A few concepts: electric force, field and potential

- **Gravitational Force**
  - What is the force on a mass produced by other masses?
  - Kepler’s Laws & Circular Motion

- **Gravitational Potential Energy**
  - Conservation of Energy

- **Electric force**
  - What is the force on a charge produced by other charges?
  - What is the force on a charge when immersed in an electric field?

- **Electric field**
  - What is the electric field produced by a system of charges? (Several point charges, or a continuous distribution)
Plus a few other items…

• Electric field lines

• Electric dipoles: field and potential produced BY a dipole, torque ON a dipole by an electric field, torque and potential energy of a dipole

• Gauss’s Law: For conductors, planar symmetry, cylindrical symmetry, spherical symmetry. Return of the Shell Theorems!

What? The Flux! $\Phi = q/\varepsilon_0$. Given the field, what is the charge enclosed? Given the charges, what is the flux? Use it to deduce formulas for electric field.
- Constants, definitions:
  \[ g = 9.8 \text{ m/s}^2 \]
  \[ G = 6.67 \times 10^{-11} \text{ m}^3 \text{kg}^{-1} \text{s}^{-2} \]
  \[ M_{Sun} = 1.99 \times 10^{30} \text{ kg} \]
  \[ \epsilon_0 = 8.85 \times 10^{-12} \text{ C}^2 \text{Nm}^{-2} \]
  \[ c = 3.00 \times 10^8 \text{ m/s} \]
  \[ m_p = 1.67 \times 10^{-27} \text{ kg} \]
  \[ m_e = 9.11 \times 10^{-31} \text{ kg} \]

- Kinematics (constant acceleration):
  \[ v = v_o + at \quad x - x_o = \frac{1}{2}(v_o + v)t \quad x - x_o = v_o t + \frac{1}{2}at^2 \quad v^2 = v_o^2 + 2a(x - x_o) \]

- Circular motion:
  \[ F_c = ma_c = \frac{mv^2}{r}, \quad T = \frac{2\pi r}{v}, \quad v = \omega r \]

- General (work, def. of potential energy, kinetic energy):
  \[ K = \frac{1}{2}mv^2 \quad F_{\text{net}} = m\ddot{a} \quad E_{\text{mech}} = K + U \]
  \[ W = -\Delta U \text{ (by field)} \]
  \[ W_{\text{ext}} = \Delta U \text{ (if objects are initially and finally at rest)} \]

- Earth-Sun distance: \(1.50 \times 10^{11} \text{ m}\)
- Earth-Moon distance: \(3.82 \times 10^{8} \text{ m}\)
- \(e = 1.60 \times 10^{-19} \text{ C}\)
- \(1 \text{ eV} = e(1 \text{ V}) = 1.60 \times 10^{-19} \text{ J}\)
- Charge densities: \(\lambda = \frac{Q}{L}, \quad \sigma = \frac{Q}{A}, \quad \rho = \frac{Q}{V}\)
- \(\mu_0 = 4\pi \times 10^{-7} \text{ T} \cdot \text{m/A}\)
- Volume of a sphere: \(V = \frac{4}{3}\pi r^3\)
- Volume element: \(A = 4\pi r^2 dr\)
- **Gravity:**

  Newton’s law: \(|\vec{F}| = G \frac{m_1 m_2}{r^2}\)

  Law of periods: \(T^2 = \left(\frac{4\pi^2}{GM}\right) r^3\)

  Potential Energy of a System (more than 2 masses): \(U = -\left(G \frac{m_1 m_2}{r_{12}} + G \frac{m_1 m_3}{r_{13}} + G \frac{m_2 m_3}{r_{23}} + \ldots\right)\)

- **Electrostatics:**

  Coulomb’s law: \(|\vec{F}| = k \frac{|q_1| \cdot |q_2|}{r^2}\)

  Electric field of a point charge: \(|\vec{E}| = k \frac{|q|}{r^2}\)

  Electric field of a dipole on axis, far away from dipole: \(\vec{E} = \frac{2k\vec{p}}{z^3}\)

  Electric field of an infinite line charge: \(|\vec{E}| = \frac{2k\lambda}{r}\)

  Electric field at the center of uniformly charged arc of angle \(\phi\): \(|\vec{E}| = \frac{\lambda \sin(\phi/2)}{2\pi \varepsilon_0 R}\)

  Torque on a dipole in an \(\vec{E}\) field: \(\vec{\tau} = \vec{p} \times \vec{E}\),

