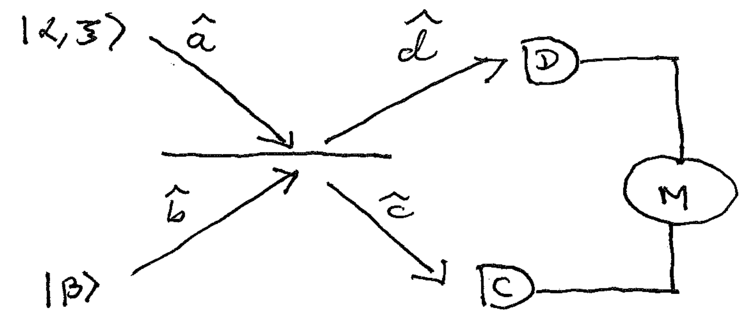
Balanced Homodyne Detection



The idea is to mix unknown squeezed state $|\alpha, \xi\rangle$, where $\alpha = e^{i\varphi}|\alpha|$ and $\xi = re^{i\theta}$, with a known "local oscillator" $|\beta\rangle = |\beta|e^{-i\psi}$

$|\beta\rangle$ is a coherent state and typically $|\beta| \gg |\alpha|$ and $r \gg 1$ for this to work

For a 50:50 BS we have:

$$\hat{c} = (\hat{a} + i\hat{b})/\sqrt{2}$$
$$\hat{d} = (i\hat{a} + \hat{b})/\sqrt{2}$$

where $\hat{BS} = \begin{bmatrix} 1 & i \\ i & 1 \end{bmatrix}/\sqrt{2}$ and $\det|\hat{BS}| = 1$

giving $\pi/2$ vs 0 phase shift trans vs. ref. as reciprocity requires.

The detectors D and C measure intensities

$$I_D = \langle \hat{d}^+\hat{d} \rangle \quad \text{and} \quad I_C = \langle \hat{c}^+\hat{c} \rangle$$

The "subtracter" or difference counter M counts

$$M = I_C - I_D$$
$$= \langle c^+c \rangle - \langle d^+d \rangle$$
$$= \langle c^+c - d^+d \rangle \equiv \langle \hat{M} \rangle$$

Now let's include time dependence

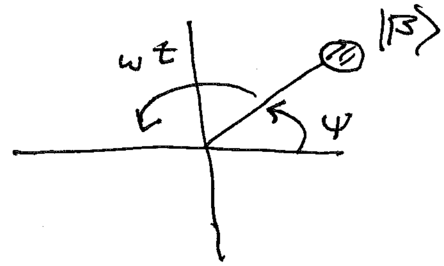
$$|\beta e^{-i\psi}\rangle \to \left||\beta| e^{-i\omega t - i\psi}\right\rangle$$

$$|\Psi\rangle_{IN} = |\alpha_J\rangle_A |\beta\rangle_B$$

$$\hat{a}(t) \to \hat{a}_0 e^{-i\omega t}$$

This assumes what comes in A is same ω as what comes in B.

[Same laser]



We detect $\hat{OP}$:

$$\hat{M} = c^\dagger c - d^\dagger d$$

$$= \tfrac{1}{2}\left\{ (a^\dagger - i b^\dagger)(a + i b) - (-i a^\dagger + b^\dagger)(i a + b) \right\}$$

$$= \boxed{i\,[\,a^\dagger b - b^\dagger a\,]}$$

Hence

$$\langle \hat{M} \rangle = i\, {}_A\langle \alpha_J |\, {}_B\langle |\beta| e^{-i(\omega t+\psi)} |\, a^\dagger b - b^\dagger a \,|\, |\beta| e^{-i(\omega t + \psi)} \rangle_B\, |\alpha_J\rangle_A$$

$$= i \left\{ \langle a^\dagger \rangle_A \langle b \rangle_B - \langle b^\dagger \rangle_B \langle a \rangle_A \right\}$$

But

$$i \langle \hat{b} \rangle_B = i \langle |\beta| e^{-i\omega t} e^{-i\psi} \,|\, \hat{b} \,|\, |\beta| e^{-i\omega t} e^{-i\psi} \rangle$$

$$= i\,|\beta|\, e^{-i\omega t} e^{-i\psi}$$

$$= |\beta|\, e^{-i\omega t} e^{-i\psi} e^{i\pi/2}$$

$$= |\beta|\, e^{-i\omega t} e^{-i[\psi - \pi/2]}$$

$$= \boxed{|\beta|\, e^{-i\omega t} e^{-i\theta}} \qquad \boxed{\theta \equiv \psi - \pi/2}$$

and

$$\boxed{-i \langle \hat{b}^\dagger \rangle_B = |\beta|\, e^{i\omega t} e^{i\theta}}$$

