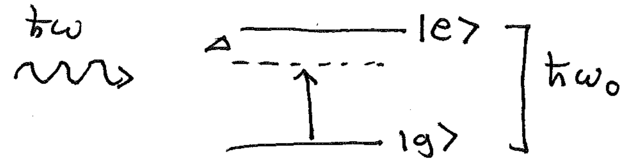
8.0 DECOHERENCE — OPTICAL BLOCH EQ.

It is typical to approximate decoherence in semiclassical theory phenomenologically

Recall Rabi Model

$E_g = \hbar\omega_g \qquad \omega_0 = \omega_e - \omega_g$

$E_e = \hbar\omega_e$



$$|t\rangle = C_g(t)\, e^{-i\omega_g t} |g\rangle + C_e(t)\, e^{-i\omega_e t} |e\rangle \qquad \text{Eq. 4.70}$$

$$C_e(t) = \frac{iU}{\Omega} e^{i\Delta t/2} \sin[\Omega t/2]$$

$$C_g(t) = e^{i\Delta t/2}\left[\cos \Omega t/2 - i \frac{\Delta}{\Omega} \sin \Omega t/2\right]$$

Let's take $\Delta = \omega - \omega_0 = 0$

$$\Omega_R = [\Delta^2 + U^2]^{1/2} \rightarrow U = -\hbar \langle e|\hat{\vec{d}}\cdot\vec{\mathcal{E}}_0|g\rangle = \Omega_0$$

$$\Rightarrow |t\rangle = \cos \Omega_0 t/2\, |g\rangle e^{-i\omega_g t} + \sin[\Omega_0 t/2]\, |e\rangle e^{-i\omega_e t}$$

we may construct density matrix

$$\hat{\rho}(t) = |t\rangle\langle t| = \begin{array}{ccc} |e\rangle & |g\rangle & \\ \left[\begin{array}{c} \sin^2 \Omega_0 t/2 \\ \tfrac{1}{2}\sin \Omega_0 t\, e^{+i\omega_0 t} \end{array}\right. & \left.\begin{array}{c} \tfrac{1}{2}\sin\Omega_0 t\, e^{-i\omega_0 t} \\ \cos^2 \Omega_0 t/2 \end{array}\right] & \begin{array}{c} \langle e| \\ \langle g| \end{array} \end{array}$$

Recall $W(t) = \rho_{ee} - \rho_{gg} = \sin^2 - \cos^2 = -\cos \Omega_0 t$ is the inversion. The off diagonal terms $\frac{1}{2}\sin \Omega_0 t\, e^{\mp i\omega_0 t}$ are the coherence terms. These are interference terms.

Note $\hat{\rho}_{MIX} = \begin{bmatrix} \sin^2 \Omega_0 t/2 & 0 \\ 0 & \cos^2 \Omega_0 t/2 \end{bmatrix}$

Desribes a classical statistical mixture,

$$P_e = \sin^2 \Omega t/2 \qquad P_g = \cos^2 \Omega t/2$$

and so measurements produce these randomly like coin flipping. The off-diagonal terms describe quantum interference.

If the atom is coupled to a heat bath or any system with an ∞ number of modes the coherence will decay with a rate $\gamma = 1/T$. We model this as $e^{\pm i\omega_0 t} \to e^{\pm i\omega_0 - \gamma t}$ and so the off diagonal terms experience exponential decay

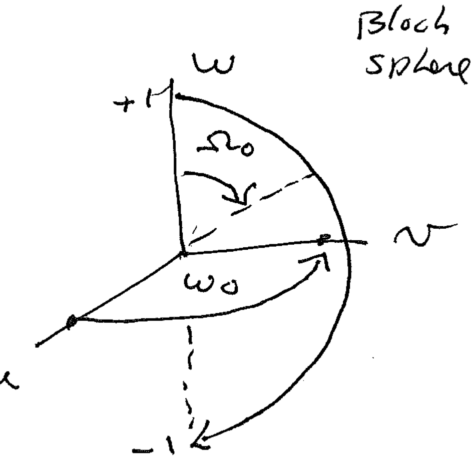
$$\hat{\rho} = \begin{bmatrix} \sin^2 \Omega_0 t/2 & \frac{1}{2}\sin(\Omega_0 t) e^{-i\omega_0 t} e^{-\gamma t} \\ \frac{1}{2}\sin(\Omega_0 t) e^{+i\omega_0 t} e^{-\gamma t} & \cos^2 \Omega_0 t/2 \end{bmatrix} \xrightarrow{t\to\infty} \hat{\rho}_{mix}$$

Note in decoherence probability is still conserved a measurement will produce les or 19S.

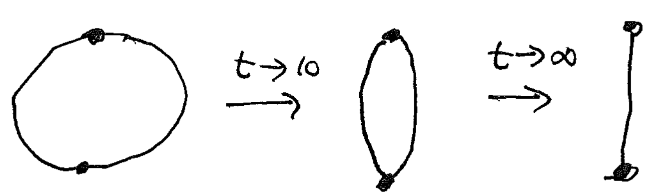
Typical we write $u + iv = \mathrm{Re}\,[\rho_{eg}] + \mathrm{Re}\,(\rho_{ge})$

$$\Rightarrow \quad u = \tfrac{1}{2}\sin(\Omega_0 t)\cos\omega_0 t\, e^{-\gamma t}$$
$$v = \tfrac{1}{2}\sin(\Omega_0 t)\sin\omega_0 t\, e^{-\gamma t}$$

Bloch sphere



As u and v decay to zero



Indicating only North / South pole possible.

