Chapter 19: Electric Potential Energy & Electric Potential

Why electric field contains energy?

Is there an alternative way to understand electric field?

Concepts:

- **Work done by conservative force**
- **Electric potential energy**
- **Electric potential**

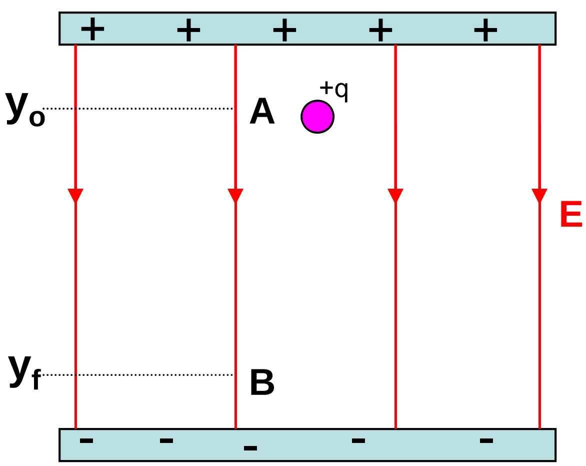
19.1 Potential Energy

The electric force, like gravity, is a conservative force:

Recall Conservative Forces

1. **The work done on an object by a conservative force depends only on the object's initial and final position, and not the path taken.**
2. **The net work done by a conservative force in moving an object around a closed path is zero.**

Let's place a positive point charge q in a uniform electric field and let it move from point A to B (no gravity):



How much work is done by the field in moving the charge from A to B?

*** Remember, $W = F_d \times d$, where F_d is the component of the constant force along the direction of the motion.**

Here, $F = qE$ **, so** $W = qE(y_f - y_o) = \underbrace{qE\Delta y}$

Electrostatic Potential Energy (EPE)

Thus, $\boxed{W_{AB} = \Delta EPE}$ **The work done is equal to the change in electrostatic potential energy!**

Now let's divide both sides by the charge, q:

$$\frac{W_{AB}}{q} = \boxed{\frac{\Delta EPE}{q}}$$

The quantity on the right is the potential energy per unit charge. We call this the Electric Potential, V:

$$\boxed{V = \frac{EPE}{q}}$$

The electric potential is a scalar!

Units?

$$\left[\frac{\text{Energy}}{\text{Charge}}\right] = \left[\frac{J}{C}\right] = [\text{Volt}] = [V]$$

Review of Work: **1. Work is not a vector, but it can be either positive or negative:**

Positive – Force is in the same direction as the motion

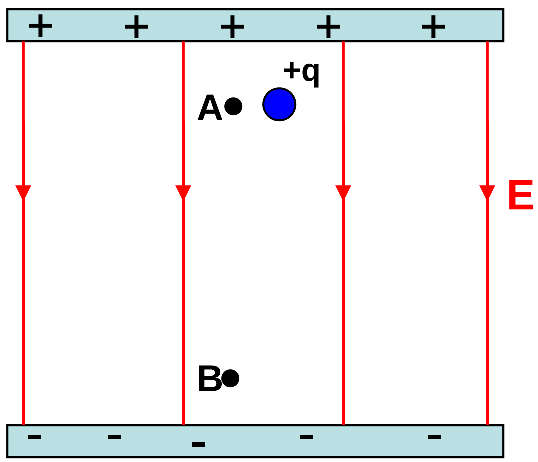
Negative – Force is in the opposite direction as the motion

2. If positive work is done on an object, the object speeds up.

3. If negative work is done on an object, the object slows down.

19.2 Electric Potential Difference

We can talk about the value of the potential at different points in space:
For example, what is the difference in electrostatic potential between two points, A and B, in an electric field???



$$V_B - V_A = \frac{EPE_B}{q} - \frac{EPE_A}{q} = \frac{-W_{AB}}{q}$$

So, $$\boxed{\Delta V = V_B - V_A = \frac{-W_{AB}}{q}}$$

Why is there a minus sign???