Also

$$\langle \hat{a} \rangle_A = \langle \hat{a}_0 e^{-i\omega t} \rangle_A$$

$$= e^{-i\omega t} \langle \hat{a}_0 \rangle_A$$

$$\Rightarrow \langle \hat{a}^\dagger \rangle_A = e^{+i\omega t} \langle \hat{a}_0^\dagger \rangle_A$$

Hence all time dependence cancels out!

$$\langle \hat{M} \rangle = \langle \hat{a}_0^\dagger \rangle_A \cancel{e^{i\omega t}} |\beta| \cancel{e^{-i\omega t}} e^{-i\theta} + |\beta| \cancel{e^{i\omega t}} e^{i\theta} \langle \hat{a}_0 \rangle_A \cancel{e^{-i\omega t}}$$

$$= |\beta| \left\{ \langle \hat{a}_0^\dagger \rangle_A e^{-i\theta} + \langle \hat{a}_0 \rangle_A e^{i\theta} \right\}$$

$$= \boxed{2|\beta| \; {}_A\langle \Psi_{IN} | \frac{\hat{a}_0^\dagger e^{-i\theta} + \hat{a}^{0} e^{i\theta}}{2} | \Psi_{IN} \rangle_A}$$

while we are considering $|\Psi_{IN}\rangle_A = |\alpha\rangle_A$, $|\Psi_{IN}\rangle_A$ could be anything! If we compare this to Eq. 7.7

$$\hat{X}(\theta) \equiv \frac{1}{2} \left[\hat{a}_0 e^{-i\theta} + \hat{a}_0^\dagger e^{+i\theta} \right]$$

the generalized quadrature operator then

$$\langle \hat{M} \rangle = \langle \Psi_{IN} | \hat{M} | \Psi_{IN} \rangle = {}_A\langle \Psi_{IN} | \, {}_B\langle \underbrace{\Psi_{IN}}^{\beta e^{-i\phi}} | \hat{M} | \Psi_{IN} \rangle_A | \Psi_{IN} \rangle_B$$

$$= \boxed{2|\beta| \; {}_A\langle \Psi_{IN} | \hat{X}(\theta) | \Psi_{IN} \rangle_A}$$

In the limit $|\beta| \gg |\alpha|$ we may replace

$\hat{b} \to \beta e^{-i\psi}$ and $\hat{b}^+ = \beta^* e^{i\psi} = |\beta| e^{i(\omega t + \psi)}$

$$\Rightarrow \quad \hat{M} \cong i\,[\,a^+ b - b^+ a\,]$$

$$= [\,\hat{a}_0^+ |\beta| e^{-i\theta} + \hat{a}_0 |\beta| e^{i\theta}\,]$$

$$\cong 2|\beta| \, \hat{X}(\theta)$$

Hence

$$\boxed{\Delta M^2 \cong 4|\beta|^2 \Delta X^2(\theta)}$$

Fluctuations in $|\Psi_{IN}\rangle$ show up in $\langle \hat{M} \rangle$

where $\Delta M^2 = \langle \Psi_{IN} | \hat{M}^2 | \Psi_{IN} \rangle - \langle \Psi_{IN} | M | \Psi_{IN} \rangle^2$

but $\Delta X^2 = {}_A\langle \Psi_{IN} | \hat{X}^2 | \Psi_{IN} \rangle_A - {}_A\langle \Psi_{IN} | \hat{X} | \Psi_{IN} \rangle_A^2$

Let us take:

$\cancel{|\Psi_{IN}\rangle_A = |\xi\rangle}$ ~~SQUEEZED VACCUM~~

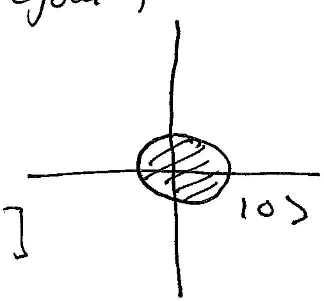
$\cancel{\alpha = 0 \Rightarrow |\beta| \gg 0}$