In this same model we can introduce loss

$|e\rangle$ —— $\rightsquigarrow \gamma_e$

$|g\rangle$ —— $\rightsquigarrow \gamma_g$

That is population losses electrons at rate $\gamma_e = 1/T_e$ & $\gamma_g = 1/T_g$ two same third level uncoupled from field

In this case probability is not conserved!

$\gamma_t \equiv \gamma_e + \gamma_g$

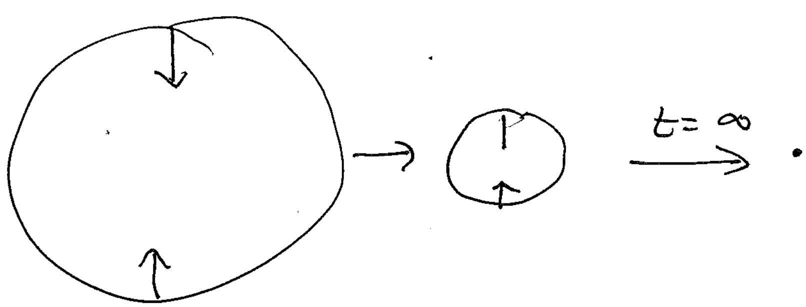
$$\hat{\rho} = \begin{bmatrix} e^{-\gamma_e t} \sin^2 \Omega t/2 & \frac{1}{2} \sin \Omega t \, e^{-i\omega_0 t} e^{-\gamma_t t} \\ e^{-\gamma_t t} \frac{1}{2} \sin \Omega t \, e^{+i\omega_0 t} & e^{-\gamma_g t} \cos^2 \Omega t/2 \end{bmatrix}$$

$$P_e = e^{-\gamma_e t} \sin^2 \Omega t/2$$

$$P_g = e^{-\gamma_g t} \cos^2 \Omega t/2$$

$$\boxed{P_e + P_g \leq 1}$$

Here sphere shrinks!



$\omega = \omega_0$

Let us go back and revist

$V = V_0 \cos\omega_0 t$

$$i\hbar\dot{\hat{\rho}} = [\hat{H}, \hat{\rho}]$$

$$\Rightarrow \quad i\hbar\dot{\hat{\rho}} = \begin{bmatrix} \hbar\omega_0 & V \\ V & \hbar\omega_0 \end{bmatrix}\begin{bmatrix} \rho_{11} & \rho_{12} \\ \rho_{21} & \rho_{22} \end{bmatrix} - \begin{bmatrix} \rho_{11} & \rho_{12} \\ \rho_{21} & \rho_{22} \end{bmatrix}\begin{bmatrix} \hbar\omega_0 & V \\ V & \hbar\omega_0 \end{bmatrix}$$

$$\Rightarrow i\hbar \begin{bmatrix} \dot{\rho}_{11} & \dot{\rho}_{12} \\ \dot{\rho}_{21} & \dot{\rho}_{22} \end{bmatrix} = \begin{bmatrix} \hbar\omega_0\rho_{11} + V\rho_{21} & \hbar\omega_0\rho_{12} + V\rho_{22} \\ V\rho_{11} + \hbar\omega_0\rho_{21} & \end{bmatrix}$$

$\vdots$

$$\boxed{\begin{aligned} \frac{d\rho_{22}}{dt} &= -\frac{\partial\rho_{11}}{\partial t} = -\frac{1}{2} i\Omega_0 \rho_{12} + \frac{1}{2} i\Omega_0 \rho_{21} \\ \frac{d\rho_{12}}{dt} &= -\frac{\partial\rho_{21}^*}{\partial t} = \frac{1}{2} i\Omega_0 [\rho_{11} - \rho_{22}] \end{aligned}}$$

These are also called optical Bloch Eqs.

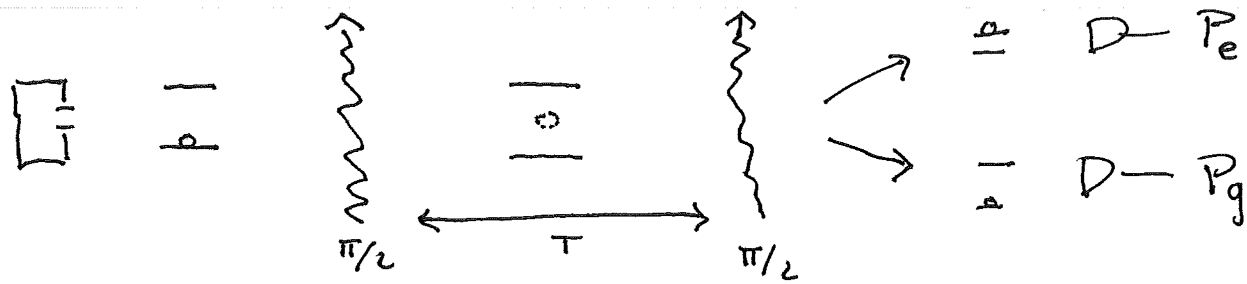
with $\rho_{11} - \rho_{22} = w$ and $U + iV = 2\rho_{12}$

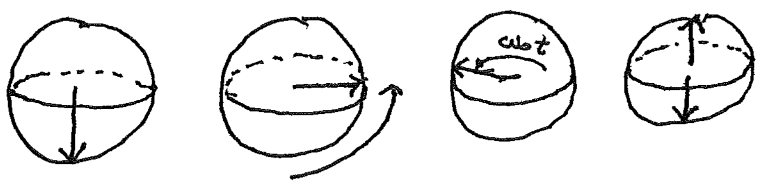
we recover same results

However let's take simple case of

Loss.