Let's say the charge at point A is positive:

If I release it, which way will it move? **It moves down toward B!**

Since the force is down and the motion is down, positive work is done on the charge. Thus, W_{AB} is positive.

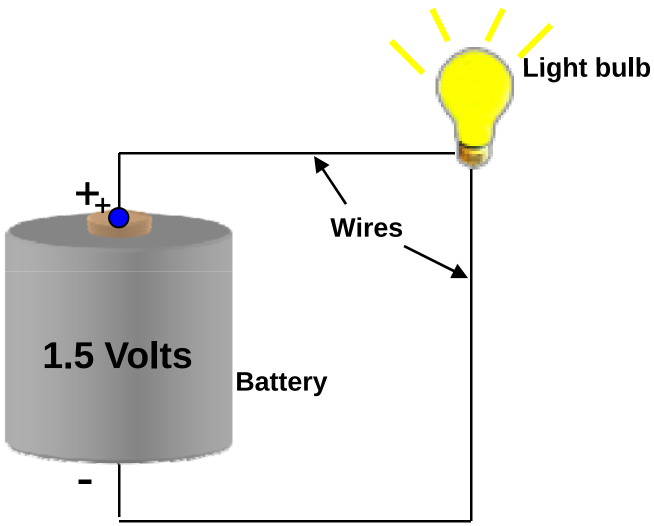
This means that ($V_B - V_A$) is negative, or $V_A > V_B$.

We say that point A is at a higher potential than point B.

Positive charges, starting from rest, will accelerate from regions of high potential and move toward regions of low potential.

Negative charges, starting from rest, will accelerate from regions of low potential and move toward regions of high potential.

One common object associated with voltages is a battery:



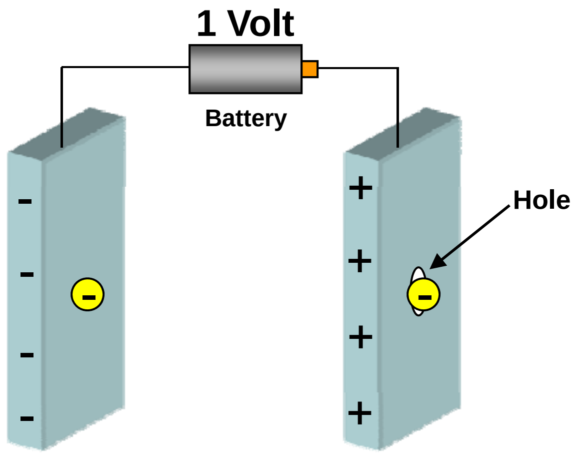
$$1.5\,\mathrm{V} = 1.5\,\frac{\mathrm{J}}{\mathrm{C}} \longrightarrow$$

The battery supplies 1.5 Joules of energy for every coulomb of charge.

Notice that the positive charge moves from higher potential (+) to lower potential (-).

The word “volt” also appears in a unit of energy:

Let’s accelerate an electron from rest through a potential difference of 1 Volt:



The electron gets accelerated from low potential to high potential → It gains kinetic energy.

The energy gained by an electron when accelerated through a potential difference of 1 Volt = 1 electron volt = 1 eV.

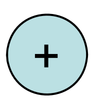
**If I accelerated an electron from rest through a potential difference of 50,000 V, then I know immediately that its kinetic energy is 50,000 eV.

Energy is usually expressed in Joules: $\boxed{1 \text{ eV} = 1.602 \times 10^{-19} \text{ J}}$

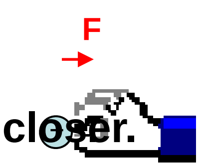
Just like in a gravitational field, in an electric field, potential energy (PE) can be converted into kinetic energy (KE):

<u>Example</u>: Let's bring a small positive test charge from very far away in toward a fixed, positive point charge:

As I push the charge in closer and closer, the repulsive force on it gets bigger and bigger:

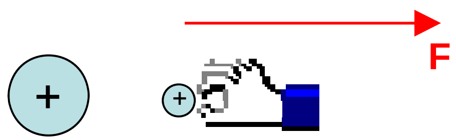


*Thus, I have to do work on the charge to move it closer.



