Physics 2101, Exam #3, Spring 2010

March 24, 2010

Name: Key

Section: (Circle one)

1 (Rupnik, MWF 8:40 AM)

2 (Rupnik, MWF 10:40 AM)

3 (Zhang, MWF 12:40 PM)

4 (Plummer, TTh 9:10)

5 (Jin, TTh 12:10)

6 (González, TTh 4:40)

7(Sprunger, TTh 1:40)

- Please be sure to write (print) your name and circle your section above.
- Please turn OFF your cell phone and MP3 player!
- Feel free to detach, use, and keep the formula sheet. No other reference material is allowed during the exam.
- You may use either a scientific or a graphing calculator...
- GOOD LUCK!

SHOW WORK on all problems that are NOT multiple choice

1) [5 pts] A flywheel, initially at rest, has a constant angular acceleration. After 9 s the flywheel has rotated 450 rad. Its angular acceleration in rad/s^2 is:

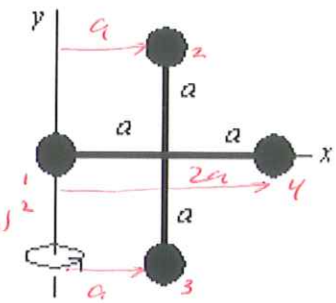
a) 11.1
b) 100
c) 15.9
d) 1.77
e) 50

$\omega_0 = 0 \quad t = 9\,\text{sec} \quad \Delta\theta = 450\,\text{rad}$

$\theta = \omega_0 t + \frac{1}{2}\alpha t^2$

$\alpha = \frac{2\theta}{t^2} = \frac{2(450\,\text{rad})}{(9\,\text{s})^2}$

2) [10 pts] Four identical point particles, each with mass $\boldsymbol{m}$, are arranged in the x, y plane as shown. They are connected by light (massless) sticks, with length $\boldsymbol{a}$, to form a rigid body. The rotational inertia of this array about the y axis is:



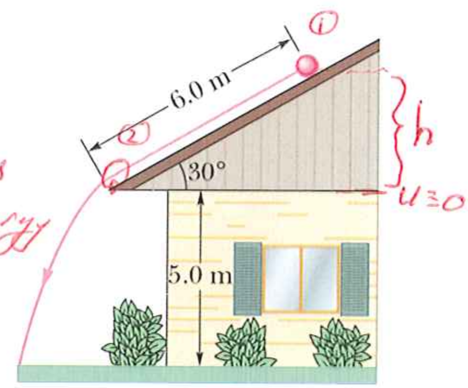
a) $4ma^2$
b) $6ma^2$
c) $7ma^2$
d) $8ma^2$
e) $16ma^2$

$I_{tot} = \sum_{i=1}^{4} m_i r_i^2$

$= m_1(0)^2 + m_2 a^2 + m_3 a^2 + m_4 (2a)^2$

$= 6ma^2$

3) A bowling ball ($I_{com} = \frac{2}{5}MR^2$) of radius 11.0 cm and mass 6.00 kg starts from rest and rolls without slipping a distance of 6.00 m down a house roof that is inclined at 30°.



(a) [10 pts] What is the *angular speed* of the bowling ball as it leaves the roof?

No work done by non-conservative forces
⟺ Conservation of mechanical energy

$E_{mech,\,initial} = E_{mech,\,edge}$

$KE_0 + U_0 = KE_1 + U_1$

$0 + mgL\sin\theta = \left(\frac{1}{2}mv_{com}^2 + \frac{1}{2}I_{com}\omega^2\right) + 0$

$I_{com} = \frac{2}{5}mr^2$ & for rolling $v_{com} = \omega r$

$\cancel{m}gL\sin\theta = \frac{1}{2}\cancel{m}(\omega_2 r)^2 + \frac{1}{2}\left(\frac{2}{5}\cancel{m}r^2\right)\omega_2^2$