ATOMIC CLOCK Ramsey Interferometer





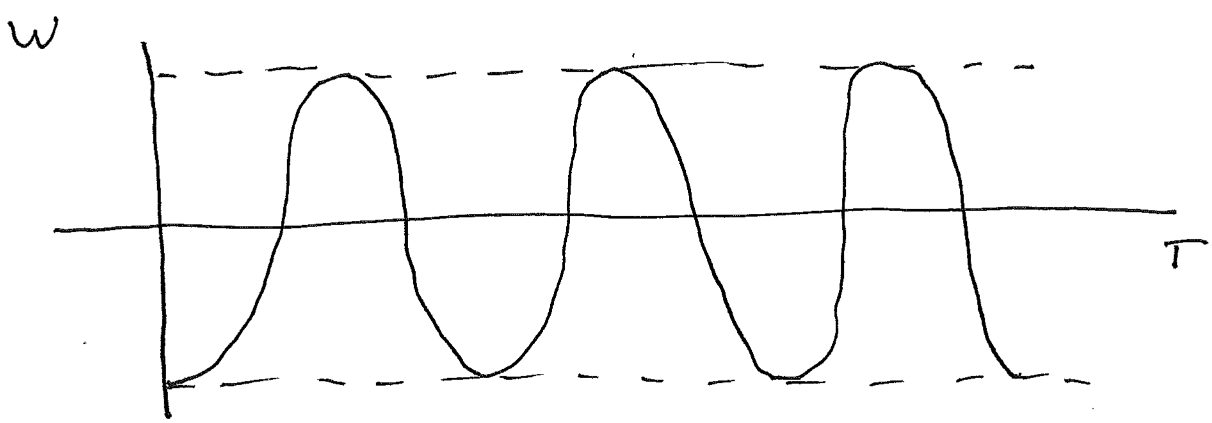
$W=-1$

$$P_e = \sin^2(\omega_0/2\, T)$$

$$P_e + P_g = 1$$

$$P_g = \cos^2(\omega_0/2\, T)$$

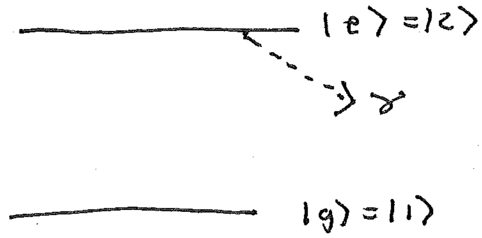
$$W = P_e - P_g = -\cos[\omega_0 T]$$



If ω_0 is a frequency standard I can measure time very accurately by counting peaks

$$t = N t_0 = \frac{N 2\pi}{\omega_0}$$

we allow for spontaneous emission out of upper level into some third level, or coupling to environment



Phenomenologically we insert decay terms.

OBE

$$\boxed{\begin{aligned}
\dot{\rho}_{22} &= -\dot{\rho}_{11} = \underbrace{-\tfrac{i}{2}\Omega_0\rho_{12} + \tfrac{i}{2}\Omega_0\rho_{21}}_{\text{old}} \underbrace{- 2\gamma\rho_{22}}_{\text{loss out of } |e\rangle} \\
\dot{\rho}_{12} &= -\dot{\rho}_{21}^{*} = \underbrace{\tfrac{i}{2}\Omega_0\,[\rho_{11}-\rho_{22}]}_{\text{old}} \underbrace{- \Gamma\rho_{12}}_{\text{decoherence}}
\end{aligned}}$$

If atom in empty space $\gamma = \Gamma$ = Sp Emission rate is only source of decoherence.

However interaction with environment

$$\Gamma = \underset{\substack{\uparrow \\ \text{SpE}}}{\gamma_{SPE}} + \underset{\substack{\uparrow \\ ?}}{\gamma_{DEC}}$$

collisions
heat bath
etc.
decoherence

$\frac{1}{\gamma} = T_1$ $\qquad$ $\frac{1}{\Gamma} = T_2$ decoherence time

Exact solution of 2×2 optical Bloch equations <u>with</u> <u>loss</u> / <u>decoherence</u> not possible. Integrate 2×2 coupled 1st order <u>linear</u> diffy-Qs.