The work I do on the charge goes into increasing its potential energy!

Now release the charge…..



The charge converts its stored EPE into KE!!!

<u>Remember:</u> The total mechanical energy of a system must be conserved.

$$E_{Tot} = \underbrace{\frac{1}{2}mv^2}_{KE_{Trans}} + \underbrace{\frac{1}{2}I\omega^2}_{KE_{Rot}} + \underbrace{mgh}_{PE_{Grav}} + \underbrace{\frac{1}{2}kx^2}_{PE_{Elastic}} + \underbrace{EPE}_{PE_{Electro}}$$

And,….If the work done by nonconservative forces is zero, then:

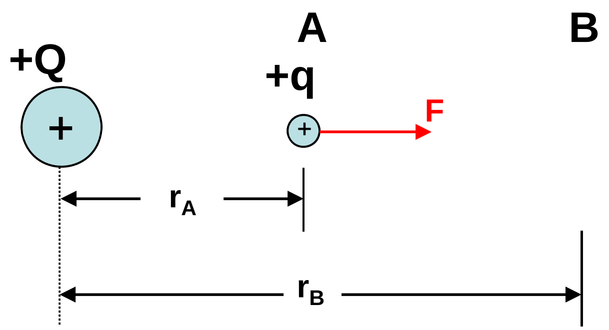
$$\boxed{E_{Tot_f} = E_{Tot_o}}$$

19.3 The Electric Potential of a Point Charge

An electric potential exists around charges.

What is the form of the potential for a point charge?

Let's place a positive test charge near a positive fixed point charge:



The electric field created by the point charge does work on the test charge and moves it to the right

What is the work done by the field in moving the charge from A to B?

Well, $W = F \cdot d$, and the force is given by Coulomb's Law: $F = k\frac{qQ}{r^2}$

But, the force is not constant as the charge moves from r_A to r_B, since the force depends on r.

Thus, we have to use calculus and integrate the force over the distance:

Result: $$W_{AB} = k\frac{qQ}{r_A} - k\frac{qQ}{r_B}$$

From earlier, we know that: $V_B - V_A = \frac{-W_{AB}}{q} = k\frac{Q}{r_B} - k\frac{Q}{r_A}$

If we let r_B be really far away, i.e. $r_B \to \infty$, then $V_B = k\frac{Q}{r_B} \to 0.$

*This sets our zero potential at infinity.

Thus, we are left with: $V_A = k\frac{Q}{r_A}.$

r_A is just some arbitrary distance from the point charge, so we drop the subscript:

$$\boxed{V = k\frac{Q}{r}}$$

This is the electric potential due to a point charge.

We now have 3 equations which kind of look similar:

$$\boxed{F = k\frac{qQ}{r^2}}$$

Force between two charges.

