Ch. 24 Electromagnetic Waves

Take a single positive charge and wiggle it up and down:

The charge changes position as a function of time, so as the electric field $E$.

But since the charge is moving, it constitutes a current:

The current points up when the charge moves up, and the current points down when the charge moves down.

This current, like all currents, creates a magnetic field!

The direction of the field is given by RHR-2.
By RHR-2, we see that when the current points up, the mag. field points into the screen, and when the current points down, the mag. field points out of the screen.

Thus, I have a changing magnetic field and a changing electric field which are oriented at right angles to each other!

The electric field is in the xz-plane, and the magnetic field is in the xy-plane.

The fields move out away from the source (our accelerating charge):

**Propagation of Electromagnetic (EM) Waves**

An EM wave is a **transverse wave**: The wave motion is at right angles to the direction of propagation.
It was James Clerk Maxwell (1831-1879) who worked out the mathematics of
the wave propagation:

In words: The changing electric field induces a magnetic field (which also
changes), and this changing magnetic field induces an electric field, etc.

This is how the wave propagates!

EM waves don’t need a medium to travel through. They can propagate
through a vacuum.

How fast do EM waves travel?

We can answer this question by looking at Maxwell’s wave equation:

$$\nabla \times B = \mu_0 J + \mu_0 \varepsilon_0 \frac{dE}{dt}$$

From partial differential equations, we identify the speed\(^2\) of the wave as
one over the coefficient on \(dE/dt\).
Only fields exist in the free space, the wave equation becomes:

\[ \nabla \times B = \frac{1}{v^2} \frac{dE}{dt} \quad \nabla \times E = -\frac{dB}{dt} \]

So \( v^2 = \frac{1}{\mu_0 \varepsilon_0} \Rightarrow v = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} \)

\( \Rightarrow v = \frac{1}{\sqrt{(4\pi \times 10^{-7})(8.85 \times 10^{-12})}} \Rightarrow v = \frac{1}{3.334 \times 10^{-9}} \)

\( \Rightarrow v = 2.999 \times 10^8 \text{ m/s} \)

Wow! EM waves propagate at the speed of light!!!!
Review:

1. Stationary charges create electric fields.
2. Moving charges (constant velocity) create magnetic fields.
3. Accelerating charges create electromagnetic waves.

24.2 Electromagnetic Spectrum

Since Maxwell discovered that EM waves move at the speed of light, he hypothesized that light itself must be an EM wave!

Like any wave, EM waves have a frequency, a period, and an amplitude.

From Ch. 16, we know that: \( v = f \lambda \)

And since \( v = c \), we get for EM waves: \( c = f \lambda \)
Higher frequencies mean shorter wavelengths!

\[ f = \frac{c}{\lambda} \]

The Electromagnetic Spectrum
24.3 The Speed of Light

Very fast!!!!.....but finite. $c = 3.00 \times 10^8 \text{ m/s}$

Moon to Earth → 1.3 seconds
Sun to Earth → 8 minutes

Distant stars and other astronomical objects are so far away that astronomers use a unit of distance called the light year (ly).

1 ly = The distance light travels in 1 year = $9.5 \times 10^{15} \text{ m}$
The FM station broadcasting at 103.3 MHz plays music for the “Diva in all of us”. What is the wavelength of these radio waves?

1. 0.344 m
2. 2.9 m
3. 3.4 × 10^{-7} m
4. 2.9 × 10^6 m
24.4 Energy Carried by EM Waves

EM waves carry energy just like any other wave.

An EM wave consists of both an electric and magnetic field, and energy is contained in both fields.

Energy density in electric field (in free space):

\[
\text{Electrical Energy Density} = \frac{\text{Electrical Energy}}{\text{Volume}} = \frac{1}{2} \varepsilon_o E^2
\]

Energy density in magnetic field (in free space):

\[
\text{Magnetic Energy Density} = \frac{1}{2\mu_o} B^2
\]

Notice that the energy goes as the square of the fields.
So the total energy density in an EM wave is the sum of these two:

\[ u = \frac{1}{2} \varepsilon_o E^2 + \frac{1}{2\mu_o} B^2 \]

But in an EM wave, the electric field and magnetic field carry the same energy.

Thus,

\[ \frac{1}{2} \varepsilon_o E^2 = \frac{1}{2\mu_o} B^2 \]

This allows me to write the total energy density in terms of just \( E \) or just \( B \):

\[ u = \varepsilon_o E^2 = \frac{1}{\mu_o} B^2 \]

Since,

\[ \frac{1}{2} \varepsilon_o E^2 = \frac{1}{2\mu_o} B^2 \Rightarrow E^2 = \frac{1}{\mu_o \varepsilon_o} B^2 \Rightarrow E^2 = \frac{1}{c^2} B^2 \]
So in an EM wave, the magnitude of the electric field is proportional to the magnitude of the magnetic field, and the proportionality constant is \( c \), the speed of light!

The magnitude of the electric and magnetic fields in an EM wave fluctuate in time. It is useful to consider an *average value* of the two fields:

This is called the rms or *root-mean-square* value of the fields:

\[
E_{rms} = \frac{E_o}{\sqrt{2}} \quad B_{rms} = \frac{B_o}{\sqrt{2}}
\]

Here, \( E_o \) and \( B_o \) are the peak values of the two fields.