  Potential energy of a dipole in \(\vec{E}\) field: \(U = -\vec{p} \cdot \vec{E}\)

- **Electric flux:** \(\Phi = \int \vec{E} \cdot d\vec{A}\)

  - Gauss’ law: \(\varepsilon_0 \oint \vec{E} \cdot d\vec{A} = q_{enc}\)

- Electric field of an infinite non-conducting plane with a charge density \(\sigma\): \(E = \frac{\sigma}{2\varepsilon_0}\)

- Electric field of infinite conducting plane or close to the surface of a conductor: \(E = \frac{\sigma}{\varepsilon_0}\)
13.2 Newton’s Law of Gravitation: May The Force Be With You

\[ F = G \frac{m_1 m_2}{r^2} \]

(Newton’s law of gravitation).

Here \( m_1 \) and \( m_2 \) are the masses of the particles, \( r \) is the distance between them, and \( G \) is the gravitational constant.

\[ G = 6.67 \times 10^{-11} \text{ Nm}^2/\text{kg}^2 \]
\[ = 6.67 \times 10^{-11} \text{ m}^3/\text{kg s}^2. \]

---

Fig. 13-2 (a) The gravitational force on particle 1 due to particle 2 is an attractive force because particle 1 is attracted to particle 2. (b) Force is directed along a radial coordinate axis \( r \) extending from particle 1 through particle 2. (c) is in the direction of a unit vector along the \( r \) axis.
13.2 Gravitation and the Principle of Superposition

For $n$ interacting particles, we can write the principle of superposition for the gravitational forces on particle 1 as

$$\vec{F}_{1,\text{net}} = \vec{F}_{12} + \vec{F}_{13} + \vec{F}_{14} + \vec{F}_{15} + \cdots + \vec{F}_{1n}.$$ 

Here $\vec{F}_{1,\text{net}}$ is the net force on particle 1 due to the other particles and, for example, $\vec{F}_{13}$ is the force on particle 1 from particle 3, etc. Therefore,

$$\vec{F}_{1,\text{net}} = \sum_{i=2}^{n} \vec{F}_{1i}.$$ 

The gravitational force on a particle from a real (extended) object can be expressed as:

$$\vec{F}_1 = \int d\vec{F},$$ 

Here the integral is taken over the entire extended object.

ICPP: How would you calculate total force on central mass if all masses equal?
1. A uniform shell of matter exerts no net gravitational force on a particle located **inside** it.

2. A uniform shell of matter exerts a force on a particle located **outside** it as if all the mass was at the center.

\[ \text{density} = \rho = \frac{M_{\text{tot}}}{V_{\text{tot}}} \]

\[ M_{\text{ins}} = \rho V_{\text{ins}} = \frac{M}{V_{\text{tot}}} V_{\text{ins}} = M \frac{r^3}{R^3} \]

\[ \text{force} = F = \frac{GmM_{\text{ins}}}{r^2} = \frac{Gm}{r^2} \left( M \frac{r^3}{R^3} \right) = \frac{GmM}{R^3} r \]

\[ \text{field} = g = \frac{GM_{\text{ins}}}{r^2} = \frac{G}{r^2} \left( M \frac{r^3}{R^3} \right) = \frac{GM}{R^3} r \]

Inside the Earth the Force and Field Scale LINEARLY with \( r \). This is like Hooke’s Law for a Mass on a Spring.
Escape Speed

If you fire a projectile upward, usually it will slow, stop momentarily, and return to Earth. There is, however, a certain minimum initial speed that will cause it to move upward forever, theoretically coming to rest only at infinity. This minimum initial speed is called the (Earth) escape speed.

Consider a projectile of mass \( m \), leaving the surface of a planet (or some other astronomical body or system) with escape speed \( v \). The projectile has a kinetic energy \( K \) given by \( \frac{1}{2}mv^2 \) and a potential energy \( U \) given by Eq. 13-21:

\[
U = -\frac{GMm}{R},
\]

in which \( M \) is the mass of the planet and \( R \) is its radius.

When the projectile reaches infinity, it stops and thus has no kinetic energy. It also has no potential energy because an infinite separation between two bodies is our zero-potential-energy configuration. Its total energy at infinity is therefore zero. From the principle of conservation of energy, its total energy at the planet’s surface must also have been zero, and so

\[
K + U = \frac{1}{2}mv^2 + \left(-\frac{GMm}{R}\right) = 0.
\]

This yields

\[
v = \sqrt{\frac{2GM}{R}}. \tag{13-28}
\]
Conservation of Mechanical Energy!

