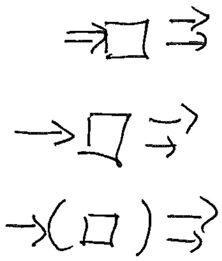
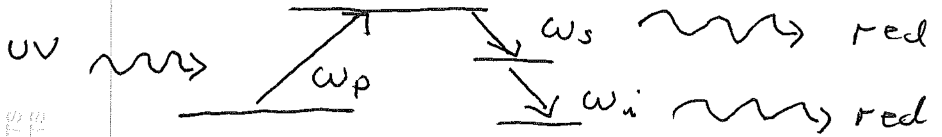
7.2 The SQUEEZE GENERATION

SPDC = spontaneous parametric down conversion

OPA = optical parametric amplifier

OPO = " " oscillator

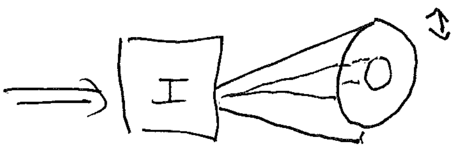


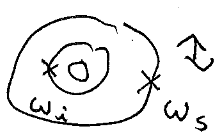


To conserve energy: $\hbar\omega_p = \hbar\omega_s + \hbar\omega_i$

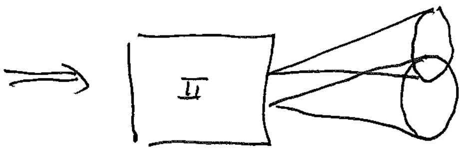
To conserve momentum: $\hbar\vec k_p = \hbar\vec k_s + \hbar\vec k_i$

Type I Polarization all the same





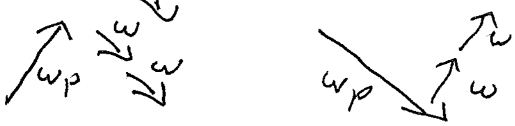
Type II Polarization ⊥



$|\updownarrow\leftrightarrow\rangle + |\leftrightarrow\updownarrow\rangle$

Let $\omega_s = \omega_i = \omega = \omega_p/2$

$$\hat H = \underbrace{\hbar\omega\hat a^{+}\hat a}_{\text{signal}} + \underbrace{\hbar\omega_p b^{+} b}_{\text{pump}} + i\hbar\chi^{(2)}\left[\underbrace{\hat a^{2}\hat b^{+}} - \underbrace{\hat a^{+2}\hat b}\right]$$



$\chi^{(2)}$ is second order nonlinearity

$$\vec P = \underbrace{(\chi^{(0)} + \chi^{(1)})}_{n}\vec E + (\chi^{(2)}\vec E)\vec E + (\chi^{3}E^{2})\vec E \cdots\cdots$$

Expansion of polarization $\vec P$ of a nonlinear dielectric

SHO $$\vec{F}(x) = \underbrace{kx}_{\text{linear}} + \underbrace{\underbrace{\alpha x^2}_{\chi^{(2)}} + \underbrace{\beta x^3}_{\chi^{(3)}}}_{\text{anharmonic oscillator}} + \cdots$$

Parametric nondepleted pump approximation

$$\hat{b} \to \beta e^{-i\omega_p t}\,\mathbb{C} \quad \Rightarrow \quad \hat{b}^{\dagger} \to \beta^{*} e^{+i\omega_p t}$$

Pump is ∞ reservoir of $\hbar\omega_p$ photons

(Typical: watts → kilowatts)

constant does not contribute

$$\Rightarrow \quad \hat{H} = \hbar\omega\,\underset{\overset{\uparrow}{\hat{a}^{\dagger}\hat{a}}}{\hat{n}} + \cancel{\hbar\omega_p|\beta|^2} + i\hbar\chi^{(2)}\left[\hat{a}^2\beta^{*}e^{+i\omega_p t} - \hat{a}^{\dagger 2}\beta e^{-i\omega_p t}\right]$$

Recall Schrödinger → Heisenberg

$$\hat{a} \to \hat{a}(t) = \hat{a}e^{-i\omega t}$$

$$\Rightarrow \quad \hat{H}_{\text{Interaction}} = i\hbar\chi^{(2)}\left[\hat{a}^2\beta^{*}e^{i(\omega_p-2\omega)t} - \hat{a}^{\dagger 2}\beta e^{-i(\omega_p-2\omega)t}\right]$$

But $\omega_p = \omega_s + \omega_i = 2\omega$ Erg. Cons.

So $\omega_p - 2\omega = 0$ Let $\xi \equiv \chi^{(2)}\beta$

$$\Rightarrow \quad \boxed{\hat{H}_I = i\hbar\left[\xi^{*}\hat{a}^2 - \xi\hat{a}^{\dagger 2}\right]}$$

