Physics 2113
Lecture 03
Coulomb’s Law I

Charles-Augustin de Coulomb (1736–1806)

Version: 9/1/17
Let’s Get Started!
Electric Charges...

- Two Types of Charges: Positive/Negative
- **Like** Charges Repel
- **Opposite** Charges Attract

**Atomic Structure:**
- Negative Electron Cloud
- **Nucleus of Positive Protons, Uncharged Neutrons**

The Unit of Electric Charge is the “Coulomb” which is “C”.
Proton Charge: $e = 1.60 \times 10^{-19} \text{ C}$
Rules of Electric Attraction and Repulsion
Discovered by Benjamin Franklin:
Electrical Insulators

Benjamin Franklin
(1705–1790)

**Opposite charges attract**

- 
- 
+ 
+

**Like charges repel**

Glass

Plastic

Glass
Rules of Electric Attraction and Repulsion
Discovered by Benjamin Franklin: Electric Conductors

Opposite charges attract

Like charges repel

A negatively charged object is brought near to a neutral, conducting sphere. Electrons in the sphere are forced from the left side of the sphere to the right side.
Rules of Electric Attraction and Repulsion: ICPP

CHECKPOINT 1

The figure shows five pairs of plates: A, B, and D are charged plastic plates and C is an electrically neutral copper plate. The electrostatic forces between the pairs of plates are shown for three of the pairs. For the remaining two pairs, do the plates repel or attract each other?

Opposite charges attract

C and D attract

B and D attract

Like charges repel
Coulomb’s Law — the Force Between Point Charges:

- Lies Along the Line Connecting the Charges.
- Is Proportional to the Product of the Magnitudes.
- Is Inversely Proportional to the Distance Squared.
- Note That Newton’s Third Law Says $|F_{12}| = |F_{21}|$!!
Force Between Pairs of Point Charges: Coulomb’s Law

Something funny about shells!
Electric Field Outside as If a point charge at core. Electric field inside is zero!
Astronomers: “It’s the shell theorem! That implies a $1/r^2$ Law!”
Coulomb’s Law

\[ |F_{12}| = \frac{k |q_1| |q_2|}{r_{12}^2} \]

The “\(k\)” is the electric constant of proportionality.

\[ k = 8.99 \times 10^9 \, \frac{\text{Nm}^2}{\text{C}^2} \propto \frac{\text{kg m}^3}{\text{s}^2 \text{C}^2} \]

Usually, we write:

\[ k = \frac{1}{4\pi \varepsilon_0} \]

with \( \varepsilon_0 = 8.85 \times 10^{-12} \, \frac{\text{C}^2}{\text{Nm}^2} \)

Units: \( F = [\text{N}] = [\text{Newton}] \);

\( r = [\text{m}] = [\text{meter}] \);

\( q = [\text{C}] = [\text{Coulomb}] \)
**Force Between Pairs of Point Charges: ICPP**

\[
\begin{align*}
|F_{12}| &= \frac{k |q_1| |q_2|}{r_{12}^2} \\
F_{12} &= +q_1 \quad F_{21} = -q_2 \\
F_{12} &= -q_1 \quad F_{21} = +q_2
\end{align*}
\]

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**CHECKPOINT 2**

The figure shows two protons (symbol p) and one electron (symbol e) on an axis. What is the direction of (a) the electrostatic force on the central proton due to the electron, (b) the electrostatic force on the central proton due to the other proton, and (c) the net electrostatic force on the central proton?

(a) \[\vec{F}_{pe}\]  
(b) \[\vec{F}_{pp}\]  
(c) \[\vec{F}_{net} = \vec{F}_{pe} + \vec{F}_{ee}\]
CHECKPOINT 3

The figure here shows three arrangements of an electron e and two protons p. (a) Rank the arrangements according to the magnitude of the net electrostatic force on the electron due to the protons, largest first. (b) In situation c, is the angle between the net force on the electron and the line labeled d less than or more than 45°?

\[ F \approx \frac{1}{r^2} \quad \text{Double the distance } 1/4^{th} \text{ the force!} \]

\[
q_e = -1.60 \times 10^{-19} \text{ C} \quad (a) \ a > c > b
\]

\[
q_p = +1.60 \times 10^{-19} \text{ C} \quad (b) \ \text{less}
\]
Coulomb’s Torsion Balance Experiment For Electric Force Identical to Cavendish’s Experiment For Gravitational Force!