Sample Problem

Asteroid falling from space, mechanical energy

An asteroid, headed directly toward Earth, has a speed of 12 km/s relative to the planet when the asteroid is 10 Earth radii from Earth's center. Neglecting the effects of Earth's atmosphere on the asteroid, find the asteroid's speed \( v_f \) when it reaches Earth's surface.

Because we are to neglect the effects of the atmosphere on the asteroid, the mechanical energy of the asteroid–Earth system is conserved during the fall. Thus, the final mechanical energy (when the asteroid reaches Earth's surface) is equal to the initial mechanical energy. With kinetic energy \( K \) and gravitational potential energy \( U \), we can write this as

\[
K_f + U_f = K_i + U_i.
\]

Also, if we assume the system is isolated, the system's linear momentum must be conserved during the fall. Therefore, the momentum change of the asteroid and that of Earth must be equal in magnitude and opposite in sign. However, because Earth's mass is so much greater than the asteroid's mass, the change in Earth's speed is negligible relative to the change in the asteroid's speed. So, the change in Earth's kinetic energy is also negligible. Thus, we can assume that the kinetic energies in Eq. 13-29 are those of the asteroid alone.

Calculations: Let \( m \) represent the asteroid's mass and \( M \) represent Earth's mass \( (5.98 \times 10^{24} \text{ kg}) \). The asteroid is initially at distance \( 10R_E \) and finally at distance \( R_E \), where \( R_E \) is Earth's radius \( (6.37 \times 10^6 \text{ m}) \). Substituting Eq. 13-21 for \( U \) and \( \frac{1}{2}mv^2 \) for \( K \), we rewrite Eq. 13-29 as

\[
\frac{1}{2}mv_f^2 - \frac{GMm}{R_E} = \frac{1}{2}mv_i^2 - \frac{GMm}{10R_E}.
\]

Rearranging and substituting known values, we find

\[
\begin{align*}
v_f^2 &= v_i^2 + \frac{2GM}{R_E} \left( 1 - \frac{1}{10} \right) \\
&= (12 \times 10^3 \text{ m/s})^2 + \frac{2(6.67 \times 10^{-11} \text{ m}^3/\text{kg} \cdot \text{s}^2)(5.98 \times 10^{24} \text{ kg})}{6.37 \times 10^6 \text{ m}} \times 0.9 \\
&= 2.56 \times 10^8 \text{ m}^2/\text{s}^2,
\end{align*}
\]

and

\[
v_f = 1.60 \times 10^4 \text{ m/s} = 16 \text{ km/s} \quad \text{(Answer)}
\]

At this speed, the asteroid would not have to be particularly large to do considerable damage at impact. If it were only 5 m across, the impact could release about as much energy as the nuclear explosion at Hiroshima. Alarmingly, about 500 million asteroids of this size are near Earth's orbit, and in 1994 one of them apparently penetrated Earth's atmosphere and exploded 20 km above the South Pacific (setting off nuclear-explosion warnings on six military satellites). The impact of an asteroid 500 m across (there may be a million of them near Earth's orbit) could end modern civilization and almost eliminate humans worldwide.
Charged Insulators & Conductors

- Will two charged objects attract or repel?
- Can a charged object attract or repel an uncharged object?

**CHECKPOINT 1**

The figure shows five pairs of plates: $A$, $B$, and $D$ are charged plastic plates and $C$ is an electrically neutral copper plate. The electrostatic forces between the pairs of plates are shown for three of the pairs. For the remaining two pairs, do the plates repel or attract each other?
Electric forces and fields: point charges

- Coulomb’s law: \( F = k \frac{|q_1||q_2|}{r^2} \)
- Electric field measured with a test charge: \( \vec{E} = \frac{\vec{F}}{q_0} \)
- Force on a charge in an electric field: \( \vec{F} = q\vec{E} \)
- Electric field of a point charge: \( E = \frac{F}{q_0} = k \frac{|q|}{r^2} \)

Figure 22N-14 shows an arrangement of four charged particles, with angle \( \theta = 34^\circ \) and distance \( d = 2.20 \) cm. The two negatively charged particles on the \( y \) axis are electrons that are fixed in place; the particle at the right has a charge \( q_2 = +5e \)

(a) Find distance \( D \) such that the net force on the particle at the left, due to the three other particles, is zero.

(b) If the two electrons were moved further from the \( x \) axis, would the required value of \( D \) be greater than, less than, or the same as in part (a)?