$$\boxed{E = k\frac{Q}{r^2}}$$

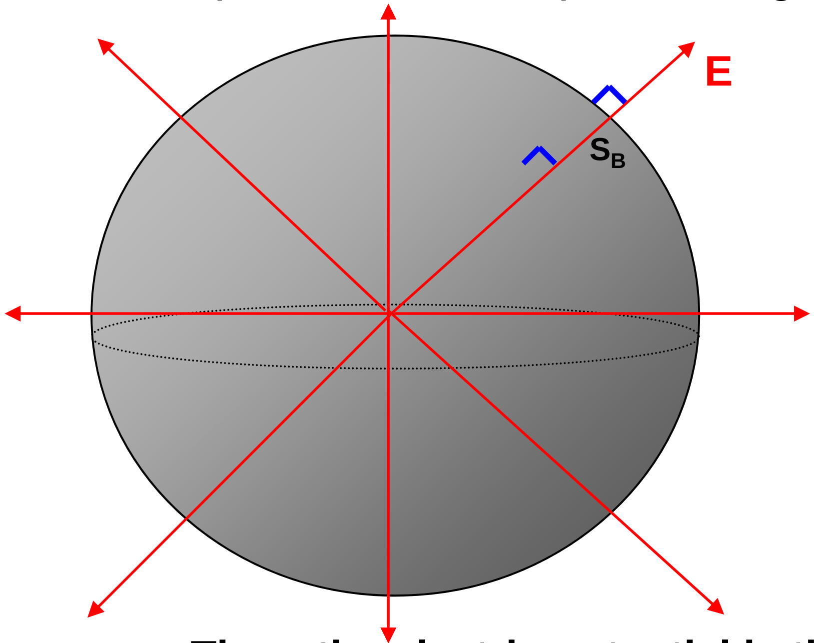
Electric field of a point charge.

$$\boxed{V = k\frac{Q}{r}}$$

Electric potential of a point charge.

<u>**19.4 Equipotential Surfaces**</u>

So the potential from a point charge is $V = k\dfrac{Q}{r}$



This means the potential is the same in every direction around the point charge at a distance r away.

In 3D, this forms a spherical shell of radius r around the charge.

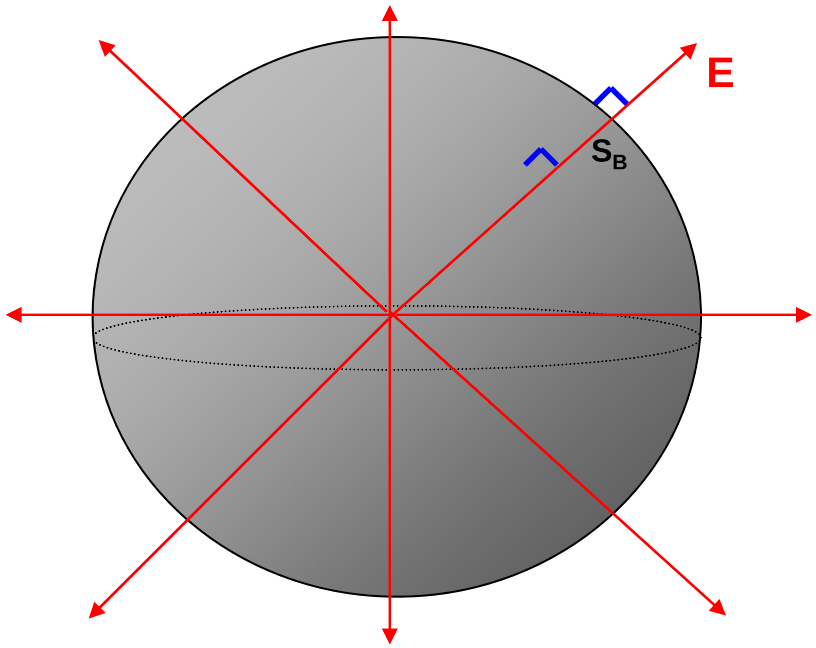
Thus, the electric potential <u>is the same everywhere</u> on this spherical surface (S_A). It is called an <u>equipotential surface.</u>

Equipotential surfaces are surfaces of constant potential.

Let's look at another equipotential surface (S_B) around the point charge:

We know the electric field lines point everywhere radially outward:

<u>Notice:</u> The electric field lines are <u>perpendicular to</u> the equipotential surfaces.



Since S_A is closer to the positive charge than S_B, S_A is at a higher potential than S_B.

Thus, electric field lines point in the direction of decreasing potential, i.e. they point from high potential to low potential.