Now we can calculate an average energy density using the rms values:

\[
\bar{u} = \frac{1}{2} \varepsilon_o E_{rms}^2 + \frac{1}{2\mu_o} B_{rms}^2
\]
So as the EM waves propagate thru space, they carry energy along with them. The transport of energy is related to the intensity of the wave.

Back in Ch. 16 we defined the intensity of a wave as the power per unit area:

\[ S = \frac{P}{A} \Rightarrow \frac{W}{tA} \Rightarrow \frac{\text{Energy}}{tA} \]

What is the relationship between the intensity, \( S \), and the energy density, \( u \)?

Sparing you the derivation, we find that \( S = cu \)

So the intensity and energy density are just related by the speed of light, \( c \).

In terms of the fields then:

\[ S = cu = \frac{1}{2} c \varepsilon_o E^2 + \frac{c}{2 \mu_o} B^2 \]

\[ S = c \varepsilon_o E^2 \]

\[ S = \frac{c}{2 \mu_o} B^2 \]

If we use the rms values for the fields, then \( S \rightarrow \bar{S} \), the average intensity.
Both the electric and magnetic field of an EM are doubled in magnitude. What happens to the intensity of the wave?

1. Nothing
2. It doubles
3. **It quadruples**
4. It decreases by a factor of 2
5. It decreases by a factor of 4
The Doppler Effect

When the observer of a wave, or source of the wave (or both) is moving, the observed wave frequency is different than that emitted by the source.

EM waves also exhibit a Doppler effect. But….

1. They don’t require a medium thru which to propagate, and..

2. Only the relative motion of the source to the observer is important, since the speed at which all EM waves move is the same, the speed of light.

So how do we calculate the shift in frequency?

If the EM wave, the source, and the observer all travel along the same line, then:

\[
fo = fs \left(1 \pm \frac{v_{rel}}{c}\right)
\]

- \(f_o\) is the observed frequency
- \(f_s\) is the frequency emitted by the source
- \(v_{rel}\) is the relative velocity between observer and source

The + sign is used when the object and source move toward each other.

The – sign is used when the object and source move away from each other.

(*This is valid for speeds \(v_{rel} << c\).)
Astronomers can use the Doppler effect to determine how fast distant objects are moving relative to the earth.

**Example**

A distant galaxy emits light that has a wavelength of 500.7 nm. On earth, the wavelength of this light is measured to be 503.7 nm. (a) Decide if the galaxy is moving away from or toward the earth. (b) Find the speed of the galaxy relative to earth.

**Solution**

We start with the Doppler equation: 

\[ f_o = f_s \left(1 \pm \frac{v_{rel}}{c}\right) \]

The light is shifted to longer wavelengths, which means smaller frequencies: 

\[ f = \frac{c}{\lambda} \]

Thus, \( f_o < f_s \). Which means that the parenthesis \( \left(1 \pm \frac{v_{rel}}{c}\right) \) must be < 1.

Therefore, the correct sign in the parentheses is the – sign: the galaxy is moving away from earth.

(b) From the Doppler equation: 

\[ v_{rel} = c \left(1 - \frac{f_o}{f_s}\right) \]

Thus, 

\[ v_{rel} = c \left(1 - \frac{\lambda_s}{\lambda_o}\right) = 3 \times 10^8 \left(1 - \frac{500.7 \text{ nm}}{503.7 \text{ nm}}\right) = 1.8 \times 10^6 \text{ m/s} \]
24.6 Polarization

EM waves are transverse waves and can be polarized.

Consider the electric field part of an EM wave.

It oscillates up and down as the wave propagates:

Thus, the wave oscillations are perpendicular to the direction of travel and occur in only one direction.

We refer to this wave as **linearly polarized**.

A vertical slit would allow the wave to pass through, since the slit is parallel to the oscillations:

A horizontal slit would block the wave and not allow it to pass, since the slit is perpendicular to the oscillations:
So how is polarized light produced?

We could use the antenna on a radio station:

Antenna

Direction of wave travel

Incandescent light is **unpolarized**, resulting from many atoms vibrating in all possible orientations.
We can convert **unpolarized** light into **polarized** light by blocking all but one of the electric field orientations.

A device that does this is called a **polarizer or polaroid**.

The one direction that a polarizer allows light to pass thru it is called the **transmission axis**.

Let’s start with unpolarized light and pass it thru a polarizer:

If the intensity of the unpolarized light is $S$ before it passes thru the polarizer, then the intensity of the polarized light coming out will be $\frac{1}{2}S$. 
**Malus’ Law**

Once the light has been polarized, it’s possible for another polarizer (called the **analyzer**) to change the direction and intensity of the polarized light.

We know from our earlier discussion on intensity that:  
\[ S = c \varepsilon_o E^2 \]

Therefore:  
\[ S \propto E^2 \].  
Out of the analyzer then,  
\[ S \propto E^2 \cos^2 \theta \].

So both the intensity and polarization direction can be adjusted by changing the angle of the analyzer.

The average intensity of the light leaving the analyzer is:  
\[ \overline{S} = \overline{S_o} \cos^2 \theta \]

\( \overline{S_o} \) is the average intensity of the light entering the analyzer.