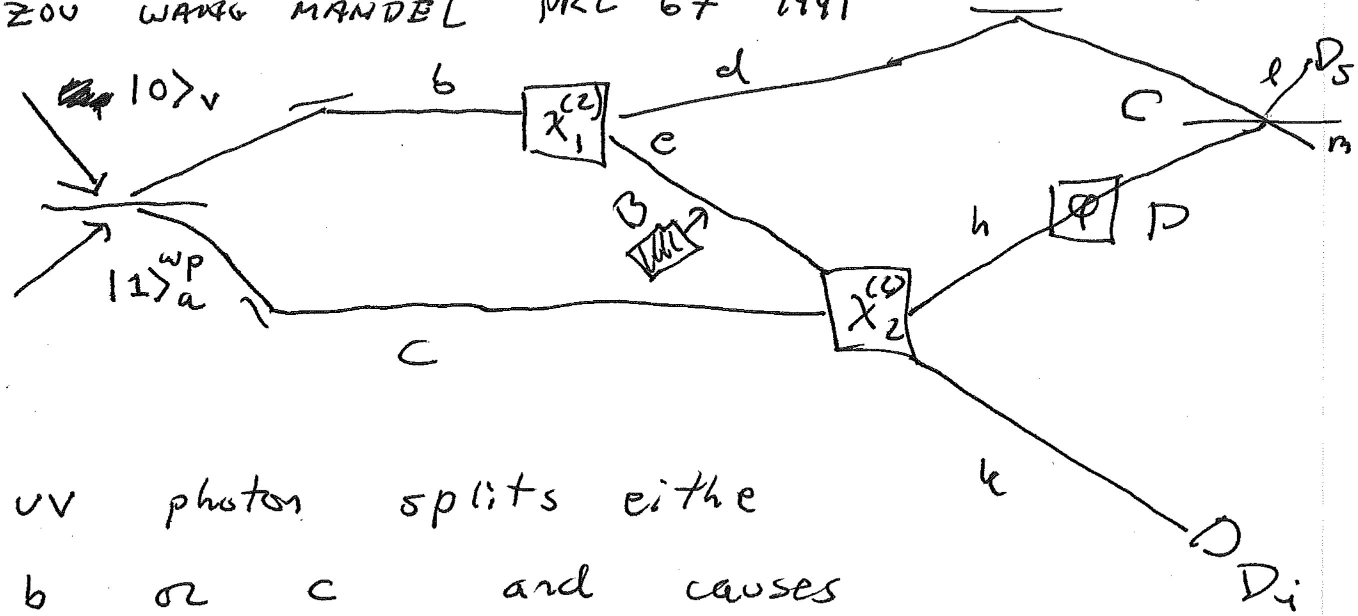
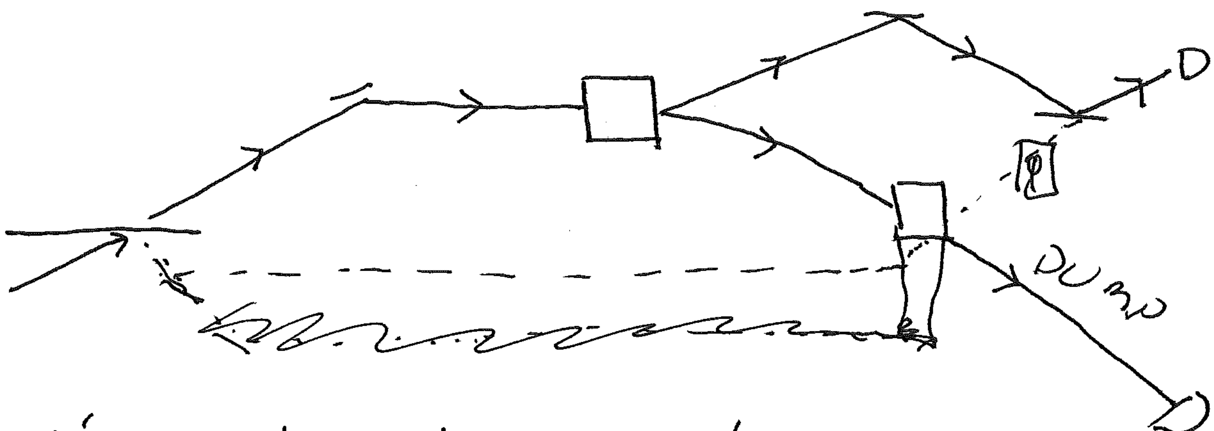
9.4 Q. Eraser II & "CRAZY PAPER"