Other possible questions: what’s the electric field produced by the charges XXX at point PPP? what’s the electric potential produced by the charges XXX at point PPP? What’s the potential energy of this system?
Electric dipoles

- Electric field of a dipole on axis: \( E = \frac{2kp}{z^3} \)
- Torque on a dipole in an electric field: \( \tau = \vec{p} \times \vec{E} \)
- Potential energy of a dipole in an electric field: \( U = -\vec{p} \cdot \vec{E} \)

- What’s the electric field at the center of the dipole? On axis? On the bisector? Far away?
- What is the force on a dipole in a uniform field?
- What is the torque on a dipole in a uniform field?
- What is the potential energy of a dipole in a uniform field?
Electric fields of distributed charges

- Electric field of an infinite non-conducting plane with a charge density \( \sigma \): \( E = \frac{\sigma}{2\epsilon_0} \)
- Electric field of an infinite line charge: \( E = \frac{2k\lambda}{r} \)

Possible problems, questions:

- What’s the electric field at the center of a charged circle?
- What’s the electric field at the center of \( \frac{1}{4} \) of a charged circle?
- What’s the electric field far from the ring? far from the disk?
- What’s the DIRECTION of an electric field of an infinite disk?
CHECKPOINT 3

(a) In the figure, what is the direction of the electrostatic force on the electron due to the external electric field shown?
(b) In which direction will the electron accelerate if it is moving parallel to the y axis before it encounters the external field? (c) If, instead, the electron is initially moving rightward, will its speed increase, decrease, or remain constant?

2 In Fig. 22-29 the electric field lines on the left have twice the separation of those on the right. (a) If the magnitude of the field at A is 40 N/C, what is the magnitude of the force on a proton at A? (b) What is the magnitude of the field at B?
Exam Review Continued

• Questions: from checkpoints and questions in the textbook!
Sample Problem 22.03  Electric field of a charged circular rod

Figure 22-13a shows a plastic rod with a uniform charge \(-Q\), bent in a 120° circular arc of radius \(r\) and symmetrically placed across an \(x\) axis with the origin at the center of curvature \(P\) of the rod. In terms of \(Q\) and \(r\), what is the electric field \(\vec{E}\) due to the rod at point \(P\)?

**KEY IDEA**

Because the rod has a continuous charge distribution, we must find an expression for the electric fields due to differential elements of the rod and then sum those fields via calculus.

**An element:** Consider a differential element having arc length \(ds\) and located at an angle \(\theta\) above the \(x\) axis (Figs. 22-13b and c). If we let \(\lambda\) represent the linear charge density of the rod, our element \(ds\) has a differential charge of magnitude

\[
dq = \lambda\, ds. \tag{22-18}
\]

**The element's field:** Our element produces a differential electric field \(d\vec{E}\) at point \(P\), which is a distance \(r\) from the element. Treating the element as a point charge, we can rewrite Eq. 22-3 to express the magnitude of \(d\vec{E}\) as

\[
d\vec{E} = \frac{1}{4\pi\varepsilon_0} \frac{dq}{r^2} = \frac{1}{4\pi\varepsilon_0} \frac{\lambda\, ds}{r^2}. \tag{22-19}
\]

The direction of \(d\vec{E}\) is toward \(ds\) because charge \(dq\) is negative.

**Symmetric partner:** Our element has a symmetrically located (mirror image) element \(ds'\) in the bottom half of the rod. The electric field \(d\vec{E}'\) set up at \(P\) by \(ds'\) also has the magnitude given by Eq. 22-19, but the field vector points toward \(ds'\) as shown in Fig. 22-13d. If we resolve the electric field vectors of \(ds\) and \(ds'\) into \(x\) and \(y\) components as shown in Figs. 22-13e and f, we see that their \(y\) components cancel (because they have equal magnitudes and are in opposite directions). We also see that their \(x\) components have equal magnitudes and are in the same direction.