Limiting case $\Omega_0 >> \Gamma$ loss is small

$$\hat{\rho}(t) \approx \begin{bmatrix} \sin^2(\Omega_0 t/2)\, e^{-\gamma t} & -\frac{i}{2}\sin[\Omega_0 t]\, e^{-\Gamma t} e^{i\omega_0 t} \\ \frac{i}{2}\sin[\Omega_0 t]\, e^{-\Gamma t} e^{-i\omega_0 t} & \cos^2(\Omega_0 t/2)\, e^{-\gamma t} \end{bmatrix}$$

$$= \begin{bmatrix} \rho_{ee} & \rho_{eg} \\ \rho_{ge} & \rho_{gg} \end{bmatrix}$$

Optical Bloch

$$w = -\cos[\Omega_0 t]\, e^{-\gamma t} \qquad \text{population loss}$$

$$\left.\begin{aligned} u &= \sin[\Omega_0 t]\, e^{-\Gamma t} \sin\omega_0 t \\ v &= -\sin[\Omega_0 t]\, e^{-\Gamma t} \cos\omega_0 t \end{aligned}\right\} \text{decoherence}$$

population loss $\Rightarrow$ decoherence

decoherence $\not\Rightarrow$ population loss

Special Case $\gamma_{SPE} = 0$ (neg. spontaneous emission)

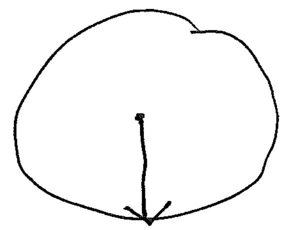
$\Gamma = 0 + \gamma_{DEC}$ (atomic collissions)

$w = -\cos[\Omega_0 t]$ ← population preserved

$$\left.\begin{aligned} u &= \sin[\Omega_0 t]\sin[\omega_0 t]\,e^{-\gamma_{DEC} t} \\ v &= \sin[\Omega_0 t]\sin[\omega_0 t]\,e^{-\gamma_D t} \end{aligned}\right]$$

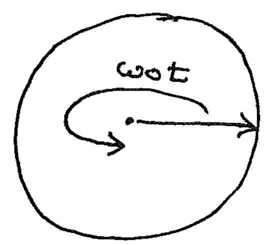
decoherence

$$\rho(0) = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$$



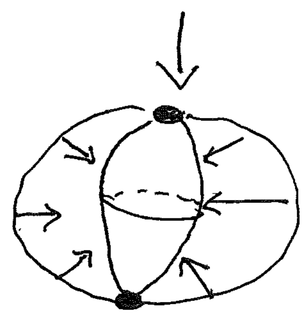
$\pi/2$

$$\rho(t) = \begin{bmatrix} 1 & -\frac{i}{2}\,e^{-\Gamma t} e^{i\omega_0 t} \\ \frac{i}{2}\,e^{-\Gamma t} e^{-i\omega_0 t} & 1 \end{bmatrix}.$$



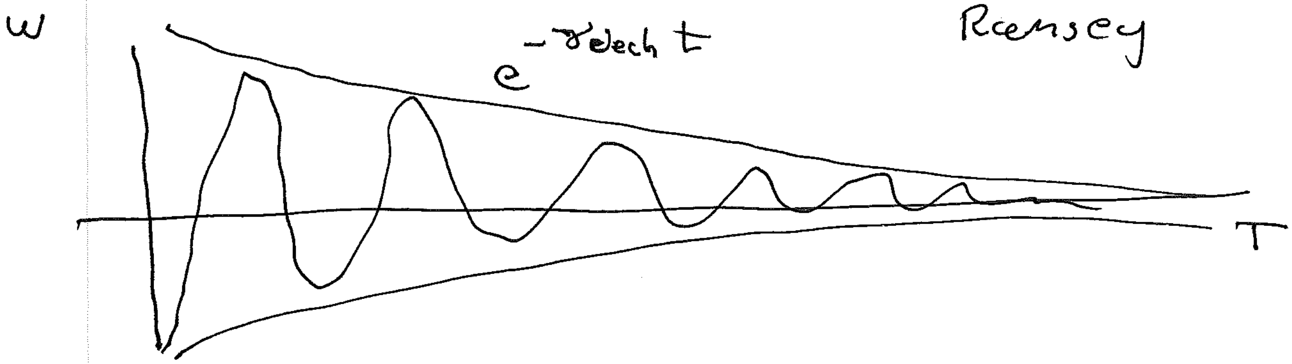
$\downarrow \; t \to \infty$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$



Mixed state! 50% up 50% down

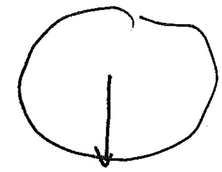
Ramsey



special case $\gamma_{spE} \neq 0$ $\gamma_{\underline{dech}} = 0$

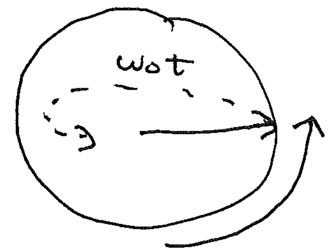
only decoherence due to loss!

$$\rho(0) = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$$



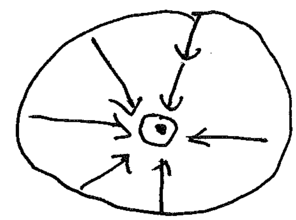
$\pi/2$

$$\rho(t) = \begin{bmatrix} e^{-\gamma t} & -\frac{i}{2} e^{-\gamma t} e^{i\omega_0 t} \\ +\frac{i}{2} e^{-\gamma t} e^{-i\omega_0 t} & e^{-\gamma t} \end{bmatrix}$$



$$\Downarrow \quad t \to \infty$$

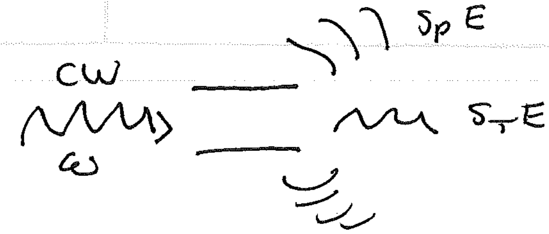
$$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$



Eventually population is all lost to inaccessible levels.

