Bootstrapping Approach for Generating Maximally Path-Entangled Photon States

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We propose a *bootstrapping* approach to the generation of maximally path-entangled states of photons, so-called "NOON states." The strong atom-light interaction of cavity QED can be employed to generate NOON states with about 100 photons. These can then be used to boost the existing experimental Kerr nonlinearities based on quantum coherence effects, to facilitate NOON generation with an arbitrarily large number of photons. We also offer an alternative scheme that uses an atom-cavity dispersive interaction to obtain a sufficiently high Kerr nonlinearity necessary for arbitrary NOON generation.

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NOON states, path-entangled states of N photons of the form $|N0::0N\rangle \equiv (|N\rangle_a|0\rangle_b + |0\rangle_a|N\rangle_b)/\sqrt{2}$, are important for quantum lithography [1] and Heisenberg-limited interferometry with photons [2]. Several theoretical proposals exist for NOON-state generation; nevertheless, experimentally, it seems to be a formidable task. So far, NOON states with only three and four photons have been generated [3]. There exists a proposal that simulates a six-photon NOON state by postselection [4] and hence may not be directly useful for the quantum lithography application. In this context, it is imperative to develop practical strategies for generation of high-NOON states. In this Letter, we propose a couple of such routes.

To recollect, existing proposals for NOON-state generation use either the linear optical quantum computing (LOQC) approach or some kind of optical nonlinearity. The LOQC approaches would be unsuitable in this quest, as the larger the required number of particles in the entangled state, the lower the success probability [5]. Thus, the routes using optical nonlinearities seem to be promising in the long run. Nevertheless, experimental nonlinearities are not as strong as required for NOON generation, even in the case where quantum coherence effects such as electromagnetically induced transparency [6] are employed. Here we propose a *bootstrapping* approach to boost the existing experimental nonlinearities in order to eventually have a large number of particles in the generated NOON state.

The bootstrapping technique proposed here involves preparation of a NOON state with a small number of photons (up to 100), which can be used to boost the conditional nonlinearities necessary to obtain NOON states with an arbitrarily large number of photons N, provided the N-photon Fock states are available. The approach is primarily based on the scheme, described in Fig. 1(a), proposed by Gerry and Campos [7]. The scheme involves two Mach-Zehnder interferometers (MZIs) coupled via a cross-Kerr nonlinearity that can be obtained via schemes based on quantum coherence effects [8]. The presence of a single photon in mode c is required to give a phase shift of π to photons in mode b; this phase shift is, however, not within the reach of current experimental cross-Kerr nonlinearities. The best current experimental cross-Kerr phase shifts are of the order of 0.1 radians, obtained via atom-light interaction in systems using quantum coherence effects [9]. This suggests that further enhancement in the nonlinearity—by roughly 2 orders of magnitude—is necessary for NOON-state generation. It is, however, important to note that the scheme requires a conditional phase shift of either 0 or π, which would require a NOON state (of about $K \approx 10\pi$ photons) to act as a control. The necessary setup is shown in Fig. 1(b). The presence of a large number of photons would boost the quantum-coherence-based cross-Kerr nonlinearity, enough to obtain a phase shift of 0 or π if all of the K control photons occupy mode d or c, all at once. Once the enhanced nonlinearity is used, measurement of the number of photons n_{D1} and n_{D2} would give the output state $[(-i)^{n_{D2}}|N_a0_b\rangle + i(-i)^{n_{D1}}|0_aN_b\rangle]\sqrt{2}$, which can be trivially corrected for the relative phase of the two components to obtain a NOON state. The condition $n_{D1} + n_{D2} = K$ if satisfied means a successful NOON generation even with inefficient detectors, provided they are photon number resolving [10].