Work? The net electric force does no work as a charge moves on an equipotential surface.

Why? We defined $V_B - V_A = \frac{-W_{AB}}{q}$. But, if we are on an equipotential surface, then $V_A = V_B$, and $W_{AB} = 0$.

Or….

In order for the charge to feel a force along an equipotential surface, there must be a component of the field along the surface, but E is everywhere perpendicular to the equipotential surface.

Fields, Potentials, and Motion of Charges - Summary

Electric field lines start on positive charges and end on negative ones.

Positive charges accelerate from regions of high potential toward low potential.

Negative charges accelerate from regions of low potential toward high potential.

Equipotential surfaces are surfaces of constant potential.

Electric field lines are perpendicular to an equipotential surface.

Electric field lines are perpendicular to the surface of a conductor, thus a conductor is an equipotential surface!

Electric field lines point from regions of high potential toward low potential.

Therefore, positive charges move in the same direction as the electric field points, and negative charges move in the opposite direction of the electric field.

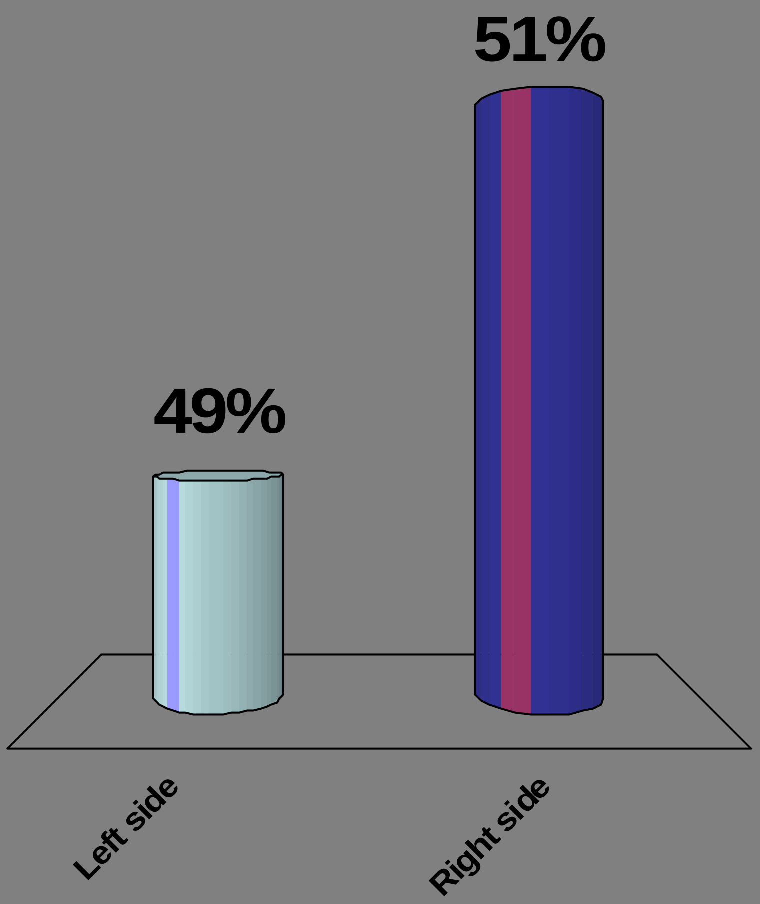
The electric force does <u>no</u> work as a charge moves on an equipotential surface.