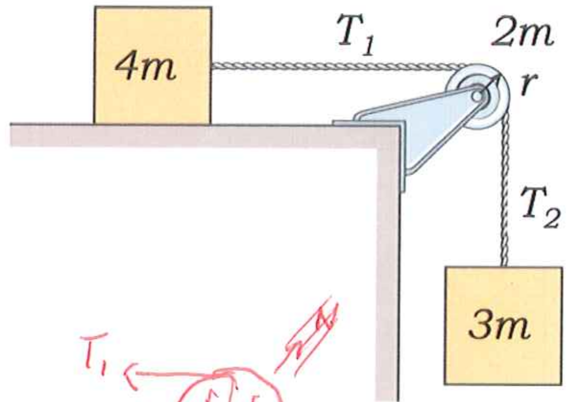
$\omega^2 = \frac{2gL\sin\theta}{(1 + 2/5)} \Rightarrow \omega_{edge} = \sqrt{\frac{10gL\sin\theta}{7}} = 59\,\text{rad/sec}$

(b) [5 pts] What is the *angular speed* of the bowling ball as it hits the ground [hint: what would cause an angular acceleration]?

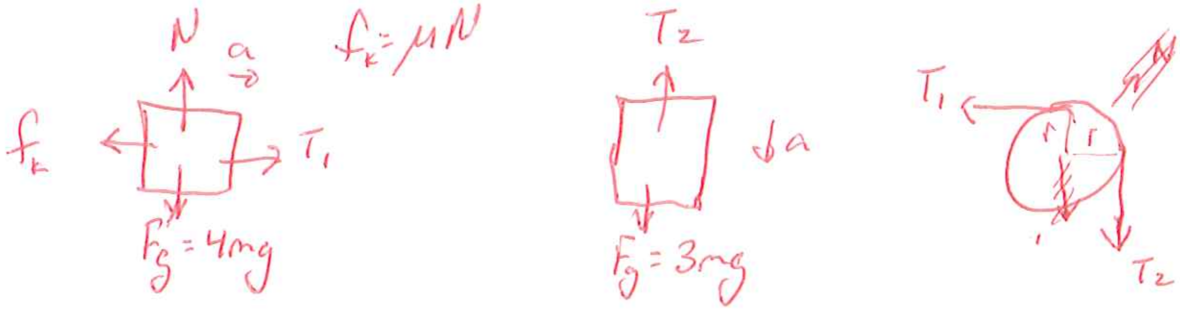
After leaving roof, there is no torque so that the angular acceleration is zero.
Thus $\boxed{\omega_{edge} = \omega_{ground} = 59\,\text{rad/s}}$

4) Two masses with mass **4*m*** and **3*m*** are set up as shown in the figure, with the mass **4*m*** on a surface with coefficient of kinetic friction $\mu_k = \frac{1}{4}$.

The pulley is a solid disk ($I_{disk} = \frac{1}{2}MR^2$) of mass **2*m*** and radius ***r***. The system starts accelerating from rest.



(a) [6 pts] Draw free body diagrams for the two masses and the pulley. Label all the forces.



(b) [7 pts] What is the linear acceleration of the system?

a) $\frac{2}{7}g$

b) $\frac{1}{4}g$

c) $\frac{3}{7}g$

d) $\frac{1}{2}g$

e) $\frac{4}{7}g$

$T_1 - f_s = 4ma \Rightarrow T_1 - \overset{1/4}{\mu}(4mg) = 4ma$ ①

$\hookrightarrow T_1 - \cancel{4}mg = 4ma$

$\tau_{net} = I\alpha$ & $a = r\alpha \rightarrow r(T_2 - T_1) = I\frac{a}{r}$

$T_2 - T_1 = \cancel{\tfrac{1}{2}(2m)a\frac{r^2}{r^2}}$

$T_2 - T_1 = ma$ ②

$T_2 - 3mg = -3ma$ ③

①+②+③ $\Rightarrow$ $3\cancel{m}g - \cancel{m}g = 4\cancel{m}a + \cancel{m}a + 3\cancel{m}a$

