## Announcements March 16, 2010 Tues ( $16^{\text {th }}$ )

- Midterm grades are posted : they're more of a guide ... you can do better (or worse...)
- HW \#7 is due Wed. night.
- HW \#8 WebAssign AND Supplemental has been posted (due on Monday) : NOTE - START NOW
- There will be a quiz over Supplemental \#7 on Thurs... Any questions??
- Remember that TEST \#3 will be next Wed. covering primarily Chapt. 10-13.4



## Class material

- Today: FINISH Chapt. 12 ... Start Chapt. 13
- Thurs: Finish Chapt. 13 (Gravitation) ..... Oh an quiz, too .... !!


## Chapt 12:

## Equilibrium (and Elasticity*)



* will not cover


## Again: Requirement of Equilibrium

$$
\begin{aligned}
& \vec{F}_{n e t}=\frac{d \vec{P}}{d t}=0 \\
& \vec{\tau}_{n e t}=\frac{d \vec{L}}{d t}=0
\end{aligned}
$$

1) Vector sum of all external forces that act on the body must be zero
2) Vector sum of all external torques that act on the body, measured about any possible point, must also be zero.

For motion only in $x-y$ plane

| Balance of <br> forces | Balance of <br> torques |
| :--- | :--- |
| $F_{n e t, x}=0$ | $\tau_{n e t, x}=0$ <br> $F_{n e t, y}=0$ |
| $\tau_{n e t, y}=0$ |  |
| $F_{n e t, z}=0$ | $\tau_{n e t, z}=0$ |



## Checkpoint 12-1

The figure gives an overhead view of a uniform rod in static equilibrium.
a) Can you find the magnitudes of $F_{1}$ and $F_{2}$ by balancing the forces?
b) If you wish to find the magnitude of $F_{2}$ by using a single equation, where should you place a rotational axis?


1) Draw a free-body diagram showing all forces acting on body
and the points at which these forces act.
2) Draw a convenient coordinate system and resolve forces into components.
3) Using letters to represent unknowns, write down equations for:
$\sum F_{x}=0, \sum F_{y}=0$, and $\sum F_{z}=0$
4) For $\sum \tau=0$ equation, choose any axis perpendicular to the xy plane. But choose judiciously!

Pay careful attention to determining lever arm and sign! [for xy-plane, ccw is positive \& cw is negative]
5) Solve equations for unknowns.

Checkpoint 1+2-4 A stationary $5 \mathrm{~kg} \operatorname{rod} A C$ is held against a wall by a rope and friction between the rod and the wall. The uniform rod is 1 m long and $\theta=30^{\circ}$.
(a) If you are to find the magnitude of the force $T$ on the rod from the rope with a single equation, at what labeled point should a rotational axis be placed?
(b) About this axis, what is the sign of the torque due to the rod's weight and the tension?


A small aluminum bar is attached to the side of a building with a hinge. The bar is held in the horizontal via a fish line with 14 lb test. The bar has a mass of 3.2 kg .
a) Will the $14 \mathrm{lb}(62.4 \mathrm{~N})$ hold the bar?
b) A nut is placed on the end and a squirrel of mass 0.6 kg is tries to climb out and get the nut. Does the squirrel get to the nut before the string breaks? If not, how far does he get?

## Example:

Squirrel trap

3m

## 

Sample Problem A ladder of length $\mathbf{L}$ and mass $\boldsymbol{m}$ leans against a slick (frictionless) wall. Its upper end is at a height $\boldsymbol{h}$ above the payment on which the lower end rests (the payment is not fricionless). The ladder's center of mass is $\boldsymbol{L} / \mathbf{3}$ from the lower end. A firefighter of mass $\boldsymbol{M}$ climbs the ladder until her center of mass is $\mathbf{L} / \mathbf{2}$ from the lower end.
a) What are the magnitudes of the forces exerted on the ladder?
b) Now assume that the wall is again frictionless and the pavement has a coefficient of friction $\mu_{\mathrm{s}}$. What is the maximum height the firefighter climbs ( $H_{\max }$ )

## Problem 12-37

What magnitude of force $\boldsymbol{F}$ applied horizontally at the axle of the wheel is necessary to raise the wheel over an obstacle of height $\boldsymbol{h}$ ? The wheel's radius is $\boldsymbol{r}$ and its mass is $\boldsymbol{m}$.


