentum of a Particle: $$\vec{p} = m\vec{v}$$

$$\frac{d\vec{p}}{dt} = m\frac{d\vec{v}}{dt} = m\vec{a} = \vec{F}_{net,ext}$$

Linear Momentum

- **It is a vector, having the same direction as velocity;**
- **Any change in p is due to external force F ≠ 0;**
- **Conservation of linear momentum when F = 0.**

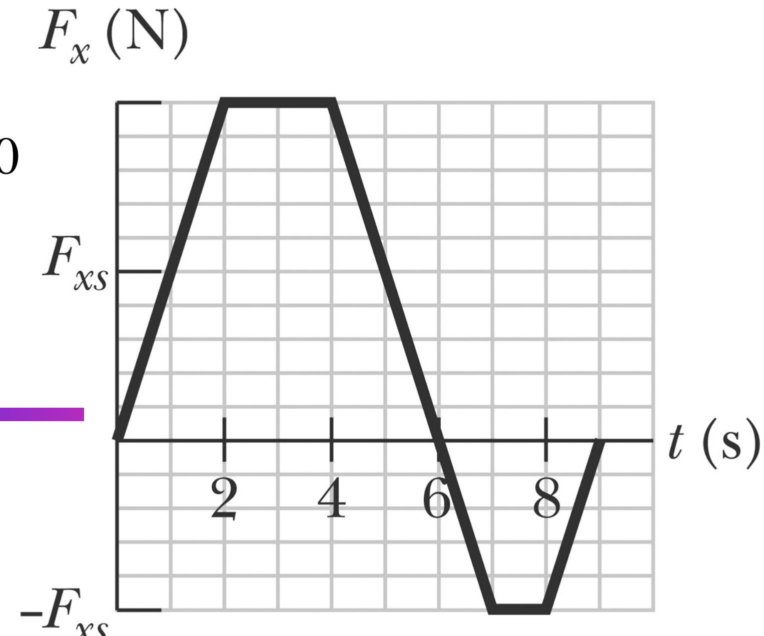
Problem #30: A toy car of mass m_t= 5 kg moves along the x axis under the influence of a force F_x shown in the figure as a function of time t. At t=0 F_x=0, and F_{xs}=5.0 N.

(a) what is **p** at 4.0, 7.0 seconds?

(b) What is **v** at 9.0 s?

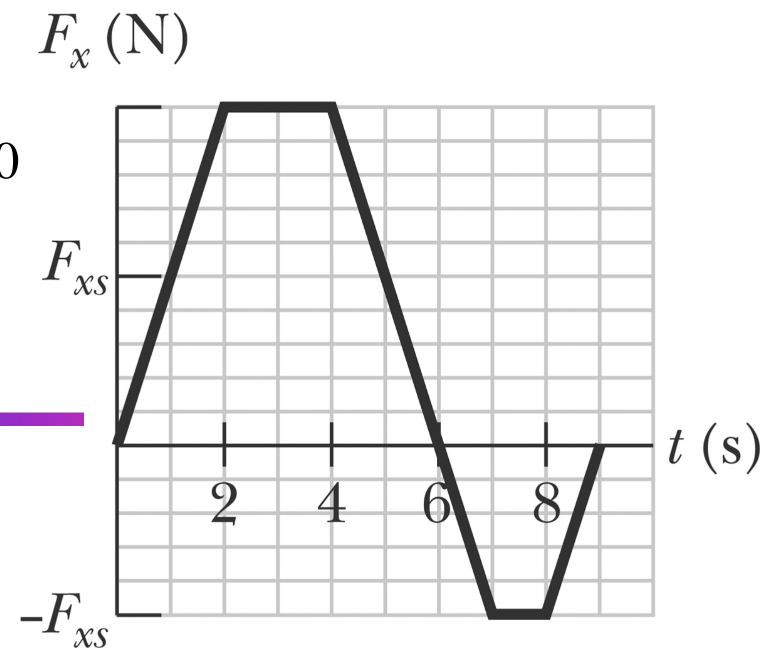
$$\vec{F} = \frac{d\vec{p}}{dt}$$

$$\vec{p} = \int_i^f \vec{F}dt$$



[redacted]

Problem #30: A toy car of mass m_t= 5 kg moves along the x axis under the influence of a force F_x shown in the figure as a function of time t. At t=0 F_x=0, and F_{xs}=5.0 N.
(a) what is **p** at 4.0, 7.0 seconds?
(b) What is **v** at 9.0 s?



$$\vec{F} = \frac{d\vec{p}}{dt}$$

$$\vec{p} = \int_i^f \vec{F}dt$$

From t=0 to t=2: $\vec{p} = \int_0^2 5t\,dt = \frac{5t^2}{2} = 10kg{\bullet}m/\sec$

From t=2 to t=4: $\vec{p} = \int_2^4 10\,dt = 20kg{\bullet}m/s$

From t=4 to t=6: $\vec{p} = \int_4^6 \left(10 - 5(t-4)\right)dt$

So $\vec{\text{p}}(4s) = 30\hat{i}kg{\bullet}m/s$

$$30(2) - \frac{5(6)^2}{2} + \frac{5(4)^2}{2} = 10kg{\bullet}m/s$$

From t=6 to t=7: $\vec{p} = \int_6^7 \left(0 - 5(t-6)\right)dt$

$$30(1) - \frac{5(7)^2}{2} + \frac{5(6)^2}{2} = -2/5kg{\bullet}m/s$$

Linear Momentum of a System of Particles

Total Linear Momentum of N particles:

$$\vec{P}_{tot} = \vec{p}_1 + \vec{p}_2 + \vec{p}_3 + \ldots + \vec{p}_N = \underbrace{M}_{\text{total mass } M}\underbrace{\vec{v}_{com}}_{\text{velocity of COM}}$$

$$\frac{d\vec{P}_{tot}}{dt} = M\,\frac{d\vec{v}_{com}}{dt} = M\vec{a}_{com} = \vec{F}_{net,ext}$$

Conservation of Linear Momentum

If External force is zero (isolated, closed system)…

$$\boxed{0 = \vec{F}_{net,ext} = \frac{d\vec{P}}{dt}}$$

$$\vec{P} = const$$

$$\Delta\vec{P} = 0$$

Question

Question 9-1

Two objects have the same momentum.

1. Their velocities must have the same magnitude and direction.
2. Their velocities have the same magnitude but direction can differ.
3. Their velocities can differ in magnitude but must have the same direction.
4. Their velocities can differ in magnitude and direction.