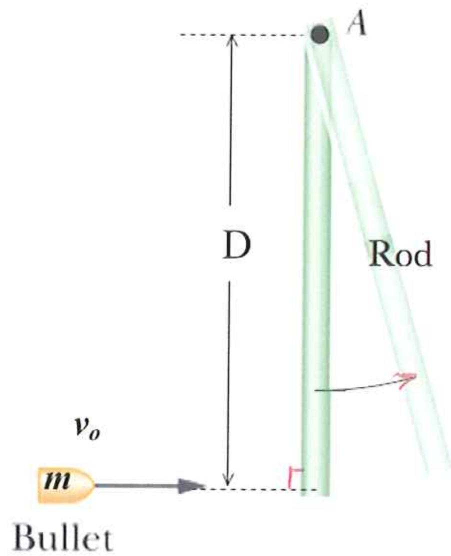
$\frac{2}{8}g = \boxed{a = \frac{1}{4}g}$

c) [2 pts] Is the tension below the pulley, ***T***$_2$, larger, equal to, or smaller than the tension to the left of the pulley, T_1? Circle the correct answer.

i) $T_1 > T_2$ ii) $T_1 = T_2 = 0$ iii) $T_1 = T_2 \neq 0$ iii) $T_1 < T_2$

from ② $\Rightarrow T_2 > T_1$

5) A *sticky* bullet of mass m is fired with speed v_0 toward the bottom end of a uniform steel ROD of mass M and length D hanging from a frictionless hinge at **A.** After a very short impact, the bullet sticks onto the rod. [Rotational inertial of a rod about its end is $I_{rod,end} = \frac{1}{3}ML^2$]



a) [5 pts] What is the magnitude of the angular momentum of the bullet relative to point **A** before the impact, in terms of m, v_0, D, and numerical constants, as needed.

$\vec{l} = \vec{r} \times \vec{p}$

$|l_i| = r(mv_0)\sin\theta$ 90°

$\boxed{l_i = DMv_0}$

b) [10 pts] What is the angular speed ω of the ROD just after the impact, in terms of v_o, D, m, M, and numerical constants, as needed.

No external torques during collision ⇒ conservation of angular momentum

$L_i = L_f$

$Dmv_0 = I\omega_f = (\underbrace{mD^2}_{bullet} + \underbrace{\tfrac{1}{3}MD^2}_{rod})\,\omega_f$

$\omega_f = \dfrac{Dmv_0}{mD^2 + \frac{1}{3}MD^2} = \boxed{\dfrac{mv_0}{D}\left(\dfrac{1}{m + \frac{1}{3}M}\right) = \omega_f}$

c) [5 pts] During the impact (while the bullet is sticking to the rod)

i) the frictional force between bullet and rod decreases the total angular momentum.

ii) the total angular momentum remains constant.

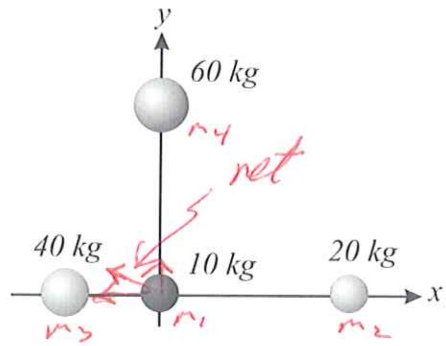
iii) the sum of the angular momentum and rotational kinetic energy remains constant.

iv) neither angular momentum nor mechanical energy is conserved because of the frictional forces between bullet and rod.

v) the frictional force between bullet and rod increases the total angular momentum.

6) The masses and coordinates of three spheres are as follows:

20 kg :	x = 2.0 m	y = 0 m
40 kg :	x = - 1.0 m	y = 0 m
60 g :	x = 0 m	y = 2.0 m



a) [8 pts] What is the *magnitude* of the x-component of the net force on a 10 kg sphere located at the origin due only to the other three spheres ($G = 6.67\times10^{-11}$ N·m^2/kg^2)?

