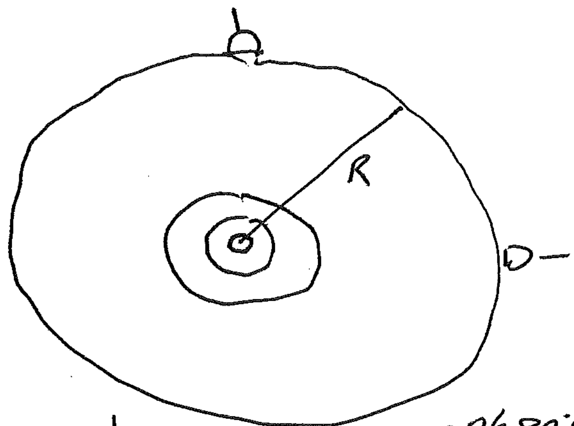
9.0 EPR & Hidden Variables

Quantum Mechanics is weird.

Dirac 1920's something weird and nonlocal going on! Imagine a source that emits a single photon in a spherical wave



at the center of a spherical shell $R = 1$ light year. We line the shell with perfect photo detectors. The state is

$$|\psi\rangle = \frac{e^{ikr}}{kr} |1\rangle \qquad \text{where} \qquad r = r(t) = ct$$

where $ct = R = 1$ light year
one detector will fire but only one with probability

$$P = \langle \psi | \hat{n} | \psi \rangle = \frac{1}{k^2 R^2}$$

In ensemble interpretation if experiment performed many times, which detector fires is completely random. However for a single event only one detector will fire!

Consider $E = \hbar\omega$ for this single photon.
For sake of argument assume detector at North Pole Fires

$$t \lesssim T = R/c \qquad E_{NP} = \frac{\hbar\omega}{k^2 R^2} = \hbar\omega \langle\psi|\hat{n}|\psi\rangle$$

That is electric and magnetic field energy is evenly distributed all over the surface of the sphere as classical E&M tells us.

However there is only <u>one</u> photon! After NP Fires we know

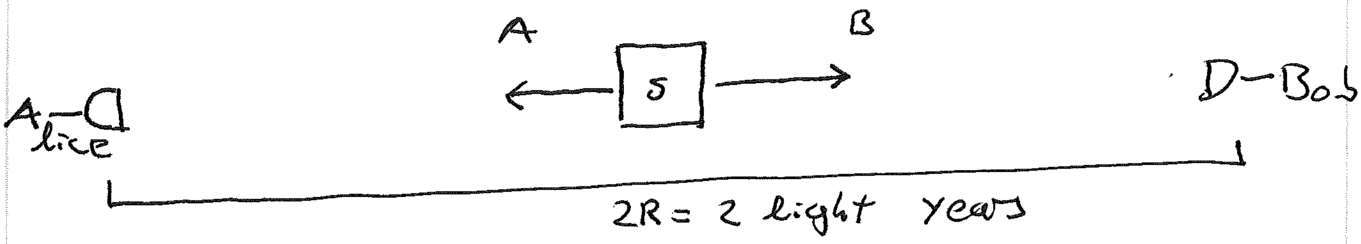
$$t \gtrsim T = R/c \qquad E_{NP} = \hbar\omega$$

That means all of the energy — spread all over the sphere — collapses <u>instantaneously</u> to the north pole!

Would seem to violate relativity!

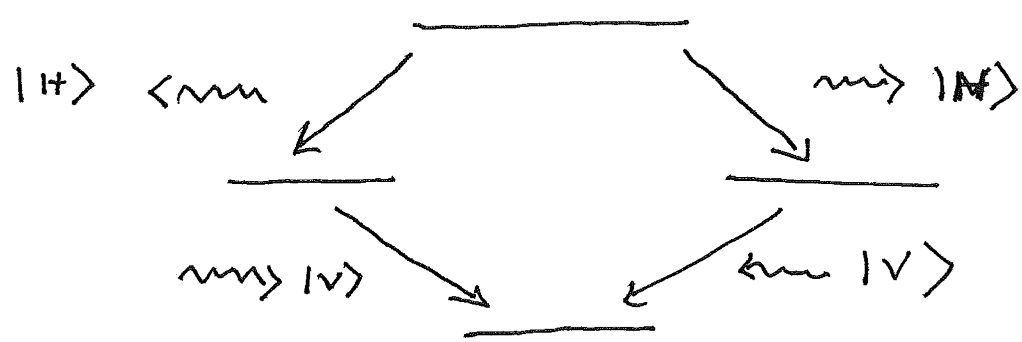
BOHM VERSION OF EPR

SUPPOSE I HAVE A SOURCE OF POLARIZATION (SPIN) ENTANGLED PHOTON PAIRS



$$|\psi\rangle = \frac{|H\rangle_A |V\rangle_B + |V\rangle_A |H\rangle_B}{\sqrt{2}}$$