## Chapt. 13: Gravitation

Isaac Newton (1687) : What keeps the moon in a nearly circular orbit about the earth?

If falling objects accelerate, they must experience a force.
Force $=$ Gravity
No contact !


## Gravitation: Notes

## Gravitation: Notes

1) Three objects -- independent of each other

Newton's 3 ${ }^{\text {rd }}$ Law
2) Gravitational Force is a VECTOR

- unit vector notation

$$
\begin{array}{ccc}
\vec{F}_{12}=G \frac{m_{1} m_{2}}{r_{12}^{2}} \hat{r}_{12} & \text { Force on } m_{1} \text { due to } m_{2} & \hat{r}_{12}=\frac{\vec{r}_{12}}{\left|r_{12}\right|} \\
\vec{F}_{21}=G \frac{m_{1} m_{2}}{r_{21}^{2}} \hat{r}_{21} & \text { Force on } m_{2} \text { due to } m_{l} & \hat{r}_{21}=\frac{\vec{r}_{21}}{\left|r_{21}\right|}=-\hat{r}_{12} \\
\longrightarrow\left|\vec{F}_{21}\right|=\left|\vec{F}_{12}\right| & \vec{F}_{21}=-\vec{F}_{12}
\end{array}
$$

3) Principle of superposition

$$
\vec{F}_{1, n e t}=\vec{F}_{12}+\vec{F}_{13}+\vec{F}_{14}+\ldots+\vec{F}_{1 n}=\sum_{i=1}^{n} \vec{F}_{1 i} \quad \text { VECTOR ADDITION!! }
$$

4) A uniform spherical shell of matter attracts an object on the outside as if all the shell's mass were concentrated at its center (note: this defines the position)

$$
\text { height }=R_{E}+h
$$

## Checkpoint

What are the gravitational forces on the particle of mass $m_{1}$ due to the other particles of mass $m$.

What is the direction of the net gravitational force on the particle of mass $m_{1}$ due to the other particles.


The figure shows four arrangements of three particles of equal mass.

## Checkpoint

Rank the arrangements according to the magnitude of the net gravitational force on the particle labeled m, greatest first.

(1)

(4)

## Newton’s Law of Gravitation


$F=G \frac{m_{1} m_{2}}{r^{2}}$ This is always attractive. $\quad G=6.67 \times 10^{-11} \frac{\mathrm{~N} \cdot \mathrm{~m}^{2}}{\mathrm{~kg}^{2}}$


$$
\begin{aligned}
F & =m_{1} g=G \frac{m_{1} m_{2}}{r^{2}} \\
& =m_{1}\left(G \frac{m_{\text {EARTH }}}{r_{\text {EARTH }}^{2}}\right)
\end{aligned}
$$



Where does " $g$ " come from? " $m_{1}\left(6.67 \times 10^{-11} \frac{\mathrm{~N} \cdot \mathrm{~m}^{2}}{\mathrm{~kg}^{2}}\right) \frac{5.98 \times 10^{24} \mathrm{~kg}}{\left(6.38 \times 10^{6} \mathrm{~m}\right)^{2}}$
What is force of gravity at surface of earth?

$$
=m_{1}\left(9.8 \frac{m}{s^{2}}\right)=m_{1} g
$$

## Acceleration of Gravity at Hubble

What is the acceleration due to gravity at the Hubble Space Telescope? It orbits at an altitude of 600 km .

$$
\begin{aligned}
g_{H S T} & =G \frac{m_{E}}{r^{2}} \\
& =\left(6.67 \times 10^{-11} \frac{\mathrm{~N} \cdot \mathrm{~m}^{2}}{\mathrm{~kg}^{2}}\right) \frac{5.98 \times 10^{24} \mathrm{~kg}}{\left(6.38 \times 10^{6} \mathrm{~m}+6 \times 10^{5} \mathrm{~m}\right)^{2}} \\
& =8.19 \frac{\mathrm{~m}}{\mathrm{~s}^{2}}
\end{aligned}
$$

Why do astronauts float? => they are in free-fall How fast are they going ? We know from before that $\quad \frac{v^{2}}{r}=g_{H S T}$
so

$$
v=\sqrt{g_{H S T} r}=7550 \frac{\mathrm{~m}}{\mathrm{~s}}\left(\frac{1 \mathrm{mph}}{0.447 \mathrm{~m} / \mathrm{s}}\right)=17,000 \mathrm{mph}
$$