Since $\hat{H}_I$ is time independent, time evolution operator is (see e.g. Merzbacher)

$$\boxed{\hat{U}_I(t,0) \equiv e^{-\frac{i}{\hbar}\hat{H}_I t}}$$

Hence

$$U_I(t) = e^{-i\hbar t\left[i\hbar(\varepsilon^* \hat{a}^2 - \varepsilon \hat{a}^{\dagger 2})\right]}$$

$$= \exp\left[\frac{2\varepsilon^* t}{2}\hat{a}^2 - \frac{2\varepsilon t}{2}\hat{a}^{\dagger 2}\right]$$

$$= \exp\left[\frac{\zeta^*(t)}{2}\hat{a}^2 - \frac{\zeta(t)}{2}\hat{a}^{\dagger 2}\right]$$

$$\equiv \hat{S}_{\zeta(t)} \quad \therefore$$

Hence $\boxed{\zeta = 2\varepsilon t = 2\beta\chi^{(2)} t}$

is the squeezed prm

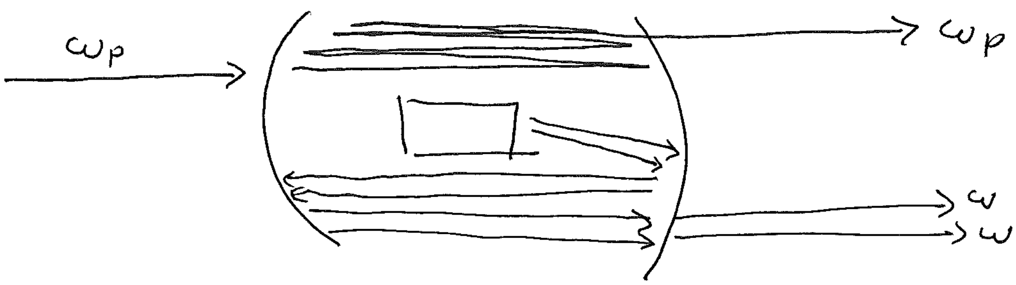
To maximize $\zeta \gg 1$

$\Rightarrow$ $\beta \gg 1$ STRONG PUMP AMPLITUDE

~~or~~ $t \gg 1$ LONG INTERACTION TIME (USE LONG XTAL OR PUT XTAL IN FABRY PEROT)

or $\chi^{(2)} \gg 1$ CHEMISTRY! CHINESE MAKE XTALS WITH LARGE $\chi^{(2)}$!

OPO



OPO Make mirrors at ωp or ω or <u>both</u>!

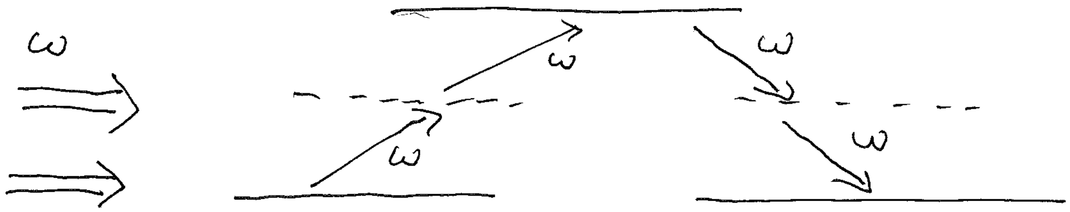
Four Wave Mixing

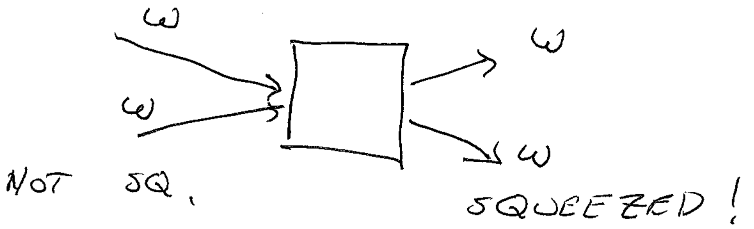
We can also use χ_3 for squeezing

Less efficient $(\chi_3 \ll \chi_2)$

but advantage for quantum lithography that

$(\omega_p, \omega_p) \to (\omega_p, \omega_p)$ no change in ω





$$\hat{H}_{\chi_3} = \underbrace{\hbar\omega a^{+} a + \hbar\omega}_{\text{same}} b^{+} b + i\hbar\chi_3 [\hat{a}^2 \hat{b}^{+2} - \hat{a}^{+2} \hat{b}^2]$$

Again assume strong classical parametric nondepleted pump

$\Rightarrow$ $\hat{b} = \beta e^{-i\omega t}$

Interaction picture $\hat{a} \to \hat{a}(t) = \hat{a} e^{-i\omega t}$

$$\Rightarrow \hat{H}_I^{\chi_3} = i\hbar\chi_3 [\beta^{*2} \hat{a}^2 - \beta^2 \hat{a}^{+2}]$$

$$= \boxed{i\hbar [\xi_3^* \hat{a}^2 - \xi_3 \hat{a}^{+2}]}$$

This is identical to previous result with

$$\xi_2 = \chi_2 \beta \to \xi_3 = \chi_3 \beta^2$$

Hence

$$e^{i4\hat{H}_I^{(3)} t} = \hat{S}_{\xi_3(t)}$$

where $\xi_3(t) = 2\chi_3\beta^2 t$ is FWMix squeeze param.

Again want

$\chi_3 >> 1$

$\beta^2 >> 1$

$t >> 1$

note $\chi_3 << \chi_2$ but $|\beta|^2 >> |\beta|$ so with strong pump squeezing nearly as good. First squeezing done w/ χ_3 (silica)

χ_2	χ_3
crystal symmetry	any materel
Must Be non axio semmetric	quartz
	glaess
hard	easy
special birefringent XTALS	