Such a state can be produced in two-photon decay process in Cs atoms



Two possible paths down are indistinguishable

$$\Rightarrow \quad |\psi\rangle = |H\rangle|V\rangle + |V\rangle|H\rangle$$

$$\hbar = c = G = \sqrt{2} = 1$$ ☺

According to Q.M. Alice and Bob's
POLARIZATION IS ANTI CORRELLATED

A —[	]— B
H	V
V	H
V	H
H	V
⋮	⋮

Alice will get H or V with 50% prob
Bob will get V or H with 50% prob

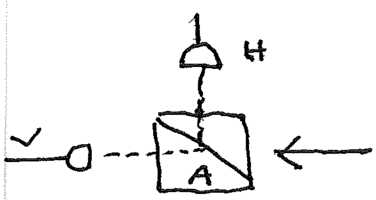
When Alice gets H Bob always gets V
" " " V " " " H

Anticorellation

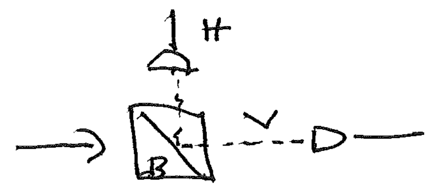
Nonlocal weirdness: when Alice makes a measurement and gets "H" Bob's states "collapses" to "V" and vice versa — even if A and B are 2 light years apart

More nonlocal weirdness

We assume Alice and Bob set their polarization analysers to H or V



Polarizing BS

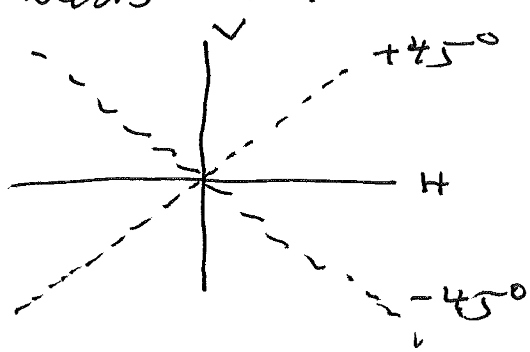


Polarizing BS

Alice	Bob
H	V
V	H
V	H
H	V
V	H
⋮	⋮

Different Detectors Go click!

However Alice and Bob could choose a different basis $|\pm\rangle$



IT IS EASY TO SHOW

$$|+\rangle = |H\rangle + |V\rangle \qquad |H\rangle = |+\rangle + |-\rangle$$

$$|-\rangle = |H\rangle - |V\rangle \qquad |V\rangle = |+\rangle - |-\rangle$$

OK BUT NOTE

$$|\psi\rangle = |H\rangle|V\rangle - |V\rangle|H\rangle$$

$$= [\,|+\rangle + |-\rangle\,][\,|+\rangle - |-\rangle\,] - [\,|+\rangle - |-\rangle\,][\,|+\rangle + |-\rangle\,]$$

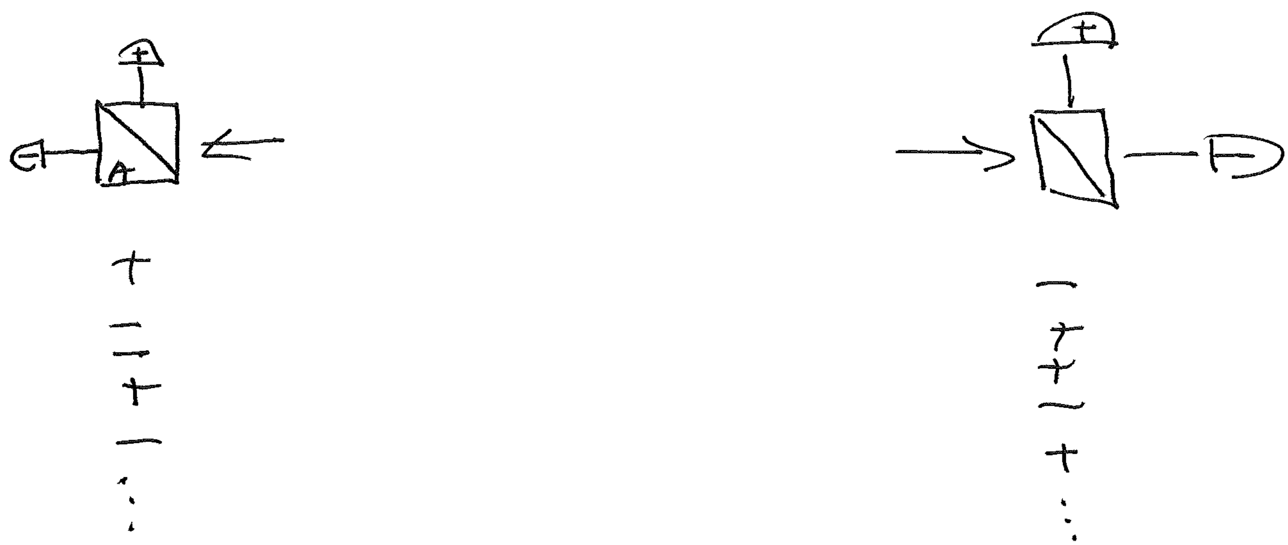
$$= \cancel{|+\rangle|+\rangle} - |+\rangle|-\rangle + |-\rangle|+\rangle - \cancel{|-\rangle|-\rangle}$$
$$-[\,\cancel{|+\rangle|+\rangle} + |+\rangle|-\rangle - |-\rangle|+\rangle - \cancel{|-\rangle|-\rangle}\,]$$

$$= \boxed{|+\rangle_A|-\rangle_B - |-\rangle_A|+\rangle_B}$$

up to overall phase $e^{i\pi} = -1$ and $\sqrt{2} = 1$

so $|\psi\rangle$ is a singlet state also

anticorrelated in $\pm$ BASIS



So now here's where it gets weird!