What is the time for one orbit? $T=\frac{2 \pi r_{O R B I T}}{v}=\left(\frac{2 \pi\left(6.98 \times 10^{6} \mathrm{~m}\right)}{7550 \mathrm{~m} / \mathrm{s}}\right)=5800 \mathrm{~s}=1.6$ hours

## Gravitational Force

Force on 1 due to 2:

$$
\vec{F}_{12}=G \frac{m_{1} m_{2}}{r_{12}^{2}} \hat{r}_{12} \quad G=6.67 \times 10^{-11} \mathrm{~N} \cdot \mathrm{~m}^{2} / \mathrm{kg}^{2}
$$

Force on 1 due to many: $\quad \vec{F}_{1, \text { et }}=\vec{F}_{12}+\vec{F}_{13}+\vec{F}_{14}+\ldots+\vec{F}_{1 n}=\sum_{i=1}^{n} \vec{F}_{1 i} \quad$ VECTOR ADDITION!!

On Earth

$$
\left.\begin{array}{rl}
\left|F_{g}\right| & =\left(G \frac{M_{E}}{R_{E}^{2}}\right) m_{\text {apple }}=m_{\text {apple }} g
\end{array} \begin{array}{r}
\text { Net force points towards } \\
\text { center of earth }
\end{array}\right\}
$$

## Problem 12-6

Three 5 kg spheres are located in the xy-plane. What is the magnitude and direction of the net gravitational force on the sphere at the origin due to the other two spheres?


## Gravitation and the earth



$$
\left|F_{g}\right|=\left(G \frac{M_{E}}{R_{E}^{2}}\right) m_{\text {apple }}=m_{\text {apple }} g
$$

Net force points towards center of earth

## $g$ differs around the earth (equator-9.780 \& north pole-9. $832 \mathrm{~m} / \mathrm{s}^{2}$ )

1) Earth is not a perfect sphere - height ( $R_{E}$ is not constant):

- On Mount Everest ( 8.8 km ) $g=9.77 \mathrm{~m} / \mathrm{s}^{2} \quad(0.2 \%$ smaller)
- At Equator earth bulges by 21 km

2) Earth is not uniform density: "gravity irregularities" $\left(10^{-6}-10^{-7}\right) g$ gravimeters can measure down to $10^{-9} g$
3) Earth is rotating: centripetal force makes apparent weight change

$$
\begin{array}{rr}
\text { At poles: } & \text { At equator: } \\
W-m g=0 & W-m g=-m\left(\frac{v^{2}}{R_{E}}\right) \\
W=m g & W=m\left(g-\frac{v^{2}}{R_{E}}\right)
\end{array}
$$

## Gravitational potential energy

From Section $8.3 \Rightarrow \quad-\Delta U=W_{\text {done by force }} \quad \Rightarrow$ Conservative force-path independent

$$
\begin{aligned}
& \Delta U_{g}=-W_{\text {done }}=-\int_{x_{i}}^{x} \vec{F}_{g} \bullet d \vec{x} \\
& W=\int_{r_{i}}^{r}\left(-G \frac{m M}{r^{2}}\right) d r=-G m M \int_{n_{i}}^{r}\left(\frac{1}{r^{2}}\right) d r
\end{aligned}
$$

At Earth's surface, $F_{g} \sim$ const.

$$
\Delta U_{g}=-W_{\text {done }}=-m(-g) \int^{v} d y=m g \Delta y
$$

If we define $U=0$ at $\infty$, then the work done by taking mass $m$ from $R$ to $\infty$

$$
\begin{aligned}
U_{\infty}-U(r) & =-W=-G m M\left\lfloor 0-\left(-\frac{1}{R}\right)\right\rfloor \\
U(r) & =-\frac{G m M}{r}
\end{aligned}
$$

$$
\begin{aligned}
& F(r)=-\frac{d U(r)}{d r} \\
& -\frac{d}{d r}\left(-\frac{G m M}{r}\right)=-\frac{G m M}{r^{2}}
\end{aligned}
$$

Note:

1) As before, Grav. Pot. Energy decreases as separation decreases (more negative)
2) Path independent
3) MUST HAVE AT LEAST TWO PARTICLES TO POTENTIAL ENERGY (\& force)
4) Knowing potential, you can get force....

