Physics 2101, Exam #2, Spring 2010

March 3, 2010

Name: ____________________________

ID#: ______________________________

Section: (Circle one)

1 (Rupnik, MWF 8:40 AM) 5 (Jin, TTh 12:10)
2 (Rupnik, MWF 10:40 AM) 6 (González, TTh 4:40)
3 (Zhang, MWF 12:40 PM) 7 (Sprunger, TTh 1:40)
4 (Plummer, TTh 9:10)

- Please be sure to write (print) your name and circle your section above.

- Please turn OFF your cell phone and MP3 player!

- Feel free to detach, use, and keep the formula sheet. No other reference material is allowed during the exam.

- You may use either a scientific or a graphing calculator.

- Show all your work for the problems that are not multiple choice!

- There are a total of 10 problems (total points add to 100 pts)

- GOOD LUCK!
1. (Show all of your work) The figure at the right shows a potential energy diagram \( U \) as a function of \( x \) for an object of mass 1 kg.

(A) (5 pts) At the point \( x = 3 \text{ m} \) the object has a velocity of 2 m/sec. Find the mechanical energy \( E_{\text{mech}} \) for this system and draw a line on the figure to represent the mechanical energy.

\[
\begin{align*}
\text{at } x = 3 \text{ m} & : \quad U = 10 J \quad \text{(1)} \\
K &= \frac{1}{2} m v^2 = \frac{1}{2} (1 \text{ kg}) (2 \text{ m/s})^2 = 2 J \quad \text{(2)} \\
E_{\text{mech}} &= K + U = 12 J \quad \text{(3)}
\end{align*}
\]

(B) (5 pts) What is the allowed range of the motion along \( x \) given the mechanical energy found above (hint: you can read this off the graph)?

allowed range: \( 2.5 \text{ m} \leq x \leq 5.5 \text{ m} \) \( \text{(4)} \)

(C) (10 pts) What is the force (magnitude and direction) acting on the object in each regions of the potential energy diagram in the allowed range of motion? Your answers should first describe the region (\( x \) values) and then the magnitude and direction of the force.

\[
\begin{array}{c|c|c|c}
\text{(m) range} & \text{force} & \text{magnitude} & \text{direction} \\
\hline
2.5-4 & F = -\frac{6-14}{4-2} = +4 \text{ N} & 1F| = 4 \text{ N} & \text{dir} = +x \\
4-5 & F = 0 & \text{(\( k \) is constant there)} & \text{(2)} \\
5-5.5 & F = -\frac{18-6}{6-5} = -12 \text{ N} & 1F| = 12 \text{ N} & \text{dir} = -x \quad \text{(4)}
\end{array}
\]
2. (5 pts) A massless spring of spring constant $k = 25.0 \text{ N/m}$ hangs motionless from the ceiling. You carefully attach a mass to the end of the spring with a weight of 5 N without letting the spring stretch and then let go of the mass from rest. The spring stretches out a distance $d$ until it momentarily comes to rest. What is $d$ in meters?

(a) 0.2 m  
(b) 0.33 m  
(c) 0.40 m  
(d) 0 m  
(e) None of the above

\[ W = \Delta K = \Delta K = 0 \text{ initially and finally at rest} \]
\[ W = W_y + W_s = mgd - \frac{1}{2} kd^2 = 0 \]
\[ mgd = \frac{1}{2} kd^2 \]
\[ d = \frac{2mg}{k} = \frac{2(5 \text{ N})}{25 \text{ N/m}} = 0.4 \text{ m} \]

3. (5 pts) The figure at the right shows the position $x$ vs. $t$ for two objects of $m_1$ and $m_2$. The two particles undergo a one-dimensional collision. Which of the following choices most fully describes this collision?

(a) An inelastic collision  
(b) An elastic collision  
(c) An inelastic collision with $m_1 = m_2$.  
(d) An elastic collision with $m_1 = m_2$.  
(e) None of the above.

\[ m_2 \text{ is at rest and } m_1 \text{ moves initially} \]
\[ m_2 \text{ moves with } v_{1c} \text{ and } m_1 \text{ stops finally} \]

4. (5 pts) The figure presents the $x$ component $F_x$ of a force acting on a particle for four different situations as a function of $x$. The particle is at rest at $x = 0$. What figures have the largest (first answer) and smallest (second answer) work done by the force on the particle?