Alice and Bob can wait $t \lesssim T = R/c$

and then flip coins to choose which

basis to measure in H, V or $\pm$

Heads $\Rightarrow$ H/V Tails $\Rightarrow$ $\pm$

25% of the time they will BOTH GET HEADS AND MEASURE

H V

or

V H

25% of the time they will both get tails and measure

\+ −

or

− +

For these events information on <u>which basis</u> Alice used must instantaneously propagate to Bob so his photon collapses in the correct basis so he is always anticorrelated.

The state of A or B's photon is uncertain <u>until</u> a measurement is made!

Somehow they "communicate" what state they collapse into over light years so anticorrelation is maintained.

Einstein, Podolsky, Rosen PR $\underset{\sim}{47}$ (1935) 777 – 780

" Can quantum mechanical description of physical reality be considered complete?"

Einstein's Most Cited Paper by order of magnitude!

10,000 vs 1,000

" Einstein Attacks Quantum Theory."

NY TIMES Podolsky leaked the news and Einstein never spoke to him again!

Summary

Any physical theory must be realistic
"Einstein Realism"
that is polarization is always H, V or +/−
and not determined by what is measured, or by the observation. Does not like collapse

Any physical theory must be local
Alice's result cannot influence Bob's.

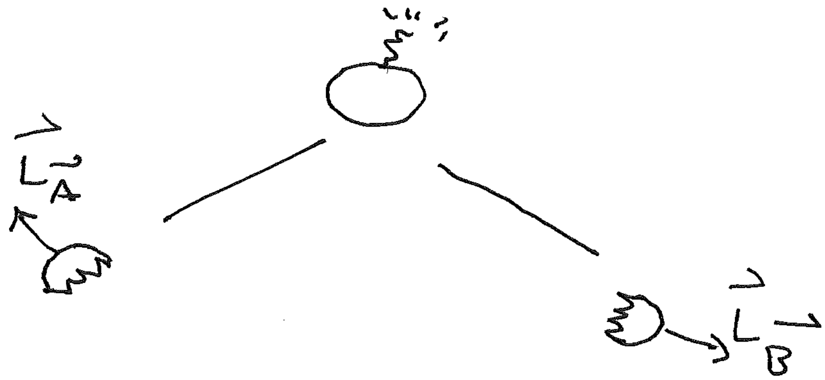
Conclusion?

Q.M. IS EITHER NONREALISTIC OR NONLOCAL OR BOTH!

Cannot be "complete" theory?

What would be a "complete" theory?

Consider a bomb always explodes in two pieces



IF ∡ momentum of bomb is initially zero, and Alice measures ∡ mom $\vec{L}_{\vec{A}}$ then Bob will <u>always</u> get $\vec{L}_{\vec{B}} = -\vec{L}_{\vec{A}}$

That is always <u>anti</u>-corellated

why is Q.M. not like this?

Hidden Variable Theory

QM is Statistical Mechanics on each run $\vec{L}_{\vec{A}} + \vec{L}_{\vec{B}} = 0$ although $\vec{L}_{\vec{A}}$ and $\vec{L}_{\vec{B}}$ are different run to run — details of explosion are "hidden" but produce anticorellations

Hidden Variable Theory is Realistic

On each run Bomb A always has a well defined $\vec{L}_A$ and Bomb B always has a well defined $\vec{L}_B$. These may be unpredictable but we are always sure

$$\vec{L}_A = -\vec{L}_B$$

From run to run. Measurement does not "create" $\vec{L}_A$ or $\vec{L}_B$ it is always there.

Hidden Variable Theory is Local

The anticorrelation is built into the experiment. Alices measurement of $\vec{L}_A$ does not determine $\vec{L}_B$ — they are predetermined — no action at a distance

EPR — can we replace QM with a statistical local, realistic hidden variable theory?

Compare predictions of Local Realistic HV theory to QM by looking at joint probabilities in non orthogonal bases

$$P_{AB}(\theta_A, \theta_B)$$

In each run Alice and Bob randomly set their polarizers to θ_A and θ_B

EG H, V $\theta_A = \theta_B = 0, 180°$

+ − $\theta_A = \theta_B = +45, -45$

Bell showed

$$\boxed{P_{AB}^{\text{Hidden Variable}} \neq P_{AB}^{Q.M}}$$

Hence one could test experimentally which is right?

Quantum Mechanics?

Local Hidden Variable?

Note Non-Local Hidden Variable Theory can reproduce QM — but this requires that A and B's detectors "conspire" with each other long after photons have left the source. Magic? Outside each other's light cones.