The experiment measures “k” the electric constant of proportionality and confirms inverse square law.

\[ k = 8.99 \times 10^9 \frac{\text{N m}^2}{\text{C}^2} \propto \frac{\text{kg m}^3}{\text{s}^2 \text{C}^2} \]

\[ |F_{12}| = \frac{k |q_1| |q_2|}{r_{12}^2} \]

Two Inverse Square Laws

Newton’s Law of Gravitational Force

\[ F_1 = F_2 = G \frac{m_1 \times m_2}{r^2} \]

Coulomb’s Law of Electrical Force

\[ F_1 = F_2 = k_c \frac{q_1 \times q_2}{r^2} \]

Area of Sphere = \(4\pi r^2\)

Number of Lines of Force is Constant.

Hence #Force Lines Per-Unit-Area is Proportional to \(1/r^2\)
Superposition

- **Question**: How Do We Figure Out the Force on a Point Charge Due to Many Other Point Charges?

- **Answer**: Consider One Pair at a Time, Calculate the Force (a Vector!) In Each Case Using Coulomb’s Law and Finally Add All the Vectors! (“Superposition”)

- Useful To Look Out for SYMMETRY to Simplify Calculations!
Feel the Force!
Example

• Three Equal Charges Form an Equilateral Triangle of Side 1.5 m as Shown
• Compute the Force on \( q_1 \)
• ICPP: What are the Forces on the Other Charges?

Solution: Set up a Coordinate System, Compute Vector Sum of \( F_{12} \) and \( F_{13} \)
Feel the Force!
Example

\[ F_{12} = F_{13} = \frac{kq_1q_2}{(r_{12})^2} \]

\[ = \frac{8.99 \times 10^9 \text{N m}^2}{C^2} \left| \begin{array}{cc} 20 \times 10^{-3} \text{C} & 20 \times 10^{-3} \text{C} \\ (0.01 \text{m})^2 & \end{array} \right| \]

\[ = 3.60 \times 10^6 \text{N} \]

\[ F_{\text{net}} = \sqrt{(F_{\text{net}}^x)^2 + (F_{\text{net}}^y)^2} \]

\[ = \sqrt{(F_{13}^x)^2 + (F_{13}^y + F_{12})^2} \]

\[ = \sqrt{(F_{13} \cos(30^\circ))^2 + (F_{13} \sin(30^\circ) + F_{12})^2} \]

\[ = \sqrt{(3.12 \times 10^6 \text{N})^2 + (5.40 \times 10^6 \text{N})^2} \]

\[ = 6.24 \times 10^6 \text{N} \]

By geometry: \( \theta = \frac{1}{2} 60^\circ + 90^\circ = 120^\circ \)

ICPP: What are the magnitudes and directions of the forces on 2 and 3?

\( q_1 = q_2 = q_3 = 20 \text{ mC} \)
\( d = r_{12} = 1.0 \text{ cm} \)
ICPP: Another Example With Symmetry

What is the Force on Central Particle?

All Forces **Cancel** Except From +2q!

\[
|\vec{F}| = \frac{k |+2q| |+q|}{r^2}
\]
Sample Problem 21.01  Finding the net force due to two other particles

(a) Figure 21-7a shows two positively charged particles fixed in place on an \( x \) axis. The charges are \( q_1 = 1.60 \times 10^{-19} \text{ C} \) and \( q_2 = 3.20 \times 10^{-19} \text{ C} \), and the particle separation is \( R = 0.0200 \text{ m} \). What are the magnitude and direction of the electrostatic force \( \overrightarrow{F}_{12} \) on particle 1 from particle 2?

This is the first arrangement.

\[ F_{12} = \frac{1}{4\pi\varepsilon_0} \frac{|q_1||q_2|}{R^2} \]
\[ = (8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2) \times \frac{(1.60 \times 10^{-19} \text{ C})(3.20 \times 10^{-19} \text{ C})}{(0.0200 \text{ m})^2} \]
\[ = 1.15 \times 10^{-24} \text{ N}. \]

Thus, force \( \overrightarrow{F}_{12} \) has the following magnitude and direction (relative to the positive direction of the \( x \) axis):

\[ 1.15 \times 10^{-24} \text{ N} \quad \text{and} \quad 180^\circ. \quad \text{(Answer)} \]

We can also write \( \overrightarrow{F}_{12} \) in unit-vector notation as

\[ \overrightarrow{F}_{12} = -(1.15 \times 10^{-24} \text{ N})\hat{i}. \quad \text{(Answer)} \]
Sample Problem 21.01  Finding the net force due to two other particles

(b) Figure 21-7c is identical to Fig. 21-7a except that particle 3 now lies on the x axis between particles 1 and 2. Particle 3 has charge \( q_3 = -3.20 \times 10^{-19} \text{ C} \) and is at a distance \( \frac{3}{4} R \) from particle 1. What is the net electrostatic force \( \vec{F}_{1,\text{net}} \) on particle 1 due to particles 2 and 3?