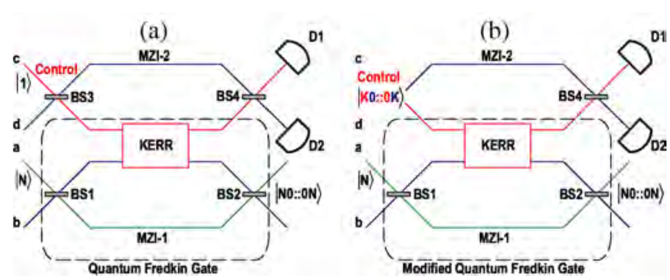

FIG. 1 (color online). Nonlinear process to generate a high NOON state $|N0::0N\rangle_{a,b}$ via a cross-Kerr nonlinearity. (a) The scheme of Ref. [7] using a single photon Fock state $|1\rangle$ as a control. (b) The bootstrapping procedure employing a small NOON state $|K0::0K\rangle_{c,d}$ as a control.

As pointed out earlier, the foremost step is to generate a low-NOON state with about a hundred photons. The system required to obtain the low-NOON state of photons is an ensemble of cold alkali atoms—roughly a few hundred of them trapped inside an optical cavity—such that the spatial extent of the cloud is much smaller than the wavelength of the light. The schematic of the device and the detailed level structure of the atoms being targeted is shown in Fig. 2. The operational steps are described below.

Step 1.—The first step is single-step generation of the Greenberger-Horne-Zeilinger (GHZ) state of atoms in two of its internal states via the Hamiltonian $H_{\mathrm{GHZ}} = \hbar\eta\sum_{j,k=1}^{N}\hat{S}_j^+\hat{S}_k^- = \hbar\eta[\frac{N}{2}(\frac{N}{2}+1) - \hat{S}_z^2 + \hat{S}_z]$, where $\hat{S}_j^+ = |a_j\rangle\langle b_j|$, $\hat{S}_k^- = |b_k\rangle\langle a_k|$, $S_z = \sum_j S_{z,j} = \sum_j |a_j\rangle \times \langle a_j| - |b_j\rangle\langle b_j|$, and $\eta = \Omega_c g\Delta/(\kappa^2 + \Delta^2)$ is the Raman Rabi frequency, signifying the coupling between the two states $|a\rangle$ and $|b\rangle$, achieved through the vacuum mode of a cavity (with decay rate κ) and a time-varying classical field (see inset 1 in Fig. 2). The approach is well studied [11] and can be used, with an initial state of all of the atoms being in the superposition $(|a\rangle + |b\rangle)/\sqrt{2}$, to generate GHZ states in the basis given by $\{|+\rangle \equiv (|a\rangle + |b\rangle)/\sqrt{2}, |-\rangle \equiv (|a\rangle - |b\rangle)/\sqrt{2}\}$ after the atom-field evolution for time t such that $\eta t = \pi$. The basis rotation can be readily performed by the application of a Raman pulse (of area $\pi/2$) coupling the two states to form the GHZ state, in the familiar basis $\{|a\rangle, |b\rangle\}$, $(|aaa\ldots\rangle + |bbb\ldots\rangle)/\sqrt{2}$. The initial state of the atoms $(|a\rangle + |b\rangle)/\sqrt{2}$ could be prepared by two classical Raman fields coupling the levels, much like the Raman process shown in Fig. 2. Direct implementation in Bose-Einstein condensates may also be possible by using the scheme of Ref. [12], which employs the interparticle interaction for generation of arbitrary Dicke states.

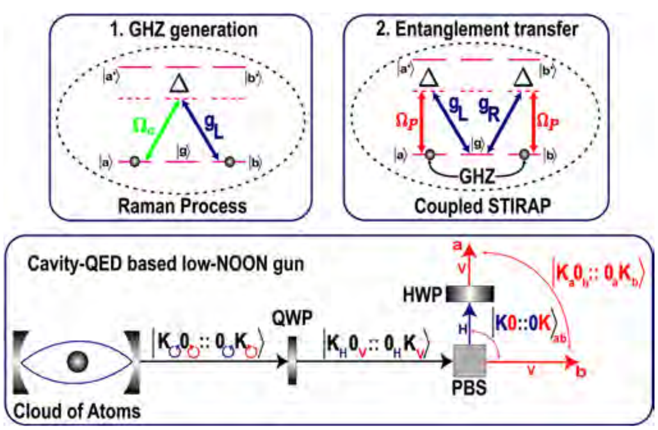

FIG. 2 (color online). A low-NOON gun ($K \approx 100$): A scheme to transfer a GHZ state of atoms into a maximally entangled state of photons into two circular polarization modes via collective interaction of atoms with light fields. By using usual optical elements such as a quarter-wave plate (QWP), a polarizing beam splitter (PBS), and a half-wave plate (HWP), the polarization-entangled photons in the two circular polarization modes emitted by the cavity can be converted to a path-entangled NOON state. The insets show the two important steps: (1) preparation of atoms in the GHZ state and (2) transfer of entanglement from atoms to the polarization modes of the cavity.