ZOU WANG MANDEL PRL 67 1991

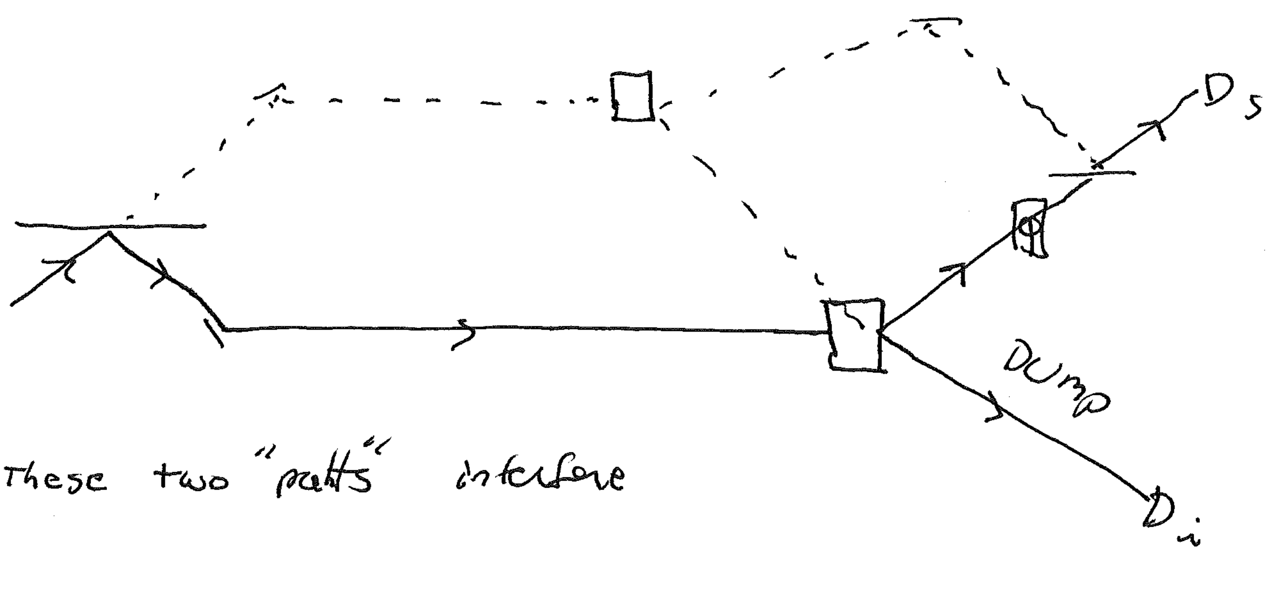


UV photon splits either b or c and causes downconversion in either $\chi_1^{(2)}$ or $\chi_2^{(2)}$

Let's consider upper route



Let's consider lower route



These two "paths" interfere

$I_s = \cos^2 \varphi$

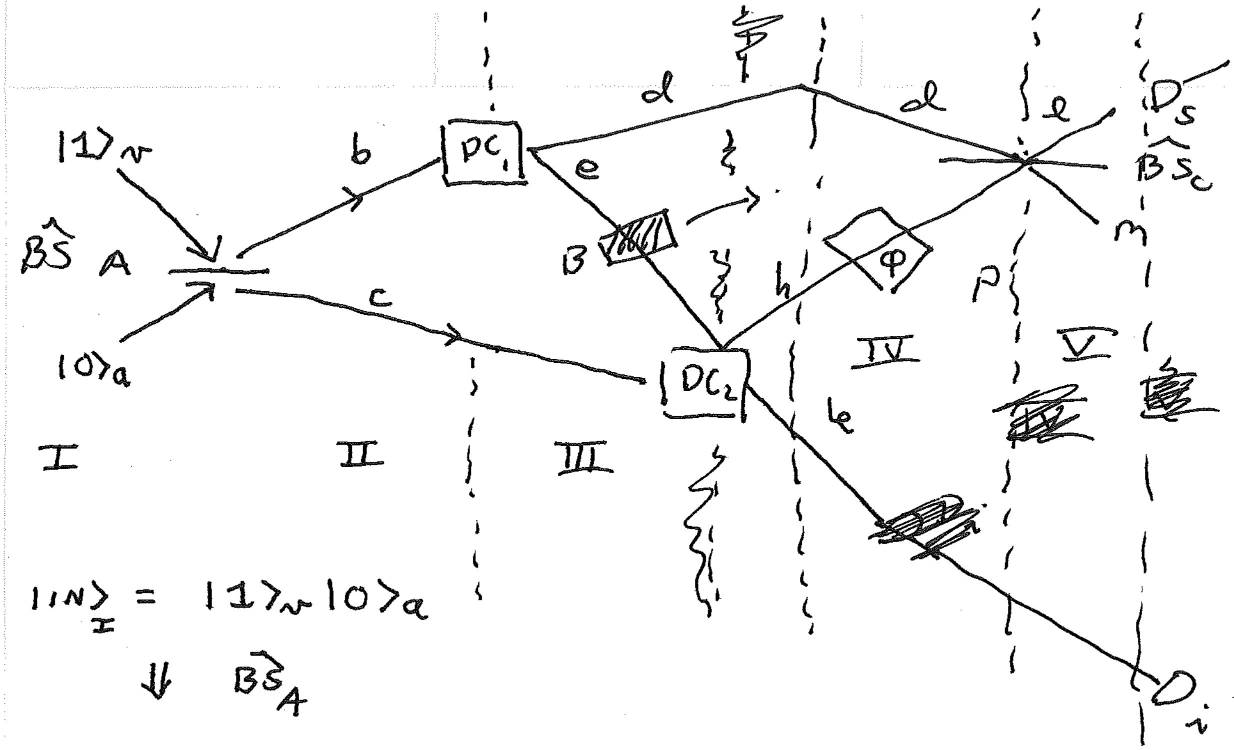
WE SEE INTERFERENCE BETWEEN UPPER AND LOWER PATH AT D_S EVEN IF D_I IS NOT THERE!