**Summing:** Thus, to find the electric field set up by the rod, we need sum (via integration) only the \(x\) components of the differential electric fields set up by all the differential elements of the rod. From Fig. 22-13f and Eq. 22-19, we can write...
the component \( dE_x \) set up by \( ds \) as

\[
dE_x = dE \cos \theta = \frac{1}{4 \pi \varepsilon_0} \frac{\lambda}{r^2} \cos \theta \, ds.
\] (22-20)

Equation 22-20 has two variables, \( \theta \) and \( s \). Before we can integrate it, we must eliminate one variable. We do so by replacing \( ds \), using the relation

\[
ds = r \, d\theta,
\]

in which \( d\theta \) is the angle at \( P \) that includes arc length \( ds \) (Fig. 22-13g). With this replacement, we can integrate Eq. 22-20 over the angle made by the rod at \( P \), from \( \theta = -60^\circ \) to \( \theta = 60^\circ \), to get the field magnitude at \( P \):

\[
E = \int dE_x = \int_{-60^\circ}^{60^\circ} \frac{1}{4 \pi \varepsilon_0} \frac{\lambda}{r^2} \cos \theta \, d\theta
\]

\[
= \frac{\lambda}{4 \pi \varepsilon_0} \left[ \sin \theta \right]_{-60^\circ}^{60^\circ}
\]

\[
= \frac{\lambda}{4 \pi \varepsilon_0} \left[ \sin 60^\circ - \sin(-60^\circ) \right]
\]

\[
= \frac{1.73 \lambda}{4 \pi \varepsilon_0 r}.
\] (22-21)

(If we had reversed the limits on the integration, we would have gotten the same result but with a minus sign. Since the integration gives only the magnitude of \( E \), we would then have discarded the minus sign.)

**Charge density:** To evaluate \( \lambda \), we note that the full rod subtends an angle of 120° and is one-third of a full circle. Its arc length is then \( 2\pi r/3 \), and its linear charge density must be

\[
\lambda = \frac{\text{charge}}{\text{length}} = \frac{Q}{2\pi r/3} = \frac{0.477Q}{r}.
\]

Substituting this into Eq. 22-21 and simplifying give us

\[
E = \left( \frac{1.73}{4 \pi \varepsilon_0} \right) \frac{0.477Q}{r} = \frac{0.83Q}{4 \pi \varepsilon_0 r^2}.
\] (Answer)

The direction of \( \vec{E} \) is toward the rod, along the axis of symmetry of the charge distribution. We can write \( \vec{E} \) in unit-vector notation as

\[
\vec{E} = \frac{0.83Q}{4 \pi \varepsilon_0 r^2} \hat{r}.
\]

**Problem-Solving Tactics  A Field Guide for Lines of Charge**

Here is a generic guide for finding the electric field \( \vec{E} \) produced at a point \( P \) by a line of uniform charge, either circular or straight. The general strategy is to pick out an element \( dq \) of the charge, find \( d\vec{E} \) due to that element, and integrate \( d\vec{E} \) over the entire line of charge.

**Step 1.** If the line of charge is circular, let \( ds \) be the arc length of an element of the distribution. If the line is straight, run an \( x \) axis along it and let \( dx \) be the length of an element. Mark the element on a sketch.

**Step 2.** Relate the charge \( dq \) of the element to the length of the element with either \( dq = \lambda \, ds \) or \( dq = \lambda \, dx \). Consider \( dq \) and \( \lambda \) to be positive, even if the charge is actually negative. (The sign of the charge is used in the next step.)

**Step 3.** Express the field \( d\vec{E} \) produced at \( P \) by \( dq \) with Eq. 22-3, replacing \( q \) in that equation with either \( \lambda \, ds \) or \( \lambda \, dx \). If the charge on the line is positive, then at \( P \) draw a vector \( d\vec{E} \) that points directly away from \( dq \). If the charge is negative, draw the vector pointing directly toward \( dq \).

**Step 4.** Always look for any symmetry in the situation. If \( P \) is on an axis of symmetry of the charge distribution, resolve the field \( d\vec{E} \) produced by \( dq \) into components that are perpendicular and parallel to the axis of symmetry. Then consider a second element \( dq' \) that is located symmetrically to \( dq \) about the line of symmetry. At \( P \) draw the vector \( d\vec{E} \) that this symmetrical element produces and resolve it into components. One of the components produced by \( dq \) is a **canceling component**; it is canceled by the corresponding component produced by \( dq' \) and needs no further attention. The other component produced by \( dq \) is an **adding component**; it adds to the corresponding component produced by \( dq' \). Add the adding components of all the elements via integration.

**Step 5.** Here are four general types of uniform charge distributions with strategies for the integral of step 4.

*Ring*, with point \( P \) on (central) axis of symmetry, as in Fig. 22-11. In the expression for \( dE \), replace \( r^2 \) with \( x^2 + R^2 \), as in Eq. 22-12. Express the adding component of \( d\vec{E} \) in terms of \( \theta \). That introduces \( \cos \theta \), but \( \theta \) is identical for all elements and thus is not a variable. Replace \( \cos \theta \) as in Eq. 22-13. Integrate over \( x \), around the circumference of the ring.

