8.5 DECOHERENCE KILLED THE CAT

IF $|\psi(0)\rangle$ is a pure state and

$$\hat{\rho}_0 = |\psi(0)\rangle\langle\psi(0)| = \begin{bmatrix} \rho_{11} & \rho_{12} & \cdots \\ \rho_{21} & \rho_{22} & \\ \vdots & & \ddots \end{bmatrix}$$ is a pure state

density matrix then $\rho_{ij} = \rho_{ji}^*$ are a measure of "coherence" the ability to perform quantum interference experiments.

Upon interaction with environment / bath / reservoir the $\rho_{ij} \to 0$ TYPICALLY MORE RAPIDLY THAN $\rho_{ii} \to 0$ (IN LAST EXAMPLE $\rho_{ii} \to 1/3$)

The rapid decay of the off diagonal terms is called decoherence. The end is

$$\hat{\rho}_0 = \begin{bmatrix} \rho_{11} & \rho_{12} & \cdots \\ \rho_{21} & \rho_{22} & \\ \vdots & & \end{bmatrix} \to \begin{bmatrix} \rho_{11} & & 0 \\ & \rho_{22} & \\ 0 & & \ddots \end{bmatrix}$$ a mixed state.

CONSIDER AN EVEN CAT AT $t=0$

$$|\psi(0)\rangle = [\,|\alpha\rangle + |\bar{\alpha}\rangle\,]$$ NEG. Normalization

$$\hat{\rho}(0) = |\alpha\rangle\langle\alpha| + |\alpha\rangle\langle\bar{\alpha}| + |\bar{\alpha}\rangle\langle\alpha| + |\bar{\alpha}\rangle\langle\bar{\alpha}|$$

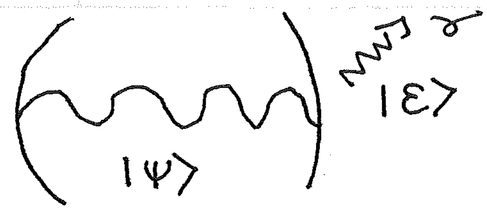
$$= \begin{array}{cc} |\alpha\rangle & |\bar{\alpha}\rangle & \\ \left[\begin{matrix} 1 \\ 1 \end{matrix}\right. & \left.\begin{matrix} 1 \\ 1 \end{matrix}\right] & \begin{matrix} \langle\alpha| \\ \langle\bar{\alpha}| \end{matrix} \end{array}$$

$$= \begin{bmatrix} \rho_{11} & \rho_{12} \\ \rho_{21} & \rho_{22} \end{bmatrix}$$

Since $\rho_{12} = \rho_{21} = 1 = \rho_{11} = \rho_{22}$

STATE IS MAXIMALLY COHERENT.

We now assume cat is coupled to the environment $|\mathcal{E}\rangle$



DUE TO LOSS AT THE MIRROR ($\gamma \neq 0$): ASSUME COUPLING TURNED ON AT $t = 0$. The combined field-environment:

$$|\Psi(0)\rangle|\mathcal{E}(0)\rangle = |\alpha\rangle|\mathcal{E}(0)\rangle + |\bar{\alpha}\rangle|\mathcal{E}(0)\rangle$$

WE NOW TURN ON COUPLING AND ALLOW STATE OF CAT <u>AND</u> ENVIRONMET TO EVOLVE. CRITICAL IS ASSUMPTION

$$|\alpha(0)\rangle|\mathcal{E}(0)\rangle \rightarrow |\alpha(t)\rangle|\mathcal{E}_1(t)\rangle$$
$$|\bar{\alpha}(0)\rangle|\mathcal{E}(0)\rangle \rightarrow |\bar{\alpha}(t)\rangle|\mathcal{E}_2(t)\rangle$$

That is environment evolves <u>differently</u> for <u>LIVE</u> CAT $|\alpha\rangle$ THAN FOR DEAD CAT $|\bar{\alpha}\rangle$

IF $\mathcal{E}$ HAS # DEGREES OF FREEDOM/ MODES/ DIMENSION OF HILBERT SPACE <u>VERY</u> LARGE THEN

$$\boxed{\langle \mathcal{E}_1(t)|\mathcal{E}_2(t)\rangle \simeq \delta_{12} \quad \forall\, t>0}$$

THAT IS DIFFERENT STATES OF ENV. ARE ORTHOGONAL

WE ALSO KNOW THAT:

$$\left.\begin{aligned} |\alpha(t)\rangle &\simeq |\alpha(0)\, e^{-\gamma t/2}\rangle \\ |\bar{\alpha}(t)\rangle &= |\bar{\alpha}(0)\, e^{-\gamma t/2}\rangle \end{aligned}\right\}\ \text{SHRINKING CATS}$$