Step 2: Entanglement transfer.—Once a GHZ state of the atoms is prepared, a coupled stimulated Raman adiabatic passage (STIRAP) process (see Fig. 2) can be achieved by controlling the time variation of the pump-field Rabi frequency $\Omega_P(t)$ to generate an entangled state of cavity photons $|N_\circlearrowleft 0_\circlearrowright :: 0_\circlearrowleft N_\circlearrowright\rangle$ that is entangled in the two counterrotating polarization modes. This polarization mode entanglement can be readily converted into path entanglement with the help of simple optical elements as shown in detail in Fig. 2.

Step 2, described above, requires a coupled STIRAP operation for transfer of entanglement from the atomic GHZ state to the polarization modes of the photons. This is the most important result of this Letter, which offers a device that we call a low-NOON gun. This device, however, cannot be directly used to generate arbitrarily large number of path-entangled photons. The atom-light interaction needs to be collective, requiring that all of the atoms see the same strength for both the cavity field and the classical field. Furthermore, the cavity decay and the atomic spontaneous emission increase as the number of atoms increases. Thus, to limit the integrated noise to less than one photon, within the time required for the adiabatic entanglement-transfer process, the total number of atoms needs to be restricted to about 100. The aforementioned error estimates are studied in great detail by Brown *et al.* in the context of a Fock-state generator [13] and are directly applicable to our scheme, which can be thought of as a superposition of two Fock-state generators. The actual requirement of our bootstrapping approach is to have an entangled state of about 32 photons as a control to boost the experimental phase shift of 0.1 radians to π radians. This would require only 32 atoms to be trapped in the cavity, making the scheme highly feasible. An experimental scheme of Sauer *et al.* [14], for cavity QED with optically transported atoms, allows control over the number of atoms interacting with the cavity field for up to 100 atoms and could be directly used for implementation of our scheme. In the following, we discuss the physics of our NOON gun in detail.

The initial state of the atomic cloud is the GHZ state with the component states $|a\rangle$ and $|b\rangle$ being the hyperfine sublevels $m_F = -1$ and $m_F = 1$ of the $F = 1$ hyperfine manifold of an alkali atom, respectively. This atomic cloud is then trapped in a cavity such that the cloud size is much smaller than the wavelength of the fundamental mode of the cavity. Also, the external pumping field is assumed to couple to all of the atoms identically. These restrictions could be easily obtained within the current experimental parameters of the optical cavity, when the number of atoms is confined to about a hundred. The interaction of the atomic cloud with the two polarization modes of the cavity and the π-polarized pump field can be described by the

Hamiltonian $H = \hbar\sum_i \Omega_P(t)(|a'\rangle_i\langle a|_i + |b'\rangle_i\langle b|_i + |a\rangle_i \times \langle a'|_i + |b\rangle_i\langle b'|_i) + g_L(\hat{c}_L|a'\rangle_i\langle g|_i + \hat{c}_L^\dagger|g\rangle_i\langle a'|_i) + g_R(\hat{c}_R|b'\rangle_i \times \langle g|_i + \hat{c}_R^\dagger|g\rangle_i\langle b'|_i) = \hbar[\Omega_P(t)(\hat{d}_{a'}^\dagger\hat{d}_a + \hat{d}_a^\dagger\hat{d}_{a'} + \hat{d}_{b'}^\dagger\hat{d}_b + \hat{d}_b^\dagger\hat{d}_{b'}) + g_L(\hat{c}_L\hat{d}_{a'}^\dagger\hat{d}_g + \hat{c}_L^\dagger\hat{d}_g^\dagger\hat{d}_{a'}) + g_R(\hat{c}_R\hat{d}_{b'}^\dagger\hat{d}_g + \hat{c}_R^\dagger\hat{d}_g^\dagger\hat{d}_{b'})]$, where $\hat{c}_{L,R}$ and $\hat{c}_{L,R}^\dagger$ are the cavity mode photon annihilation and creation operators for the left (L) and right (R) circular modes of polarization, respectively, and i is the label for the atoms. We arrived at the above equation by introducing a number representation for the collective atomic states and the corresponding operators $\hat{d}$ and $\hat{d}^\dagger$ labeled by the appropriate atomic level signifying annihilation or creation of an atom in that state.