(a) a, d  
(b) d, a  
(c) b, d  
(d) c, d  
(e) b, a
5. The figure shows a particle on a track with elevated ends. The track is frictionless except for the horizontal region from $x=0$ to $x=L$. In this case $L=1$ m and the height of release $h=1$ m and $\mu_k=0.21$.

(A) (5pts) What is the maximum height the particle will achieve on the right hand side of the track after passing through the friction area once?

\[ W_{ec} = 0 \]
\[ K_i + U_i - K_f L = K_f + U_f \]

(a) 1 m
(b) 0.9 m
(c) 0.8 m
(d) 0.79 m
(e) 0.21 m
(f) 0 m

\[ h' = h - \mu_k L = 1 - (0.21)(1) = 0.79 \text{ m} \]

If 0.79 m is rounded to 0.8 m, OK

(B) (5pts) How many times will the particle pass completely through the region with friction? Left to right and right to left count as two.

(a) Zero times
(b) Once
(c) Twice
(d) Three times
(e) Four times
(f) Five times
(g) None of the above

\[ \Delta h = h - h' = \mu_k L \]
\[ \Delta h \cdot m = h \quad \Rightarrow \quad m = \frac{h}{\Delta h} = \frac{h}{\mu_k L} = \frac{1}{0.21} = 4.76 \]

\[ 4 \text{ complete passes} \]

6. (5pts) A 3 kg object is moving on a frictionless surface in the $x$ direction with a speed 10 m/sec. What is the maximum instantaneous power that can be delivered to the object by a 20 N applied force?

(a) Zero
(b) 9.8 W
(c) 30 W
(d) 200 W
(e) 60 W

\[ P = F \cdot v \]
\[ P = P_{\text{max}} \quad \text{if} \quad F \parallel v \quad \Rightarrow \quad P_{\text{max}} = |F| \cdot |v| = 20 \text{ N} \cdot 10 \text{ m/s} = 200 \text{ W} \]

\[ P_{\text{max}} = 200 \text{ W} \]
7. (5pts) Which one of the following is not a property of energy?
   (a) Energy is a vector quantity. (True)
   (b) Energy has units of Joules. (True)
   (c) Potential Energy can be positive or negative. (True)
   (d) Kinetic Energy must always be positive. (True)
   (e) Energy cannot be created nor destroyed. (True)

8. (Show your work) (30pts, 5 pts for each question). A bullet of mass \( m \) and velocity \( v_0 \) is moving in the positive x direction. At time \( t=0 \) s it collides with a block \( M \) attached to a spring with a spring constant \( k \). Initially the spring is relaxed. The bullet sticks in the block and we assume this collision is instantaneous. The block and the bullet move to the right until they momentarily come to a stop at \( x=d \) (spring compressed by distance \( d \)). The figure defines the coordinate system. Answer the following questions.

(A) This is an example of an/a __________ collision (circle one).
   Inelastic     Elastic     Completely Inelastic

(B) What is the instantaneous velocity \( v_f \) of the block and bullet immediately after the collision in terms of the variable listed above \( (m, M, v_0, \text{and } k, \text{as needed})\)?

\[ 2 \sum p_i = \sum p_f \]
\[ m \cdot v_0 = (m + M) \cdot v_f \]
\[ v_f = \frac{4m}{m + M} v_0 \]

(C) How much energy is lost during the instantaneous collision in terms of the variable listed above \( (m, M, v_0, \text{and } k, \text{as needed})\)? Where did it go?

\[ K_i - K_f = \frac{1}{2} m v_0^2 - \frac{1}{2} \frac{(m + M)^2}{m + M} \cdot \frac{m v_0^2}{M + m} \]
\[ v_0^2 = \frac{1}{2} m v_0^2 \left( 1 - \frac{m}{M + m} \right) \]
\[ = \frac{1}{2} \frac{m M v_0^2}{m + M} \]