Clicker Question 19-4

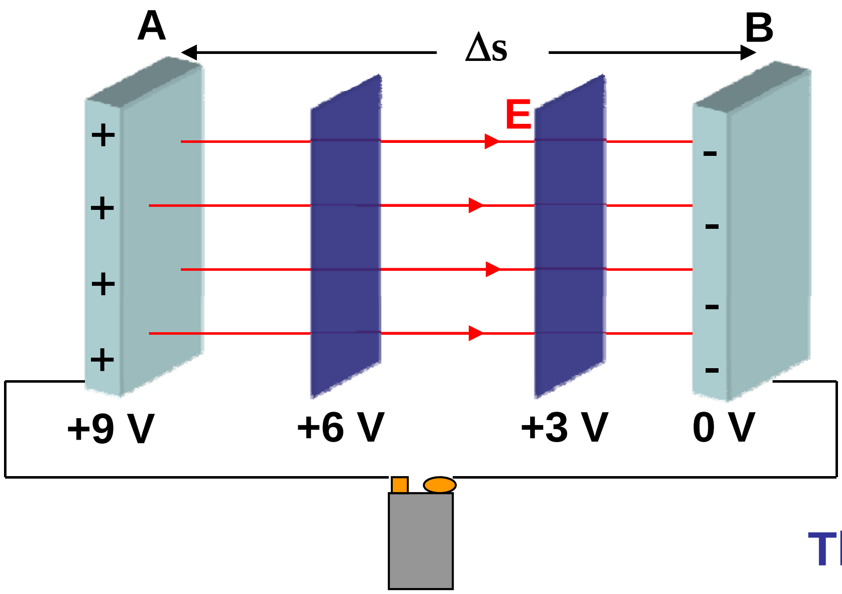
Which side of space (left or right) is at a lower potential?

1. Left side
2. Right side

The electric field points from left to right, and electric field lines start from regions of high potential, thus the right side is at a lower poential.



Parallel plate capacitor



The positive plate is at a potential of +9 V and the negative plate is at 0 V.

What would the equipotential surfaces look like between the plates?

They would be a parallel set of planes!

Let the plates be separated by a distance Δs.

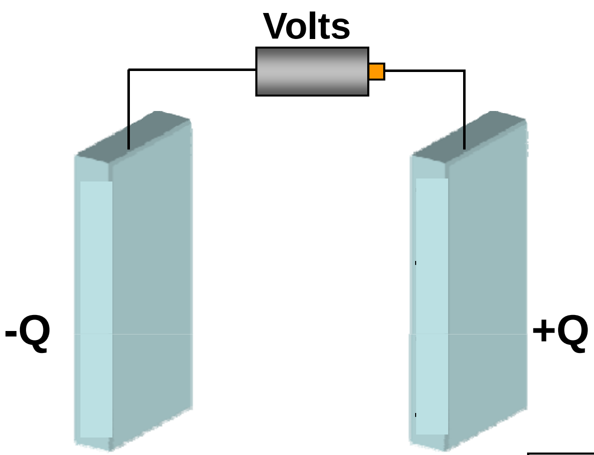
The electric field is then $= \frac{[\text{Change in voltage}]}{[\text{Change in distance}]} \Rightarrow \boxed{E = \frac{\Delta V}{\Delta s}}$

This is called the electric field gradient.

Thus, the electric field also has units of $[V/m]$

19.5 Capacitors

Two oppositely charged conductors separated by some small distance.



We can charge the plates by connecting them to a battery:

The higher the voltage on our battery, the more charge we can put on each plate.

Thus, $Q \propto V$

Make this an equality: $\boxed{Q = CV}$ **C is a new quantity called the Capacitance.**

Units?

$$C = \frac{Q}{V} \rightarrow \frac{[\text{Charge}]}{[\text{Voltage}]} = \left[\frac{\text{C}}{\text{V}}\right] = [\text{Farad}] = [\text{F}]$$

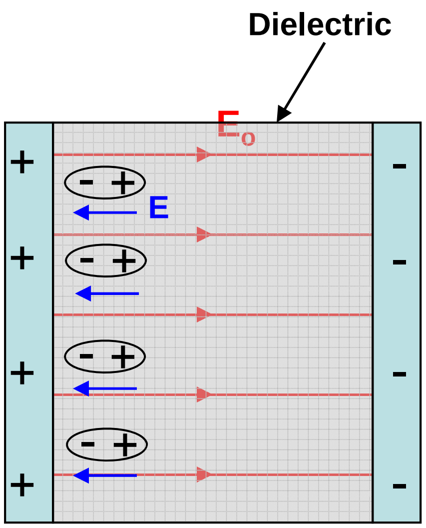
***A farad is a very large capacitance. We often use microfarads (μf) and picofarads (pf).**

1×10^{-6} F 1×10^{-12} F

The larger the capacitance, the more charge it will hold!