D_I cannot tell if photon came from $\chi_1^{(2)}$ or $\chi_2^{(2)}$

NOW WE BLOCK BEAM e WITH A BEAM BLOCK. D_I CAN NOW TELL WHICH PATH (LOWER) AND INTERFERENCE AT D_S DISAPPEARS!

NOTE: BLOCK "B" IS NOT IN EITHER PATH

NOTE: D_I DOES NOT ACTUALLY NEED TO BE THERE! ONLY POTENTIAL.



$$|IN\rangle_{I} = |1\rangle_v |0\rangle_a$$

$$\Downarrow \ \widehat{BS}_A$$

$$|\psi\rangle_{II} = \frac{1}{\sqrt{2}} \left[|1\rangle_b |0\rangle_c + i\, |0\rangle_b |1\rangle_c \right]$$

$$\cancel{\Downarrow\ \widehat{DC}_1 |1\rangle_b |0\rangle_c = \gamma |0\rangle_b |0\rangle_c}$$

$$\cancel{|\psi\rangle_{III} = \frac{\gamma}{\sqrt{2}}} \qquad \gamma \propto \chi^{(2)}$$

$$\widehat{DC}_1\, |1\rangle_b |0\rangle_c \overbrace{|0\rangle_d |0_e\rangle}^{add} = \gamma |0\rangle_b |0\rangle_c\, |1\rangle_d |1\rangle_e$$

$$\widehat{DC}2\, |0\rangle_b |1\rangle_c \overbrace{|0\rangle_h |0\rangle_k}^{add} = \gamma\, |0\rangle_b |0\rangle_c\, |1\rangle_h |1\rangle_k$$

Note: if MODE e and k are aligned

they are the same mode!

$$\widehat{DC}_2 \widehat{DC}_1 |\psi\rangle_{II} = |\psi\rangle_{III} = \frac{\gamma}{\sqrt{2}} \overbrace{|0\rangle_b |0\rangle_c}^{drop} \left[|1\rangle_d |1\rangle_e |0\rangle_h |0\rangle_k + i\, |0\rangle_d |0\rangle_e |1\rangle_h |1\rangle_k \right]$$

The $\widehat{PS}_\varphi |0\rangle_h = |0\rangle_h$ and $\widehat{PS}_\varphi |1\rangle_h = e^{i\varphi} |1\rangle_h$

$$\widehat{PS}_\varphi |\psi\rangle_{III} = |\psi\rangle_{IV}$$

$$\boxed{|\psi\rangle_{IV} = \frac{\gamma}{\sqrt{2}} \left[|1\rangle_d |1\rangle_e |0\rangle_h |0\rangle_k + i\, e^{i\varphi} |0\rangle_d |0\rangle_e |1\rangle_h |1\rangle_k \right]}$$

Now: $\hat{BS}_c |1\rangle_d |0\rangle_h = \frac{1}{\sqrt{2}} [|1\rangle_m |0\rangle_\ell + i |0\rangle_m |1\rangle_\ell]$

$\hat{BS}_c |0\rangle_d |1\rangle_h = \frac{1}{\sqrt{2}} [|0\rangle_m |1\rangle_\ell + i |1\rangle_m |0\rangle_\ell]$

$$\boxed{\begin{aligned} |\psi\rangle'_{V} &= \hat{BS}_c |\psi_{IV}\rangle \\ &= \frac{\gamma}{2} [|1\rangle_e |0\rangle_k (|1\rangle_m |0\rangle_\ell + i |0\rangle_m |1\rangle_\ell) \\ &\quad + i e^{i\varphi} |0\rangle_e |1\rangle_k (|0\rangle_m |1\rangle_\ell + i |1\rangle_m |0\rangle_\ell)] \end{aligned}}$$

and no beam block B

Now if aligned λ $|\ \rangle_e$ and $|\ \rangle_k$ are the same mode and $|1\rangle_e |0\rangle_k = |0\rangle_e |1\rangle_k \equiv |1\rangle_k$

That is you can not tell even in principle is photon $|1\rangle_k$ was created in DC_1 or DC_2.