The energy went into deformation of the wood (by the bullet) and produced sound and heat.
(D). What is the maximum compression \( d \) of the spring by the combined bullet and block. Your equation should be in terms of the variables listed above \( (m, M, v_0, k, a) \), as needed?

\[
\begin{align*}
K_i + U_i^c &= K_f + U_f^c \\
\frac{1}{2} (m + M) v_i^2 &= \frac{1}{2} k d^2 \\
U_i^c &= U_f^c
\end{align*}
\]

\[
\begin{align*}
U_{i-s} &= 0 \\
U_{f-s} &= \frac{1}{2} k d^2
\end{align*}
\]

\[
d = \frac{m v_0}{\sqrt{k (m + M)}}
\]

(E). On the coordinate system at the right, draw the Force on the bullet + block as a function of \( x \) between \( x = 0 \) and \( x = d \).

(F). What happens if there is friction on the horizontal surface where the block is sliding? Plot (using a dashed line on the same graph) the resultant (net) force on the block + bullet.

\[
\text{If positive force, } F_x \]

9. (5pts) A ball of mass \( m = 0.105 \) kg moves horizontally on a frictionless surface with velocity \( v_0 = 5.0 \) m/s in the positive \( x \) direction. It strikes a vertical wall and rebounds straight back but the collision is inelastic with the ball losing 19% of its initial energy. What is the magnitude and direction of the \( \vec{J} \) acting on the ball?

\[
\begin{align*}
\vec{v}_0 &= 5 \text{ m/s} \\
v_i &= 0.105 k g \\
k_f &= 0.81 \frac{1}{2} m v_i^2 = \frac{1}{2} \frac{1}{2} m v_i^2 \Rightarrow v_f = v_0 \sqrt{0.81} = 0.9 v_0 \\
\vec{F}_b &= \Delta \vec{P}_b = \vec{P}_{f-b} - \vec{P}_{i-b} = m (-0.9 v_0 - v_0) \hat{t} \\
&= (0.105 kg)(-1.9)(5 \text{ m/s}) \hat{t} = -0.9975 \hat{t} = -1 \text{ Ns} \hat{t}
\end{align*}
\]
10. An *Okie for Muskogee* stands at the edge of a raft of mass $m_R$ that is 10 m long, of uniform density, and whose edge is against the shore of a lake in Oklahoma. He wants to go ashore so he walks slowly to the other edge. Let the mass of the Okie $m_o = m_R = 60$ kg.

(A) (5 pts) After he walks the 10 m on the raft to come to the other side, how far from the shore is he?
- (a) 0 m
- (b) 2.5 m
- (c) 5 m
- (d) 7.5 m
- (e) 10 m
- (f) none of the above

Using $x_{\text{low}} = x_{\text{comp}}$,

\[
\frac{w_R \frac{L}{2} + w_o L}{w_o + w_R} = \frac{w_{0x} + w_{Rx} (\frac{L}{2} + x)}{w_{ox} + w_{Rx}} = \frac{5m}{10m} \quad \text{m}
\]

\[
\frac{3L}{2} = 2x + \frac{L}{2} \implies L = 2x \implies x = \frac{L}{2} = \frac{10}{2} = 5 \text{ m}
\]

(B) (5 pts) This is not a dumb Okie! He walks slowly back to where he started from and then takes off running as fast as he can. When he gets to the edge of the raft closest to the shore he has reached a speed of $v_0$ with respect to the raft. How fast is he moving with respect to the shore?
- (a) $v_0$ m/s
- (b) $> v_0$
- (c) $< v_0$
- (d) 0 m/s

**Calculation:**

\[
\frac{v}{t} = \frac{L - x}{t} = \frac{x}{t} = v_0 - \frac{v_0}{2} = \frac{v_0}{2}
\]

\[
v < v_0
\]

We could just answer that $v < v_0$ because while Okie runs in our direction relative to the raft, raft moves in the opposite direction relative to the shore, so Okie's speed is smaller than $v_0$, when measured relative to the shore.