SPECIAL CASE

$\dot{\hat{\rho}} = 0$ STEADY STATE



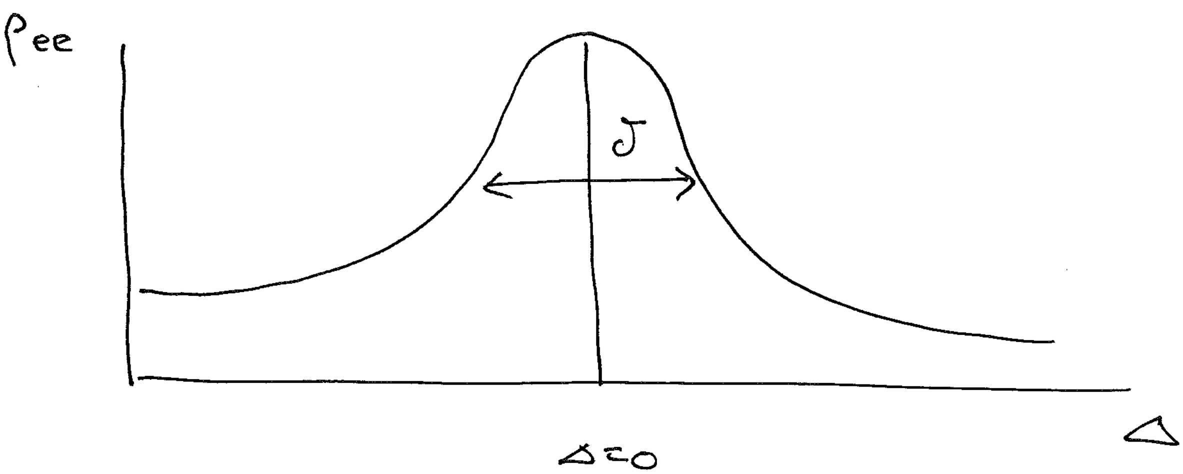
can now solve 4 linear algebra eqs.

$$\rho_{ee} = \rho_{gg} = \frac{\frac{1}{4}\Omega_0^2}{\Gamma^2 + \frac{1}{2}\Omega_0^2 + \Delta^2}$$

$$\rho_{eg} = \rho_{ge}^* = \frac{\frac{1}{2} i \Gamma \Omega_0 e^{-i\Delta t}}{\Gamma^2 + \frac{1}{2}\Omega_0^2 + \Delta^2}$$

$\Delta = \omega_0 - \omega$

This is Lorentzian as function of Δ



spectrum gives width in resonance

collision broad
↓

width of curve

$\Gamma = \gamma_{SpE} + \gamma_{decoh}$

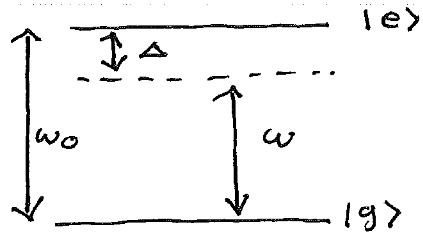
↑
Natural line width

$$\sigma = \sqrt{\Gamma^2 + \frac{1}{2}\Omega_0^2}$$

$\Omega_0 \propto E_0$ ← power broad

8.0 Decoherence in the Rabi Model / Optical Bloch Eqs.

Recall Semiclassical Rabi Model



Field classical ω $\quad$ $\hbar\omega$

Atom Quantum

$$\omega_0 = (E_e - E_g)/\hbar \qquad \Delta = \omega_0 - \omega = \text{detuning}$$

neglect semiclassical

$$\hat{H} = \hat{H}_A + \cancel{\hat{H}_F} + \hat{H}_I$$

We can derive Rabi solutions in density matrix form

$$\hat{H}_A = \hbar\omega\left[\,|e\rangle\langle e| - |g\rangle\langle g|\,\right] = \begin{array}{cc} & \begin{array}{cc} |e\rangle & |g\rangle \end{array} \\ & \begin{bmatrix} \hbar\omega_0 & 0 \\ 0 & -\hbar\omega_0 \end{bmatrix} \begin{array}{c} \langle e| \\ \langle g| \end{array} \end{array}$$

$$\hat{H}_I = \hat{V}_0 \cos\omega t = \begin{bmatrix} 0 & V^0_{eg} \\ V^{0*}_{eg} & 0 \end{bmatrix} \cos\omega t$$

where $V^0_{eg} = V^{*0}_{ge} = \langle e|\hat{V}_0|g\rangle = \langle e| -\hat{d}\cdot\vec{\mathcal{E}}\,|g\rangle$

$= -\langle e|\hat{\vec{d}}|g\rangle\cdot\vec{\mathcal{E}} = -\vec{d}_{eg}\cdot\vec{\mathcal{E}}$ are off diagonal

Hence

$$\hat{H} = \begin{bmatrix} \hbar\omega_0 & V_{eg}\cos\omega t \\ V^*_{eg}\cos\omega t & \hbar\omega_0 \end{bmatrix}$$