Hence it factorizes!

$$|\psi\rangle_{V} = e^{i\varphi/2} \frac{\gamma}{2} |1\rangle_k [-\left(\frac{e^{i\varphi/2} - e^{-i\varphi/2}}{2i}\right) 2i \, |1\rangle_m |0\rangle_\ell + 2i \left(\frac{e^{i\varphi/2} + e^{-i\varphi/2}}{2}\right) |0\rangle_m |1\rangle_\ell]$$

$$\boxed{= \gamma e^{i\varphi/2} |1\rangle_k [-\sin\varphi/2 \, |1\rangle_m |0\rangle_\ell + \cos\varphi/2 \, |0\rangle_m |1\rangle_\ell]}$$

So what is $P_{coin}(1_\ell, 1_k)$? Detect 1 at D_s and 1 at D_i

$$P_{coin}(1_\ell, 1_k) = \left| {}_{V}\langle\psi| a^\dagger_\ell a_\ell \, a^\dagger_k a_k |\psi\rangle_{V} \right|^2 = \gamma^2 \cos^2\varphi/2 = \boxed{\frac{\gamma^2}{2}(1 + \cos\varphi)}$$

So what is

$P_s(1_\ell)$ independent/ignor/trace D_i?

$$P_s(1_\ell) = {}_{\text{IV}}\langle\psi| a^\dagger_\ell a_\ell |\psi\rangle = \boxed{\frac{\gamma^2}{2}(1+\cos\varphi)}$$

Same! D_i does not even need to be there!

OK we move Beam Block B INTO PLACE

Now $|1\rangle_e|0\rangle_k \neq |0\rangle_e|1\rangle_k$ modes are distinguishable! They no longer factorize

$$|\psi\rangle'_{\text{IV}} = \frac{\gamma}{2}\cdot\Big[|1\rangle_e|0\rangle_k\,(|1\rangle_m|0\rangle_\ell + i\,|0\rangle_m|1\rangle_\ell)$$
$$+ ie^{i\varphi}|0\rangle_e|1\rangle_k\,(|0\rangle_m|1\rangle_\ell + i\,|1\rangle_m|0\rangle_\ell)\Big]$$

Note $|1\rangle_e|0\rangle_k \perp$ to $|0\rangle_e|1\rangle_k$

$$P'_{coin}(1_\ell\, 1_k) = {}_{\text{IV}}'\langle\psi| a^\dagger_\ell a_\ell\, a^\dagger_k a_k |\psi\rangle'_{\text{IV}}$$

$$= \frac{\gamma^2}{4}$$ Interference vanishes

D_i can now tell which path $|1\rangle_k$ took

Even better

$$P'_s(1_\ell) = {}'_{\mathrm{V}}\langle\psi|\, a^{+}_{\ell}\, a_{\ell}\, |\psi\rangle'_{\mathrm{V}}$$

$$= \frac{\gamma^2}{4} + \frac{\gamma^2}{4} = \frac{\gamma^2}{2}$$

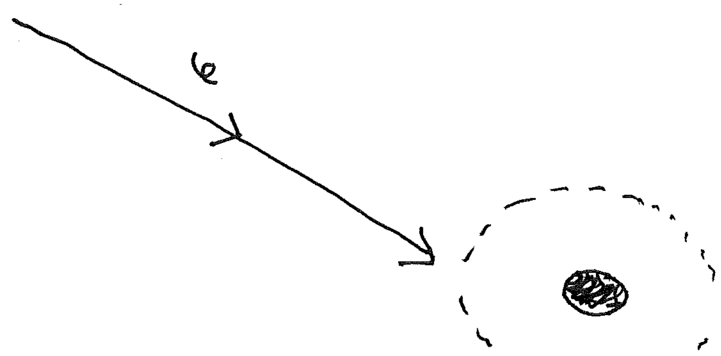
STILL NO INTERFERENCE

D_i does not have to even be there!

The fact that which-path information is available in environment destroys interference even if nobody looks!

The really crazy paper

Replace D_i with a Black Hole



EVENT HORIZON (IN SOME THEORIES) ACTS AS QUANTUM ERASER AND ERASES WHICH-PATH.

INTERFERENCE COMES BACK! Z.W.M

Interferometer can be used as a black hole detector! (Hockney & Yurtsever)