The atomic level scheme contains two coupled Λ systems. It is important to identify quantities conserved under the action of the Hamiltonian in order to understand the form of the eigenstates of the system. The conserved quantities are the total number of atoms in the five states $N = \sum_i \hat{d}_i^\dagger\hat{d}_i$, with $i \in \{a, a', b, b', g\}$, and the quantity $M = \hat{d}_g^\dagger\hat{d}_g - \hat{c}_L^\dagger\hat{c}_L - \hat{c}_R^\dagger\hat{c}_R$. It should also be noted that the initial state in the given manifold, labeled by N and M, shall always remain in that manifold under the action of the Hamiltonian. Moreover, each manifold contains a dark state, which does not contain any atoms in the excited states $|a'\rangle$ and $|b'\rangle$. We do not give the complete form of the general dark state; however, we note an important observation that is useful for the problem at hand. If the initial state of the cavity fields is chosen such that $\hat{c}_L^\dagger\hat{c}_L = \hat{c}_R^\dagger\hat{c}_R = 0$, and the atomic state is such that all of the atoms are in the Λ-type system formed by either $|g\rangle - |b'\rangle - |b\rangle$ or $|g\rangle - |a'\rangle - |a\rangle$ (e.g., the GHZ state), then the interaction Hamiltonian reduces to either $H_{\rm eff}^{(1)} = \hbar[\Omega_P(t)(\hat{d}_{b'}^\dagger\hat{d}_b + \hat{d}_b^\dagger\hat{d}_{b'}) + g_R(\hat{c}_R\hat{d}_{b'}^\dagger\hat{d}_g + \hat{c}_R^\dagger\hat{d}_g^\dagger\hat{d}_{b'})]$ or $H_{\rm eff}^{(2)} = \hbar[\Omega_P(t)(\hat{d}_{a'}^\dagger\hat{d}_a + \hat{d}_a^\dagger\hat{d}_{a'}) + g_L(\hat{c}_L\hat{d}_{a'}^\dagger\hat{d}_g + \hat{c}_L^\dagger\hat{d}_g^\dagger\hat{d}_{a'})]$ as the effective Hamiltonian governing the system dynamics. This gives two different manifolds of states with the conserved quantities N and $M^{(1)} = \hat{d}_g^\dagger\hat{d}_g - \hat{c}_R^\dagger\hat{c}_R$ or $M^{(2)} = \hat{d}_g^\dagger\hat{d}_g - \hat{c}_L^\dagger\hat{c}_L$. These manifolds do not couple to each other, and the corresponding dark states are given by

$$|\Psi^{(1)}\rangle = \frac{1}{D}\sum_{j=0}^{N}\frac{[-\Omega_P(t)/g_L]^j}{\sqrt{(N-j)!j!j!}}|(N-j)_a, 0_b, j_g, j_L, 0_R\rangle, \quad (1)$$

$$|\Psi^{(2)}\rangle = \frac{1}{D'}\sum_{k=0}^{N}\frac{[-\Omega_P(t)/g_R]^k}{\sqrt{(N-k)!k!k!}}|0_a, (N-k)_b, k_g, 0_L, k_R\rangle, \quad (2)$$