Take $V_{eg} = V_{ge} = \nu \in$ Real

Let $|\psi(t)\rangle = C_g(t)\,e^{-i\omega_g t}|g\rangle + C_e(t)\,e^{-i\omega_e t}|e\rangle$

$\Rightarrow \quad \hat{\rho}_\psi = |\psi\rangle\langle\psi|$ density operator <u>pure</u> state!

$$= \begin{bmatrix} |C_e|^2 & C_e^* C_g e^{i\omega_0 t} \\ C_g^* C_e e^{-i\omega_0 t} & |C_g|^2 \end{bmatrix} \equiv \begin{bmatrix} \rho_{ee} & \rho_{eg} \\ \rho_{ge} & \rho_{gg} \end{bmatrix}$$

Instead of solving $\frac{i\hbar \partial|\psi\rangle}{\partial t} = \hat{H}|\psi\rangle$ STATE EQ.

we now solve

$$i\hbar \frac{\partial \hat{\rho}_\psi}{\partial t} = [\hat{H}, \hat{\rho}_\psi] \quad \text{density op. eq.}$$

For pure states these approaches are equivalent! However it is much easier to model loss and decoherence in density operator approach!

The three Bloch parameters are

$$W \equiv \rho_{ee} - \rho_{gg} \qquad \text{(Inversion)}$$

$$2\rho_{eg} \equiv u + iv \qquad \text{(coherence)}$$

We can immediately lift previous solution Eq.s 4.80, which we take with $\Delta = 0$ for simplicity Eqs. 4.80

$$\boxed{\begin{aligned} C_e &= i \sin(\Omega_0 t/2) \\ C_g &= \cos[\Omega_0 t/2] \end{aligned}} \qquad \Omega_0 \equiv 4V_{eg} = \text{Rabi Freq.}$$

We may construct density matrix

$$\begin{bmatrix} \rho_{ee} = |C_e|^2 = \sin^2(\Omega_0 t/2) & \text{coherence} \quad \rho_{eg} = -\frac{i}{2}\sin[\Omega_0 t]e^{i\omega_0 t} \\ \rho_{ge} = +\frac{i}{2}\sin[\Omega_0 t]e^{-i\omega_0 t} \quad \text{population} & \rho_{gg} = |C_g|^2 = \cos^2(\Omega_0 t/2) \end{bmatrix}$$

$W \equiv \rho_{ee} - \rho_{gg} = -\cos(\Omega_0 t)$ and $u + iv = \rho_{eg} \Rightarrow$

$$\boxed{w = -\cos[\Omega_0 t]} \quad \text{Inversion}$$

$$u + iv = -\frac{i}{2}\sin[\Omega_0 t]\, e^{i\omega_0 t} = -\tfrac{1}{2}\sin\Omega_0 t\,[i\cos\omega_0 t - \sin\omega_0 t]$$

$u(t)$ RWA

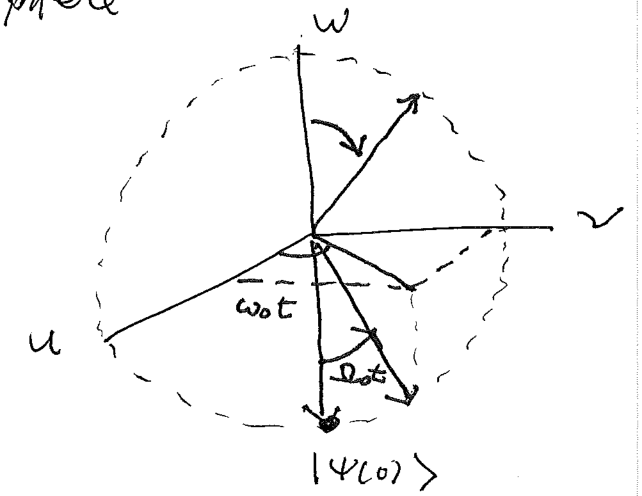
$$\Rightarrow \quad u(t) = \tfrac{1}{2}\boxed{\sin\Omega_0 t \sin\omega_0 t} = \tfrac{1}{4}\boxed{[\cos[(\Omega_0-\omega_0)t]]} - \cancel{\cos[(\Omega_0+\omega_0)t]}$$

$v(t)$ RWA

$$\text{and} \quad v(t) = -\tfrac{1}{2}\boxed{\sin\Omega_0 t\cos\omega_0 t} = -\tfrac{1}{4}\boxed{[\sin[(\Omega_0-\omega_0)t)]} + \cancel{\sin[(\Omega_0+\omega_0)t]}$$

Recall this is solution assumes $|\psi(0)\rangle = |g\rangle$

We construct a Bloch sphere of radius 1



The vector

$[u(t), v(t), w(t)]$

has length

$u^2 + v^2 + w^2$

$= \cos^2\Omega_0 t + \sin^2\Omega_0 t \sin^2\omega_0 t + \sin^2\Omega_0 t\cos^2\omega_0 t$

$= \cos^2\Omega_0 t + \sin^2[\Omega_0 t]\,[\sin^2\omega_0 t + \cos^2\omega_0 t]$

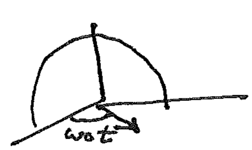
$= \cos^2\Omega_0 t + \sin^2[\Omega_0 t]$

$= 1$

So vector traces out evolution of state on sphere. $\omega_0 t$ is in equator, $\Omega_0 t$ is polar.