**Three particles:** To find the magnitude of \( \vec{F}_{13} \), we can rewrite Eq. 21-4 as

\[
F_{13} = \frac{1}{4\pi\varepsilon_0} \frac{|q_1||q_3|}{\left(\frac{3}{4} R\right)^2}
\]

\[
= (8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2) \left( \frac{1.60 \times 10^{-19} \text{ C}}{3.20 \times 10^{-19} \text{ C}} \right) \left( \frac{0.0200 \text{ m}}{\left(\frac{3}{4}\right)^2} \right) \]

\[
= 2.05 \times 10^{-24} \text{ N}.
\]

We can also write \( \vec{F}_{13} \) in unit-vector notation:

\[
\vec{F}_{13} = (2.05 \times 10^{-24} \text{ N})\hat{i}.
\]

The net force \( \vec{F}_{1,\text{net}} \) on particle 1 is the vector sum of \( \vec{F}_{12} \) and \( \vec{F}_{13} \); that is, from Eq. 21-7, we can write the net force \( \vec{F}_{1,\text{net}} \) on particle 1 in unit-vector notation as

\[
\vec{F}_{1,\text{net}} = \vec{F}_{12} + \vec{F}_{13}
\]

\[
= -(1.15 \times 10^{-24} \text{ N})\hat{i} + (2.05 \times 10^{-24} \text{ N})\hat{i}
\]

\[
= (9.00 \times 10^{-25} \text{ N})\hat{i}.
\]

(Answer)

Thus, \( \vec{F}_{1,\text{net}} \) has the following magnitude and direction (relative to the positive direction of the x axis):

\[
9.00 \times 10^{-25} \text{ N} \quad \text{and} \quad 0^\circ.
\]

(Answer)
Sample Problem 21.01  Finding the net force due to two other particles

(c) Figure 21-7e is identical to Fig. 21-7a except that particle 4 is now included. It has charge $q_4 = -3.20 \times 10^{-19}$ C, is at a distance $\frac{3}{4} R$ from particle 1, and lies on a line that makes an angle $\theta = 60^\circ$ with the $x$ axis. What is the net electrostatic force $\vec{F}_{1,\text{net}}$ on particle 1 due to particles 2 and 4?

The net force $\vec{F}_{1,\text{net}}$ is the vector sum of $\vec{F}_{12}$ and a new force $\vec{F}_{14}$ acting on particle 1 due to particle 4. Because particles 1 and 4 have charge of opposite signs, particle 1 is attracted to particle 4. Thus, force $\vec{F}_{14}$ on particle 1 is directed toward particle 4, at angle $\theta = 60^\circ$, as indicated in the free-body diagram of Fig. 21-7f.

**Four particles:** We can rewrite Eq. 21-4 as

$$F_{14} = \frac{1}{4\pi\varepsilon_0} \frac{|q_1||q_4|}{(\frac{3}{4}R)^2}$$

$$= (8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2) \times \frac{(1.60 \times 10^{-19} \text{ C})(3.20 \times 10^{-19} \text{ C})}{(\frac{3}{4})^2(0.0200 \text{ m})^2}$$

$$= 2.05 \times 10^{-24} \text{ N}.$$  

**Method 3. Summing components axis by axis.** The sum of the $x$ components gives us

$$F_{1,\text{net},x} = F_{12,x} + F_{14,x} = F_{12} + F_{14}\cos 60^\circ$$

$$= -1.15 \times 10^{-24} \text{ N} + (2.05 \times 10^{-24} \text{ N})(\cos 60^\circ)$$

$$= -1.25 \times 10^{-25} \text{ N}.$$  

The sum of the $y$ components gives us

$$F_{1,\text{net},y} = F_{12,y} + F_{14,y} = 0 + F_{14}\sin 60^\circ$$

$$= (2.05 \times 10^{-24} \text{ N})(\sin 60^\circ)$$

$$= 1.78 \times 10^{-24} \text{ N}.$$  

The net force $\vec{F}_{1,\text{net}}$ has the magnitude

$$F_{1,\text{net}} = \sqrt{F_{1,\text{net},x}^2 + F_{1,\text{net},y}^2} = 1.78 \times 10^{-24} \text{ N.} \quad \text{(Answer)}$$

To find the direction of $\vec{F}_{1,\text{net}}$, we take

$$\theta = \tan^{-1} \frac{F_{1,\text{net},y}}{F_{1,\text{net},x}} = -86.0^\circ.$$  

However, this is an unreasonable result because $\vec{F}_{1,\text{net}}$ must have a direction between the directions of $\vec{F}_{12}$ and $\vec{F}_{14}$. To correct $\theta$, we add 180°, obtaining

$$-86.0^\circ + 180^\circ = 94.0^\circ. \quad \text{(Answer)}$$
\[ E = mc^2 \]

\[ E > mb^2 \]