where D and D' are the appropriate normalization constants. These specialized dark states of a three-level Λ-type system for a collective atomic ensemble simultaneously coupled to a quantized and a classical field are well studied in Refs. [13,15]. The state notation for the complete atom-cavity field states is self-explanatory, where the excited levels $|a'\rangle$ and $|b'\rangle$ are not shown as they are not occupied. The dark states $|\Psi^{(1)}\rangle$ and $|\Psi^{(2)}\rangle$ are dynamically decoupled from each other, in the sense that if the initial state contains a certain proportion of both states, that proportion shall be left unchanged by the evolution. The evolution or relative dominance of the components of the dark states can be controlled by controlling the time evolution of the pump field via the Rabi frequency $\Omega_P(t)$. Thus, the adiabatic transformations $|N_a, 0_b, 0_g, 0_L, 0_R\rangle \rightarrow |0_a, 0_b, N_g, N_L, 0_R\rangle$ and $|0_a, N_b, 0_g, 0_L, 0_R\rangle \rightarrow |0_a, 0_b, N_g, 0_L, N_R\rangle$ can be obtained deterministically. As a result, the two components of the initially prepared atom-cavity state $(|N_a, 0_b, 0_g, 0_L, 0_R\rangle + |0_a, N_b, 0_g, 0_L, 0_R\rangle)/\sqrt{2}$ evolve independently into the state $(|0_a, 0_b, N_g, N_L, 0_R\rangle + |0_a, 0_b, N_g, 0_L, N_R\rangle)/\sqrt{2}$, just by adiabatic increase of the pump-field intensity such that $\Omega_p(t) \gg g_L, g_R$ in the long time limit like in the STIRAP processes. Moreover, choosing the detuning such that $\Delta \gg \Omega_P(t)$ guarantees that the spontaneous emission noise from the upper levels and spurious absorption events are avoided within the interaction time. The final state of all of the atoms is $|g\rangle$, and the field state is $|N_\circlearrowleft 0_\circlearrowright :: 0_\circlearrowleft N_\circlearrowright\rangle \equiv |N_\circlearrowleft 0_\circlearrowright\rangle + |0_\circlearrowleft N_\circlearrowright\rangle/\sqrt{2}$. This polarization-entangled state of photons can be readily converted into a path-entangled NOON state in a chosen polarization mode by using simple optical elements as depicted in Fig. 2. The polarization NOON state itself can be used for Heisenberg-limited measurement of polarization angle shifts such as is exploited in magnetometry [16]. Physically, the above-mentioned NOON gun is similar in operation to the experimentally demonstrated deterministic single photon source [17] and a recent theoretical proposal for Fock-state generation [13].

This completes the discussion of the bootstrapping procedure for high-NOON generation. Now we briefly discuss a different implementation of the Kerr nonlinearity based on the atom-cavity dispersive interaction [18] that can be used in place of the cross-Kerr nonlinearities obtained via quantum coherence effects. The scheme is depicted in Fig. 3, where a cavity is introduced in the path of the optical mode b after the beam splitter BS1. In comparison with the scheme of Fig. 1(a), the new scheme (see Fig. 3) uses a Ramsey interferometer in place of MZI-2. The mathematical transformation of the quantum Fredkin gate (QFG) is given by Ref. [7] as $\hat{U}_{\rm QFG} = \exp(i\chi\hat{c}^\dagger\hat{c}\hat{J}_0) \times \exp(i\chi\hat{c}^\dagger\hat{c}\hat{J}_2)$, where $\hat{J}_0 = (\hat{a}^\dagger\hat{a} + \hat{b}^\dagger\hat{b})/2$ and $\hat{J}_2 = (\hat{a}^\dagger\hat{b} - \hat{a}\hat{b}^\dagger)/2i$ are the Schwinger angular momentum representations of the photonic operators and $\chi = \pi$ is the cross-Kerr nonlinearity required for generation of the NOON state at the output of MZI-1 after detection of the control photon in one of the detectors at the output ports of MZI-2 [see Fig. 1(a)]. The Ramsey interferometer that replaces MZI-2 consists of three regions: R1 and R2 are Ramsey classical field zones giving transformations $|g_1\rangle \rightarrow (|g_1\rangle + |g_2\rangle)/\sqrt{2}$ and $|g_2\rangle \rightarrow (|g_1\rangle - |g_2\rangle)/\sqrt{2}$, respectively, with a cavity dispersively coupled (large detuning Δ) with the atomic transition $|g_1\rangle$ to $|e\rangle$ between the two Ramsey zones. The dispersive interaction imparts a phase shift of $g^2\tau_c/\Delta$ to the atomic level $|g_1\rangle$ and no phase