$\Omega_0 t = \pi/2 \Rightarrow \pi/2$ pulse $\quad [u, v, w] =$ ~~[1, sinωt~~

$= [\sin\omega_0 t, -\cos\omega_0 t, 0]$



To obtain the probability / time or rate we take Eq. 4.57

$$P_{eg}(t) = \left| \frac{i}{\hbar} \langle e|\hat{d}|g\rangle \cdot \vec{E}_0 \; \frac{e^{i(\omega+\omega_0)t} - 1}{i(\omega+\omega_0)} \right|^2$$

where we take $n=0$ so sp. EM. only

This formula was derived in free space but for one mode $\omega_{\vec{k},s}$

We can immediately use Fermi's golden rule

$$W_{eg} = \frac{2\pi V |\langle e|\hat{H}_{INT}|g\rangle|^2 \rho(\omega_0)}{\hbar^2} \qquad \text{Merzbacher 19.49}$$

where $\hat{H}_{inT} = -\hat{d}\cdot\hat{E}_0$ and $V\rho(\omega_0)$ is the free space density of modes evaluated at $\omega=\omega_0$, from Eq. 2.75 $\omega \to \omega_0$

$$\rho(\omega_0) = \frac{\omega_0^2}{\pi^2 c^3} \qquad \int d\Omega \; \hat{r}\cdot\hat{e} = 2/3$$

avg over 3 D

$$W_{eg} = \frac{2\pi}{\hbar^2} \frac{\omega_0^2}{\pi^2 c^3} \cancel{V} |d_{eg}|^2 \cdot \frac{\hbar\omega_0}{2\epsilon_0 \cancel{V}} \cdot 2/3$$

both polarizations

$$= \boxed{\frac{\omega_0^3}{\pi c^3 \hbar} |d_{eg}|^2} = A = \gamma \quad \text{Einstein } A$$

Wigner – Weisskopf

For Hydrogen $|2p\rangle - |1s\rangle$ $\qquad |e\rangle \to |e\rangle e^{-\frac{\gamma}{2}t}$

$d_{eg} = e^2 |\langle 2p|r|1s\rangle|^2 = e^2 a_0^2 \leftarrow$ Bohr radius

Notice that $A = \gamma \propto \omega_0^3$

UV transitions decay faster than IR.

So now we know the rate $|2p\rangle \to |1s\rangle$

$= T = \frac{1}{A} \approx \underline{10\ \text{ns}}$ (fast). $|2s\rangle \to |1s\rangle$

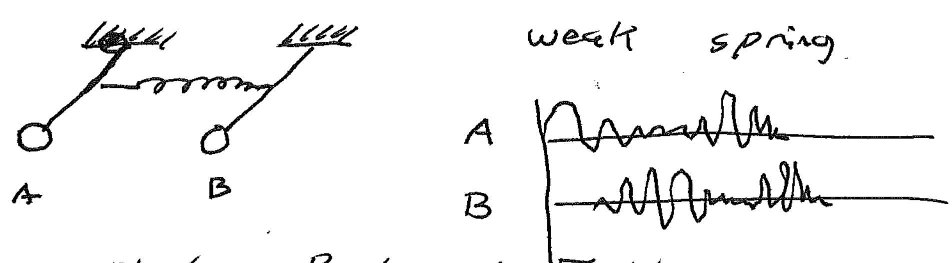
is much harder to compute (second order perturb)

but it is much slower (1 s).

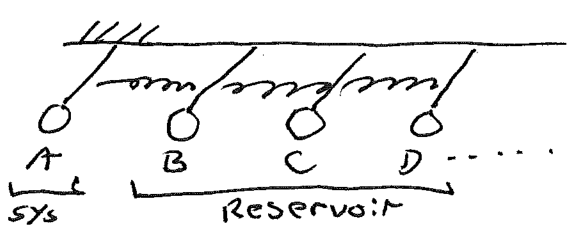
In this treatment $\underline{\text{atom}}$ is quantum system and $\underline{\text{vaccum}}$ modes is $\underline{\text{reservoir}}$ $(T=0)$. Could do $T>0$ (heat bath).

Idea is that loss/decoherence occurs when a small quantum system with a few modes is coupled to a large quantum system with many modes.

Analogy coupled pendula

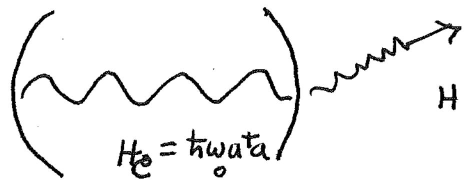


Energy sloshes back and forth



If infinite number energy never returns to A again!