Example: As our control let use

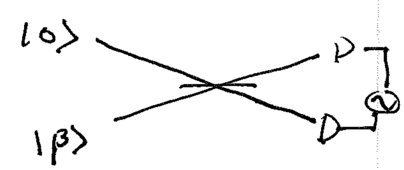
take $|\Psi_{IN}\rangle_A = |0\rangle$ vaccum (regular)

We know $\boxed{\Delta X^2_{|0\rangle}(\theta) = 1/4}$

independent of θ. [This can bee seen by taking $r \to 0$ in Eq 7.16]



Hence



$$\Delta M^2_{|\beta\rangle|0\rangle} = 4|\beta|^2 \Delta X^2(\theta)$$

$$= |\beta|^2$$

$$= \bar{n}_\beta \quad \Rightarrow$$

SHOTNOISE LIMIT

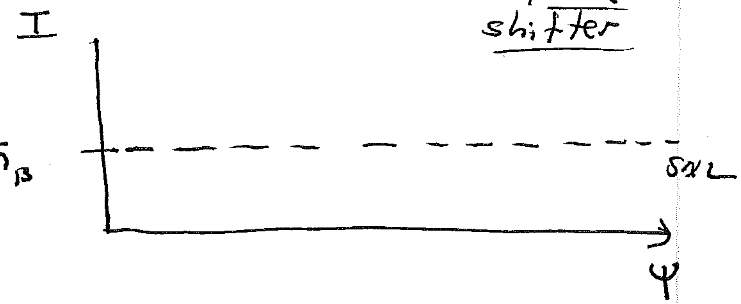
$$\boxed{\Delta M_{|\beta\rangle|0\rangle} = \sqrt{\bar{n}_\beta} \quad \text{S.N.L.}}$$

This sets our ground floor which we scan by phase shifting $\beta \propto e^{-i\psi}$ ← Tune ψ w/ phase shifter

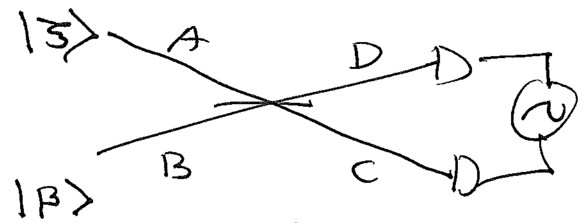
Now put

$|\psi_{IN}\rangle_A = |\xi\rangle_A$

$I_\beta = \bar{n}_\beta$

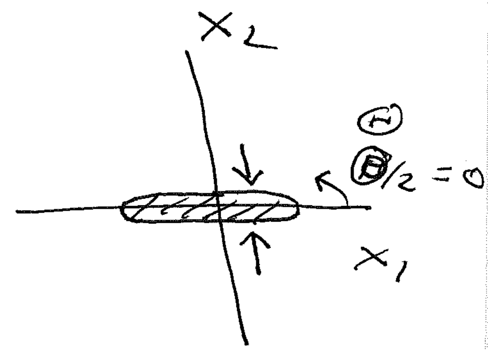


SQUEEZED VACUUM IN A PORT



with $\xi = re^{i\Theta} = r$

That is $\Theta \neq \theta$ is the squeeze angle $\Theta = 0$ means X_2 squeezed.



We now scan with coherent state ψ

using $\theta = \psi - \pi/2$

$$\Rightarrow \Delta M^2_{|\xi\rangle|\beta\rangle} = \bar{n}_\beta \left[\cosh(2r) - \sinh(2r)\cos[\psi - \pi/2]\right]$$

Using Eq. 7.16

$$\Rightarrow \Delta M^2_{|\xi\rangle|\beta\rangle} = \bar{n}_\beta \{\cosh(2r) - \sinh(2r)\sin\psi\}$$

For $r \ll 1$ (SMALL SQUEEZE)

$$\Delta M^2_{|\xi\rangle|\beta\rangle} = \bar{n}_\beta \left[1 - 2r\sin\psi\right]$$

which we plot for $\bar{n}_\beta = 1$ and $r = 0.1$

```
In[4]:= Plot[{1, 1 - .2 * Sin[f]}, {f, 0, 2 * Pi}, PlotRange → {0, 1.2}]
```

Out[4]=