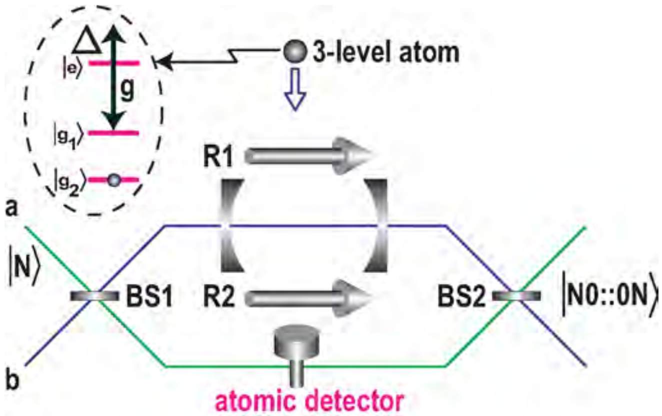

FIG. 3 (color online). Quantum Fredkin gate based on nonlinearity obtainable via cavity QED and Ramsey interferometry.

shift to state $|g_2\rangle$. Thus, after the passage of an atom initially prepared in state $|g_2\rangle$ when the photons in mode b are confined in the cavity, we obtain the following transformation $\hat{U}_{\text{Ramsey-QFG}} = \exp[i(g^2\tau_c/\Delta)\hat{\sigma}_+\hat{\sigma}_-\hat{J}_0] \times \exp[i(g^2\tau_c/\Delta)\hat{\sigma}_+\hat{\sigma}_-\hat{J}_2]$. Here $\hat{\sigma}_+ = |g_1\rangle\langle g_2|$ and $\hat{\sigma}_- = |g_2\rangle\langle g_1|$ are the atomic operators, g is the atom-cavity interaction strength, and τ_c is the time the atom spends in the cavity. With $g^2\tau_c/\Delta = \pi$, the evolution of the system is identical to the NOON generation scheme of Ref. [7] such that detection of the atom, as it comes out of the cavity, in state $|g_1\rangle$ or $|g_2\rangle$, gives the NOON state in the form $|\Psi_{1(2)}\rangle_{ab} = [|N\rangle_a|0\rangle_b \pm e^{-iN\pi/2}|0\rangle_a|N\rangle_b]/2$ at the output of the MZI. The nonlinearity in this setup is completely controllable via the atomic velocity, that is, the atomic passage time through the cavity τ_c, and the cavity parameters g and Δ. This makes it straightforward to obtain the phase shift of the order of π with just one photon present in the cavity. By introducing a delay in the path of mode a, the time spent by photons in mode b inside the cavity can be trivially compensated to obtain a balanced MZI. Within the experimental optical cavity QED parameters $g\tau_c$ can be as large as 10^5 [17]. With $\Delta = 10g$ and $g^2\tau_c/\Delta = \pi$, we need a moderate value $g\tau_c = 10\pi$, which is much easier to obtain as it requires less cooling of the atoms entering the cavity. For $g = 5\pi$ MHz, the necessary atom-cavity interaction time is $\tau_c = 2\ \mu$s, which is much smaller than the photon lifetime (few tens of milliseconds) in the cavity; thus, the approach presented here is well within the current experimental parameters.

To summarize, we have devised a bootstrapping approach to NOON-state generation for photons based on a quantum Fredkin gate using an atomic coherence effect based on cross-Kerr nonlinearity. In the process, we have proposed a device that can deterministically produce NOON states of up to 100 photons. Furthermore, we have devised a scheme for NOON-state generation based on Ramsey interferometry and a cavity-enhanced Kerr nonlinearity. It is assumed that the Fock states of an arbitrarily large number of photons are available as inputs, which could be generated by applying the proposal of Ref. [13] to cold Rydberg atoms trapped in microwave cavities or via a quantum nondemolition photon number measurement of a coherent state containing a large number of photons on average. Our strong hope is that the ideas presented here will stimulate a growth of experimental activity in the generation of entangled photon states with larger and larger numbers of photons.

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