WE'LL ASSUME t S.T. JUMPING CAT IS <u>EVEN</u>

THE DENSITY OP $\hat{\rho}$ EVOLVES TO

$$\hat{\rho}(0) \equiv |\psi(0)\rangle|\mathcal{E}(0)\rangle\langle\mathcal{E}(0)|\langle\psi(0)|$$

$$\downarrow$$

$$\left[\,|\alpha(t)\rangle|\mathcal{E}_1(t)\rangle + |\bar{\alpha}(t)\rangle|\mathcal{E}_2(t)\rangle\right]\left[\langle\alpha(t)|\langle\mathcal{E}_1(t)| + \langle\bar{\alpha}(t)|\langle\mathcal{E}_2(t)|\,\right]$$

$= (|\alpha\rangle\langle\alpha|)(|\mathcal{E}_1\rangle\langle\mathcal{E}_1|)$ DIAGONAL $= \rho_{11}(t)$

$+ (|\alpha\rangle\langle\bar{\alpha}|)(|\mathcal{E}_1\rangle\langle\mathcal{E}_2|)$ ρ_{12}

COHERENCE

$+ (|\bar{\alpha}\rangle\langle\alpha|)(|\mathcal{E}_2\rangle\langle\mathcal{E}_1|)$ ρ_{21}

$+ (|\bar{\alpha}\rangle\langle\bar{\alpha}|)(|\mathcal{E}_2\rangle\langle\mathcal{E}_2|) = \rho_{22}(t)$

$$= \begin{bmatrix} \rho_{11} & \rho_{12} \\ \rho_{21} & \rho_{22} \end{bmatrix}$$

SINCE WE DON'T MEASURE ENV. WE <u>IGNORE</u> IT!
TRACE OVER ENV GIVES REDUCED DENSITY MATRIX

$$\hat{\rho}_{RED}(\alpha,\bar{\alpha}) = \mathrm{Tr}_{ENV}\,\rho(\alpha,\bar{\alpha},\mathcal{E}_1,\mathcal{E}_2)$$

$$= \sum_{i=1}^{2} \langle\mathcal{E}_i|\,\rho(\alpha,\bar{\alpha},\mathcal{E}_1,\mathcal{E}_2)\,|\mathcal{E}_i\rangle$$

$$= |\alpha(t)\rangle\langle\alpha(t)| + |\bar{\alpha}(t)\rangle\langle\bar{\alpha}(t)|$$

$$= \begin{bmatrix} \rho_{11} & 0 \\ 0 & \rho_{22} \end{bmatrix} =$$

TRACE ENV <u>KILLS</u> ~~OFF~~ DIAGONAL <u>TERMS</u>

LEAVING $|\alpha_0 e^{-\gamma t/2}\rangle\langle\alpha_0 e^{-\gamma t/2}| + |\bar{\alpha}_0 e^{-\gamma t/2}\rangle\langle\bar{\alpha}_0 e^{-\gamma t/2}|$

AGAIN CONCLUSION IS INT. W/ENV. KILLS OFF DIAGONAL TERMS MUCH FASTER THAN $T = 1/\gamma$

SO CAT COLLAPSES TO MIXTURE

$$\hat{\rho}(t) = |\alpha(t)\rangle\langle\alpha(t)| + |\bar{\alpha}(t)\rangle\langle\bar{\alpha}(t)|$$

OF CAT DEAD OR ALIVE

TAKES MUCH LONGER $T = 1/\gamma$ FOR CAT TO DISAPPEAR ENTIRELY

$$\hat{\rho}(\infty) = |0\rangle\langle 0| \quad \text{VACUUM}$$

LET'S SEE FROM MASTER EQ.

$$\frac{d\hat{\rho}}{dt} = -i\hbar\,[\hat{H}_0, \hat{\rho}] + \frac{\gamma}{2}\left[2\hat{a}\hat{\rho}\hat{a}^{+} - \hat{a}^{+}\hat{a}\hat{\rho} - \hat{\rho}\hat{a}^{+}\hat{a}\right]$$

SLOWLY VARYING ENVELOPE $\hat{H}_0 = \hbar\omega_0\hat{n}$

$$\hat{\rho}(t) \to \hat{\rho}\, e^{-i\omega_0 t}$$

to cancell out high frequency oscillations at $t = \frac{2\pi}{\omega_0}$ leaving only slow decay at $T = 1/\gamma$

$$\Rightarrow \quad \frac{d\hat{\rho}}{dt} = \frac{\gamma}{2}\left[2\hat{a}\hat{\rho}\hat{a}^{+} - \hat{a}^{+}\hat{a}\hat{\rho} - \hat{\rho}\hat{a}^{+}\hat{a}\right] \; *$$

Assume by ansatz a coheret state: FIRST

$$\hat{\rho}(t) = |\alpha(t)\rangle\langle\alpha(t)| = |\alpha e^{-\gamma t/2}\rangle\langle\alpha e^{-\gamma t/2}|$$

$$\cancel{\dot{\rho} = -\frac{\gamma}{2}|\alpha(t)\rangle\langle\alpha(t)| - \frac{\gamma}{2}|\alpha(t)\rangle\langle\alpha(t)|}$$

Satisfies * by substitution.