*Circular arc*, with point \( P \) at the center of curvature, as in Fig. 22-13. Express the adding component of \( d\vec{E} \) in terms of \( \theta \). That introduces either \( \sin \theta \) or \( \cos \theta \). Reduce the resulting two variables \( s \) and \( \theta \) to one, \( \theta \), by replacing \( ds \) with \( r \, d\theta \). Integrate over \( \theta \) from one end of the arc to the other end.

*Straight line*, with point \( P \) on an extension of the line, as in Fig. 22-14a. In the expression for \( d\vec{E} \), replace \( r \) with \( x \). Integrate over \( x \), from end to end of the line of charge.
Straight line with point $P$ at perpendicular distance $y$ from the line of charge, as in Fig. 22-14b. In the expression for $dE$, replace $r$ with an expression involving $x$ and $y$. If $P$ is on the perpendicular bisector of the line of charge, find an expression for the adding component of $dE$. That will introduce either $\sin \theta$ or $\cos \theta$. Reduce the resulting two variables $x$ and $y$ to one, $x$, by replacing the trigonometric function with an expression (its definition) involving $x$ and $y$. Integrate over $x$ from end to end of the line of charge. If $P$ is not on a line of symmetry, as in Fig. 22-14c, set up an integral to sum the components $dE_x$, and integrate over $x$ to find $E_x$. Also set up an integral to sum the components $dE_y$, and integrate over $x$ again to find $E_y$. Use the components $E_x$ and $E_y$ in the usual way to find the magnitude $E$ and the orientation of $\overrightarrow{E}$.

**Step 6.** One arrangement of the integration limits gives a positive result. The reverse gives the same result with a minus sign; discard the minus sign. If the result is to be stated in terms of the total charge $Q$ of the distribution, replace $\lambda$ with $Q/L$, in which $L$ is the length of the distribution.

**Figure 22-14** (a) Point $P$ is on an extension of the line of charge. (b) $P$ is on a line of symmetry of the line of charge, at perpendicular distance $y$ from that line. (c) Same as (b) except that $P$ is not on a line of symmetry.

---

**Checkpoint 2**

The figure here shows three nonconducting rods, one circular and two straight. Each has a uniform charge of magnitude $Q$ along its top half and another along its bottom half. For each rod, what is the direction of the net electric field at point $P$?
Problem

• Calculate electric field at point $P$.

• Field very far away?
Problem

Field at center of arc?
**Line Of Charge: Field on bisector**

Charge per unit length \( \lambda = \frac{q}{L} \)

Distance \( d = \sqrt{a^2 + x^2} \)

\[
dE = \frac{k(dq)}{d^2}
\]

\[
dE_y = dE \cos \theta = \frac{k(\lambda \ dx)a}{(a^2 + x^2)^{3/2}}
\]

\[
\cos \theta = \frac{a}{(a^2 + x^2)^{1/2}}
\]
Line Of Charge: Field on bisector

\[ E_y = k \lambda a \int_{-L/2}^{L/2} \frac{dx}{a^2 + x^2}^{3/2} = k \lambda a \left[ \frac{x}{a^2 \sqrt{x^2 + a^2}} \right]_{-L/2}^{L/2} \]

\[ = \frac{2k \lambda L}{a \sqrt{4a^2 + L^2}} \]

What is \( E \) very far away from the line \((L << a)\)?

\( E_y \sim 2k \lambda L / a(2a) = k \lambda L / a^2 = kq / a^2 \)

What is \( E \) if the line is infinitely long \((L >> a)\)?

\[ E_y = \frac{2k \lambda L}{a \sqrt{L^2}} = \frac{2k \lambda}{a} \]
Electric fields: Example

Calculate the magnitude and direction of the electric field produced by a ring of charge $Q$ and radius $R$, at a distance $z$ on its
Sample Problem

Figure 22N-14 shows an arrangement of four charged particles, with angle \( \theta = 34^\circ \) and distance \( d = 2.20 \) cm. The two negatively charged particles on the \( y \) axis are electrons that are fixed in place; the particle at the right has a charge \( q_2 = +5e \).

(a) Find distance \( D \) such that the net force on the particle at the left, due to the three other particles, is zero.