i) 1.31×10^{-8} N

ii) 2.33×10^{-8} N

iii) 3.33×10^{-8} N

iv) 13.3×10^{-8} N

v) 21.5×10^{-8} N

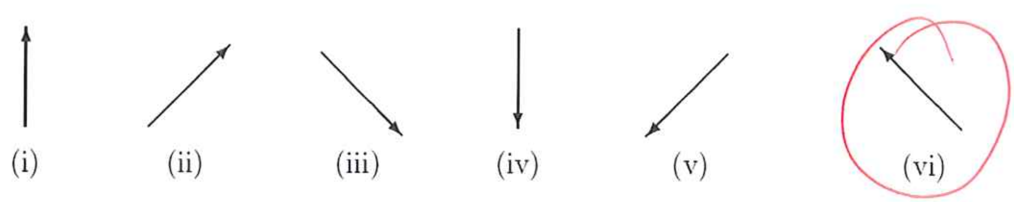
$$\vec{F}_x = \frac{Gm_1 m_2}{r_{12}^2}\hat{x} + \frac{Gm_1 m_3}{r_{13}^2}(-\hat{x})$$

$$= Gm_1\left(\frac{20\,\text{kg}}{(2m)^2} - \frac{40\,\text{kg}}{(-1m)^2}\right)\hat{x}$$

$$= -2.33\times10^{-8}\,N\,\hat{x}$$

$$|\vec{F}_x| = 2.33\times10^{-8}\,N$$

b) [2 pts] What is the approximate direction of the net gravitational force on the 10 kg sphere? Circle the correct choice. Hint: draw the forces in scale before answering.



7) [5pts] An object weighs 10 N on the earth's surface. What is the weight of the object on a planet that has one-tenth the earth's mass and one half the earth's radius?

(a) 4 N

(c) 1 N

(e) 20 N

(b) 2 N

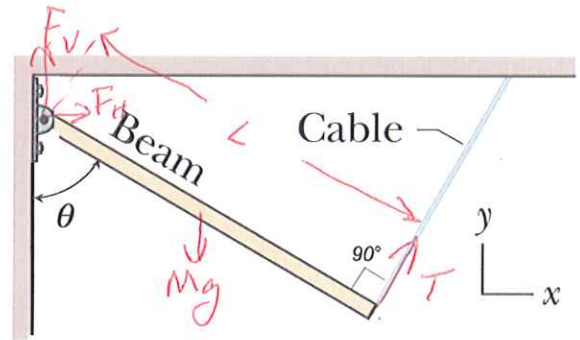
(d) 10 N

$$\text{Weight} = mg_p = m\frac{GM_p}{r_p^2} = \frac{mG\left(\frac{1}{10}M_E\right)}{\left(\frac{1}{2}R_E\right)^2}$$

$$= \underbrace{\frac{GmM_E}{R_E^2}}_{g_e}\frac{4}{10}$$

$$= \underbrace{mg_e}_{\text{Weight on earth}}\frac{4}{10} = (10N)\left(\frac{4}{10}\right) = 4N$$

8. The figure shows a uniform beam (rod) of mass M and length L that is supported on the left by a frictionless hinge attached to a wall. Originally, the rod is held stationary by a cable. The rod is at an angle θ with respect to the vertical wall. The cable is perpendicular to the rod.



a) [10 pts] While stationary, what is the magnitude of tension (T) in the cable, in terms of M, L, θ, g, and numerical constants, as needed ?

In equil'b $\Rightarrow$ $\Sigma F_x = 0$ $\quad \Sigma F_y = 0$

$+ \; \Sigma \tau_{\text{any point}} = 0$

$\Rightarrow$ Computing torque about hinge:

$0(F_V) + 0(F_H) - \left(\frac{L}{2}\right) Mg \sin\theta + TL = 0$

$$\boxed{T = \frac{Mg}{2} \sin\theta}$$

b) [10 pts] Then, the cable is cut and the beam is no longer in equilibrium. Find the initial *angular acceleration* of the beam about the hinge, in terms of M, L, θ, g, and numerical constants, as needed. [Rotational inertial of a rod about its end is $I_{rod,end} = \frac{1}{3}ML^2$]

Cut cable $\Rightarrow$ $T \to 0$

Now $\tau_{hinge} \neq 0$

$\Sigma \tau_{hinge} = I\alpha$

$\frac{\cancel{L}}{2} \cancel{M} g \sin\theta = \frac{1}{3} \cancel{M} L^{\cancel{2}} \alpha$

$$\boxed{\frac{3g \sin\theta}{2L} = \alpha}$$