Dielectrics

We can fill the space between the plates with some insulating material, say air, oil, paper, rubber, plastic, etc.



This material is called a dielectric.

So what effect does the dielectric have on the field between the plates?

Since the dielectric is an insulator, the charges in it aren't free to move, but they can separate slightly within each atom:

Each one of these atoms now produces a small internal electric field which points in the opposite direction to the field between the plates:

Thus, the net electric field between the plates is reduced by the dielectric.

The reduction of the field is represented by the following:

$$\kappa = \frac{E_o}{E}$$

E_o is the field without the dielectric
E is the field with the dielectric

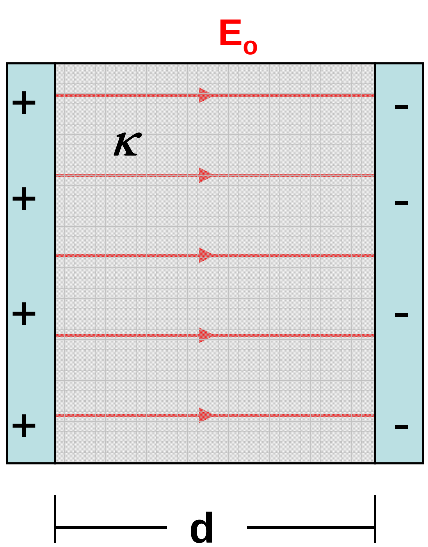
κ is called the dielectric constant, and it must be greater than 1.

$$\kappa = \frac{E_o}{E}$$

Since κ is the ratio of two electric fields, it's unitless.

Material	κ
Vacuum	1
Air	1.00054
Water	80.4

The larger κ is, the more it reduces the field between the plates!



Let's say the plates have surface area A and are separated by a distance d.

$$E = \frac{1}{\kappa} E_o = \frac{V}{d} \Rightarrow E_o = \frac{\kappa V}{d} = \frac{\sigma}{\varepsilon_o} = \frac{q}{\varepsilon_o A}$$

$$\Rightarrow \boxed{q = \left(\frac{\varepsilon_o A \kappa}{d}\right) V}$$

But, $q = CV$, **so**

$$\boxed{C = \frac{\varepsilon_o A \kappa}{d}}$$

Capacitors store charge - what about energy?

$$\boxed{EPE_{Stored} = \tfrac{1}{2}qV = \tfrac{1}{2}CV^2}$$

$$V = Ed \text{ and } C = \frac{\varepsilon_o \kappa A}{d}, \text{so} \boxed{EPE_{Stored} = \tfrac{1}{2}\left(\frac{\varepsilon_o \kappa A}{d}\right)\left(E^2 d^2\right)}$$

Rearrange this:

$$EPE = \tfrac{1}{2}\kappa\varepsilon_o E^2 \underbrace{(Ad)}_{\text{Volume between the plates}} = \tfrac{1}{2}\kappa\varepsilon_o E^2 (\mathrm{Vol})$$

$$\boxed{\frac{EPE}{\mathrm{Vol}} = \text{Energy Density} = \tfrac{1}{2}\kappa\varepsilon_o E^2}$$

Units?

$$\left[\frac{\text{Energy}}{\text{Volume}}\right] = \left[\frac{\mathrm{J}}{\mathrm{m}^3}\right]$$

***This expression holds true for any electric fields, not just for capacitors!**