## Gravitational potential energy

$$
\Delta U_{g}=-W_{\text {done }}=-\int_{x_{i}}^{x_{i}} \vec{F}_{g} \bullet d \vec{x} \Rightarrow \text { Conservative force-path independent }
$$

If we define $U=0$ at $\infty$, then the work done by taking mass $m$ from $R$ to $\infty$

$$
U(r)=-G \frac{m_{1} m_{2}}{\left|r_{12}\right|}
$$

$$
\vec{F}_{12}=G \frac{m_{1} m_{2}}{r_{12}^{2}} \hat{r}_{12}
$$

Note: Potential energy increases as you the separation gets larger:

$$
\text { If } r \uparrow \text { then }\left(\frac{G m M}{r}\right) \downarrow \text { and } \mathrm{U} \text { gets less negative (larger) }
$$

## Gravitational Potential Energy of SYSTEM

Scalar - just add up total potential energy
(BE CAREFUL: don't double count)
Worlis
$U_{t o t}=-G\left(\frac{m_{1} m_{2}}{r_{12}}+\frac{m_{1} m_{3}}{r_{13}}+\frac{m_{2} m_{3}}{r_{23}}\right)$


## Problem 12-35

Three spheres with mass $m_{A}, m_{B}$, and $m_{C}$. You move sphere B from left to right.

How much work do you do on sphere B ?
How much work is done by the gravitational force?

Gravitational Potential Energy


Escape speed: Conservation of Mechanical Energy

## Escape speed: Conservation of Mechanical Energy

Escape speed: minimum speed ( $v_{\text {excape }}$ ) required to send a mass $m$, from mass $M$ and position $R$, to infinity, while coming to rest at infinity.

At infinity: $E_{\text {mech }}=0$ because $U=0$ and $K E=0$
Thus any other place we have:

$$
\begin{aligned}
E_{\text {mech }}=\left(K E+U_{g}\right)=0 \Rightarrow E_{\text {mech }} & =\left(\frac{1}{2} m v^{2}-\frac{G m M}{R}\right)=0 \Rightarrow v_{\text {escape }}=\sqrt{\frac{2 G M}{R}} \\
& \text { Earth }=11.2 \mathrm{~km} / \mathrm{s}(25,000 \mathrm{mi} / \mathrm{hr}) \\
\text { Escape speed: } \quad & \text { Moon }=2.38 \mathrm{~km} / \mathrm{s} \\
& \text { Sun }=618 \mathrm{~km} / \mathrm{s}
\end{aligned}
$$

## Problem 10-39

A projectile is fired vertically from the Earth's surface with an initial speed of $10 \mathrm{~km} / \mathrm{s}(22,500 \mathrm{mi} / \mathrm{hr})$
Neglecting air drag, how far above the surface of Earth will it go?

$$
\begin{array}{rlrl}
\left(K E_{i}+U_{i}\right) & =\left(K E_{f}+U_{f}\right) & & R_{E}=6380 \cdot \mathrm{~km} \\
\left(\frac{1}{2} m v_{i}^{2}-G \frac{m M_{E}}{r_{i}}\right) & =\left(0-G \frac{m M_{E}}{r_{f}}\right) & G M_{E}=4 \times 10^{14} \cdot \mathrm{~m}^{3} / \mathrm{s}^{2}
\end{array}
$$

## Chapt. 13: review

Force on 1 due to 2:

$$
\vec{F}_{12}=G \frac{m_{1} m_{2}}{r_{12}^{2}} \hat{r}_{12}
$$

$$
G=6.67 \times 10^{-11} \mathrm{~N} \cdot \mathrm{~m}^{2} / \mathrm{kg}^{2}
$$

Force on 1 due to many: $\quad \vec{F}_{1, n e t}=\vec{F}_{12}+\vec{F}_{13}+\vec{F}_{14}+\ldots+\vec{F}_{1 n}=\sum_{i=1}^{n} \vec{F}_{1 i} \quad$ VECTOR ADDITION!!

## Gravitational potential energy

If we define $U=0$ at $\infty$, then the work done by taking mass $m$ from $R$ to $\infty$

$$
U(r)=-G \frac{m_{1} m_{2}}{\left|r_{12}\right|} \quad \vec{F}_{12}=G \frac{m_{1} m_{2}}{r_{12}^{2}} \hat{r}_{12}
$$