Quantum Jumps in the photon field



$$H_R = \sum_k \hbar\omega_{\vec{k}}\, \hat{b}^\dagger_{\vec{k}} \hat{b}_{\vec{k}}$$

Let us suppose some photon state $|\Psi\rangle$ is in the cavity / single mode. If $Q \neq \infty$ every once and a while a photon is lost. The vacuum modes out side the cavity are the reservoir. Scully 9.1

$$\hat{H} = \hat{H}_{CAV} + \hat{H}_{RES} + \hat{H}_{INT}$$

$$\hat{H}_{CAV} = \hbar\omega_0 a^\dagger a \qquad \text{single mode}$$

$$\hat{H}_{Res} = \sum_k \hbar\omega_k b_k^\dagger b_k \qquad \infty \text{ modes}$$

$$\hat{H}_{INT} = \hbar \sum_k g_{\vec{k}} \left[a^\dagger b_{\vec{k}} + b^\dagger_{\vec{k}} a \right]$$

g_k is coupling constant of cavity to outside

lets assume $g_{\vec{k}} = g \;\; \forall \; \vec{k}$

We can work in Heisenberg Picture

$$i\hbar\; \dot{\hat{a}} = \frac{i}{\hbar}[\hat{H}, \hat{a}] = -i\omega_0 \hat{a}(t) - ig\sum_k \hat{b}_k(t) \qquad *$$

$$\dot{\hat{b}}_k = \frac{i}{\hbar}[\hat{H}, \hat{b}_k] = -i\omega_k \hat{b}_k(t) - ig\, \hat{a}(t)$$

note $[a, b^\dagger] = 0 \qquad [b_k, b^\dagger_{k'}] = \delta_{kk'}$

Here we have an infinite # of linear coupled diffy-Q's. We can formally integrate

$$\hat{b}_{\vec{k}}(t) = \underbrace{\hat{b}_{\vec{k}}(0)\, e^{-i\omega_k t}}_{\text{Free evolution of Reservoir}} \underbrace{- ig \int_0^t dt'\, \hat{a}(t')\, e^{-i\omega_k (t-t')}}_{\text{Evolution of Reservoir perturbed by cavity.}}$$

plugging this back into * gives

$$\dot{\hat{a}}(t) = \underbrace{-i\omega_0 \hat{a}(t)}_{\text{Free Evolution of Cavity}} \underbrace{- g^2 \sum_{\vec{k}} \int_0^t dt'\, \hat{a}(t')\, e^{-i\omega_k (t-t')}}_{\text{"Driven" evolution by Reservoir}} \underbrace{+ \hat{f}(t)}_{\text{Noise Op}}$$

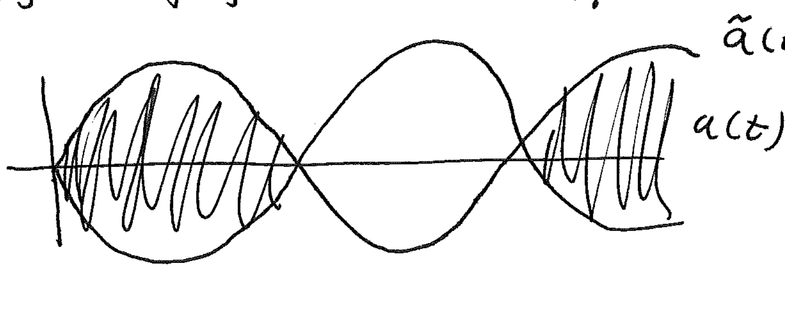
$$\hat{f}(t) = -ig \sum_{k} \hat{b}_k(0)\, e^{-i\omega_k t}$$

The effect of the "driven" term is to cause a frequency shift (Lamb shift) but the noise op causes loss (spontaneous emission)

We make the transformation

$$\boxed{\hat{\tilde{a}}(t) = \hat{a}(t)\, e^{i\omega_0 t}}$$

slowly varying envelope approximation



$$[\hat{\tilde{a}}(t), \hat{\tilde{a}}^{+}(t)] = [\hat{a}(t), \hat{a}^{+}(t)] = 1 \quad \forall \; t$$

$$\Rightarrow \quad \dot{\hat{\tilde{a}}} = -g^2 \int_0^t dt' \, \hat{a}(t') \sum_k e^{-i(\omega_k-\omega_0)(t-t')} + F(t)$$

$$F(t) = e^{i\omega t} f(t) = -ig \sum_k b_k(0) \, e^{-i(\omega_k-\omega_0)t}$$

We can again use Fermi-Golden rule

$$g^2 \sum_k \int_0^t dt' \, \hat{\tilde{a}}(t') \, e^{-i(\omega_k-\omega_0)(t-t')} \quad \propto \quad \begin{matrix}\delta(\omega-\omega_0) \\ \delta(t-t')\end{matrix}$$

$$= \boxed{\pi g^2 \rho(\omega_0) \, \tilde{a}(t)}$$

where $\rho(\omega_0) = V \cdot \dfrac{\omega^2}{\pi^2 c^3}$ is Density of Modes

Q. Langevin

$$\Rightarrow \quad \boxed{\dot{\hat{\tilde{a}}}(t) = \underbrace{-\tfrac{1}{2}\gamma \, \tilde{a}(t)}_{\text{Exponential Decay}} + \underbrace{F(t)}_{\text{Fluctuations}}}$$

Dissipation / Fluctuation

$$\boxed{\gamma = 2\pi g^2 \rho(\omega_0)}$$

corresponds to Einstein A / Spon Emission

If $F(t)$ is neg.

$$\boxed{\hat{\tilde{a}}(t) = \tilde{a}(0) \, e^{-\frac{1}{2}\gamma t}}$$

The field leaks out of the cavity with exponential decay

If reservoir at Temp $T = 0$

then Fluctuations $\hat{F}$ correspond to

vaccum fluctuations analagous to Lamb

shift

If $T > 0$ more complicated shift

$\Delta E \propto T^2$ the heat bath