(b) If the two electrons were moved further from the \( x \) axis, would the required value of \( D \) be greater than, less than, or the same as in part (a)?
At each point on the surface of the cube shown in Fig. 24-26, the electric field is in the z direction. The length of each edge of the cube is 2.3 m. On the top surface of the cube $\mathbf{E} = -38 \, \text{k N/C}$, and on the bottom face of the cube $\mathbf{E} = +11 \, \text{k N/C}$. Determine the net charge contained within the cube. 

$[-2.29\text{e-09}] \, \text{C}$
Gauss’s Law: Cylinder, Plane, Sphere

\[ \Phi = EA \cos \theta = E(2\pi rh) \cos \theta = E(2\pi rh). \]

There is no flux through the end caps because \( E \) being radially directed, is parallel to the end caps at every point.

The charge enclosed by the surface is \( A \), which means Gauss’ law,

\[ \Phi = \Phi_{\text{enc}} \]

reduces to

\[ \epsilon_0 E(2\pi rh) = \Phi_{\text{enc}}. \]

yielding

\[ E = \frac{1}{2\pi \epsilon_0} \frac{q}{r} \quad \text{(line of charge),} \tag{23-12} \]

This is the electric field due to an infinitely long, straight line of charge at a point that is a radial distance \( r \) from the line. The direction of \( E \) is radially outward from the line of charge if the charge is positive, and radially inward if it is negative. Equation 23-12 also approximates the field of a finite line of charge at points that are not too near the ends (compared with the distance from the line).

**23-8 Applying Gauss’ Law: Planar Symmetry**

**Nonconducting Sheet**

Figure 23-15 shows a portion of a thin, infinite, nonconducting sheet with a uniform (positive) surface charge density \( \sigma \). A sheet of thin plastic wrap, uniformly charged on one side, can serve as a simple model. Let us find the electric field \( E \) a distance \( r \) in front of the sheet.

A useful Gaussian surface is a closed cylinder with end caps of area \( A \), arranged to pierce the sheet perpendicularly as shown. From symmetry, \( E \) must be perpendicular to the sheet and hence to the end caps. Furthermore, since the charge is positive, \( E \) is directed away from the sheet, and thus the electric field lines pierce the two Gaussian end caps in an outward direction. Because the field lines do not pierce the curved surface, there is no flux through this portion of the Gaussian surface. Thus \( \Phi = 0 \) (as simply \( E \cdot dA \)), then Gauss’ law,

\[ \epsilon_0 \int E \cdot dA = \Phi_{\text{enc}} \]

becomes

\[ \epsilon_0 (EA + EA) = \sigma A, \]

where \( \sigma A \) is the charge enclosed by the Gaussian surface. This gives

\[ E = \frac{\sigma}{2\epsilon_0} \quad \text{(charged sheet).} \tag{23-13} \]

Since we are considering an infinite sheet with uniform charge density, this result holds for any point at a finite distance from the sheet. Equation 23-13 agrees with Eq. 22-27, which we found by integration of electric field components.

Fig. 23-10 A thin, uniformly charged, spherical shell with total charge \( q \) enclosed by a spherical surface of radius \( R \). Two Gaussian surfaces \( S_1 \) and \( S_2 \) are shown in cross section. Surface \( S_1 \) encloses the shell, and \( S_2 \) encloses only the empty interior of the shell.

Concerned at the center. Letting \( q' \) represent the enclosed charge, we can then rewrite Eq. 23-15 as

\[ E = \frac{1}{4\pi \epsilon_0} \frac{q'}{r^2} \quad \text{(spherical distribution, field at \( r < R \)).} \tag{23-17} \]

If the full charge \( q \) enclosed within radius \( R \) is uniform, then \( q' \) enclosed within radius in Fig. 23-16 is proportional to \( q \).

\[ \frac{\text{charge enclosed by}}{\text{volume enclosed by}} \frac{\text{sphere of radius} \ r}{\text{sphere of radius} \ R} \]

or

\[ \frac{q'}{q} = \left( \frac{r}{R} \right)^3 \quad \text{(23-18)} \]

This gives us

\[ q' = \frac{q}{R^3} \quad \text{(23-19)} \]

Substituting this into Eq. 23-17 yields

\[ E = \left( \frac{q}{4\pi \epsilon_0 R^2} \right) \quad \text{(uniform charge, field at \( r < R \)).} \tag{23-20} \]
Problem: Gauss’ Law to Find $E$
Two Insulating Sheets

Figure 23-17a shows portions of two large, parallel, non-conducting sheets, each with a fixed uniform charge on one side. The magnitudes of the surface charge densities are \( \sigma_+ = 6.8 \ \mu \text{C/m}^2 \) for the positively charged sheet and \( \sigma_- = 4.3 \ \mu \text{C/m}^2 \) for the negatively charged sheet.