BUT OFF DIAGONAL TERMS DECAY FASTER

$\rho_{\alpha\tilde{\alpha}} = \rho_{\tilde{\alpha}\alpha} \approx$

$$\exp\left[-2\bar{n}_0\left(1-e^{-\gamma t}\right)\right] \qquad \gamma t \ll 1$$

$$\approx \exp\left[-2\bar{n}_0 \gamma t\right] \qquad 1-e^{-\gamma t} \approx \gamma t$$

so decay rate is <u>decoherence</u> rate

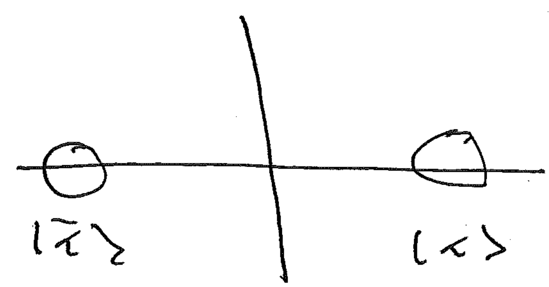
$$\Gamma = 2\bar{n}_0\gamma$$

$$T_{dech} = \frac{1}{\Gamma} = \frac{1}{2\bar{n}_0\gamma} = \frac{T_{decay}}{2\bar{n}_0}$$

which depends on size of <u>cat</u>!

$\bar{n}_0 = |\alpha|^2 \gg 1$

implies macroscopic superposition!



so decoherence <u>depends</u> on size of quantum state.

Example $\quad \gamma = 1\ Hz \qquad T_{decay} = 1\,s$

$\bar{n}_0 = 10^6 \qquad \Gamma = 1 \cdot 2 \cdot 10^6 \qquad T_{decoh} \cong 10^{-6}$ <u>sec</u>

↑

laser pointer

Hence $\bar{n}(t) = \alpha^*(t)\alpha(t) = \bar{n}(0)\, e^{-\gamma t}$

and $T = 1/\gamma$ is decay time.

consider instead $|CAT\rangle = N_e\,[\,|\alpha\rangle + |\bar{\alpha}\rangle\,]$

$$\hat{\rho}_{CAT} = |CAT\rangle\langle CAT| = N_e^2 \begin{array}{cc} & \begin{array}{cc} |\alpha\rangle & |\bar{\alpha}\rangle \end{array} \\ & \left[\begin{array}{cc} 1 & 1 \\ 1 & 1 \end{array}\right] \begin{array}{c} \langle\alpha| \\ \langle\bar{\alpha}| \end{array} \end{array} = \hat{\rho}(0)$$

This is maximally coherent $\rho_{\alpha\bar{\alpha}} = \rho_{\bar{\alpha}\alpha} = 1$

Recall normally ordered characteristic Fcn

$$C_N(\lambda, t) = \mathrm{tr}\left[\hat{\rho}(t)\, e^{\lambda \hat{a}^{+}} e^{-\lambda^* \hat{a}}\right]$$

For $\dot{\rho} = \frac{\gamma}{2}\left[2 a \rho a^{+} - a^{+} a \rho - \rho a^{+} a\right]$

one can show

$$C_N(\lambda, t) = C_N(\lambda e^{-\gamma t/2}, 0)$$

From WICH

$$\hat{\rho}_{CAT}(t) = N_e^2 \Big[\, |\alpha e^{-\gamma t/2}\rangle\langle \alpha e^{-\gamma t/2}| + e^{-2|\alpha|^2(1-e^{-\gamma t})}\, |\alpha e^{-\gamma t/2}\rangle\langle \bar{\alpha} e^{-\gamma t/2}| + e^{-2|\alpha|^2(1-e^{-\gamma t})}\, |\bar{\alpha} e^{-\gamma t/2}\rangle\langle \alpha e^{-\gamma t/2}| + |\bar{\alpha} e^{-\gamma t/2}\rangle\langle \bar{\alpha} e^{-\gamma t/2}| \,\Big]$$

so $\rho_{\alpha\alpha} = \rho_{\bar{\alpha}\bar{\alpha}} = \bar{n}_0\, e^{-\gamma t}$ and so

diagonal terms decay like coherent states

γ loss from cavity. $T_{LOSS} = \frac{1}{\gamma}$