Find the electric field \( \vec{E} \) (a) to the left of the sheets, (b) between the sheets, and (c) to the right of the sheets.

\[
\sigma_+ = +\frac{Q_+}{A}
\]
\[
\sigma_- = -\frac{Q_-}{A}
\]

\[
E_+ = \frac{\sigma_+}{2\varepsilon_0} = \frac{6.8 \times 10^{-6} \text{ C/m}^2}{2(8.85 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2)} = 3.84 \times 10^5 \text{ N/C.}
\]

\[
E_- = \frac{\sigma_-}{2\varepsilon_0} = \frac{4.3 \times 10^{-6} \text{ C/m}^2}{2(8.85 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2)} = 2.43 \times 10^5 \text{ N/C.}
\]

\[
E_L = E_+ - E_-
\]
\[
= 3.84 \times 10^5 \text{ N/C} - 2.43 \times 10^5 \text{ N/C}
\]
\[
= 1.4 \times 10^5 \text{ N/C.} \quad \text{(Answer)}
\]

\[
E_B = E_+ + E_-
\]
\[
= 3.84 \times 10^5 \text{ N/C} + 2.43 \times 10^5 \text{ N/C}
\]
\[
= 6.3 \times 10^5 \text{ N/C.} \quad \text{(Answer)}
\]

\[
E_R = E_L
\]
Two Conducting Sheets

\[ \sigma_+ = +\frac{1}{2} \frac{Q_+}{A} \quad \sigma_- = -\frac{1}{2} \frac{Q_-}{A} \]

E does not pass through a conductor

Formula for E different by Factor of 2

\[ E_{(+)} = \frac{\sigma_+}{\varepsilon_0} = \frac{6.8 \times 10^{-6} \text{ C/m}^2}{(8.85 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2)} = \frac{7.6}{8} \times 10^5 \text{ N/C.} \]

\[ E_{(-)} = \frac{\sigma_-}{\varepsilon_0} = \frac{4.3 \times 10^{-6} \text{ C/m}^2}{(8.85 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2)} = \frac{4.8}{6} \times 10^5 \text{ N/C.} \]

\[ E_L = E_{(+)} \]

\[ E_B = E_{(+)} + E_{(-)} = \frac{7.6}{8} \times 10^5 \text{ N/C} + \frac{4.8}{6} \times 10^5 \text{ N/C} = \frac{12.54}{6} \times 10^5 \text{ N/C.} \quad \text{(Answer)} \]

\[ E_R = E_- \neq E_L \]
Electric Fields With Insulating Sphere

\[ q_{\text{ins}} = Q \left( \frac{V_{\text{ins}}}{V_{\text{total}}} \right) = Q \left( \frac{4\pi r^3 / 3}{4\pi R^3 / 3} \right) = Q \frac{r^3}{R^3} \]

\[ E(R) = \frac{1}{4\pi \varepsilon_0} \frac{Q}{R^2} \]

\[ E = \frac{1}{4\pi \varepsilon_0} \frac{Qr}{r^2} \]

\[ \Phi = EA = q_{\text{ins}} / \varepsilon_0 \]

\[ r < R \rightarrow E 4\pi r^2 = Q \frac{r^3}{R^3} / \varepsilon_0 \]

\[ r > R \rightarrow E 4\pi r^2 = Q / \varepsilon_0 \]
51 SSM WWW In Fig. 23-56, a nonconducting spherical shell of inner radius \( a = 2.00 \text{ cm} \) and outer radius \( b = 2.40 \text{ cm} \) has (within its thickness) a positive volume charge density \( \rho = A/r \), where \( A \) is a constant and \( r \) is the distance from the center of the shell. In addition, a small ball of charge \( q = 45.0 \text{ fC} \) is located at that center. What value should \( A \) have if the electric field in the shell \((a \leq r \leq b)\) is to be uniform?

52 GO Figure 23-57 shows a spherical shell with uniform volume charge density \( \rho = 1.84 \text{ nC/m}^3 \), inner radius \( a = 10.0 \text{ cm} \), and outer radius \( b = 2.00a \). What is the magnitude of the electric field at radial distances (a) \( r = 0 \); (b) \( r = a/2.00 \); (c) \( r = a \); (d) \( r = 1.50a \); (e) \( r = b \), and (f) \( r = 3.00b \)?