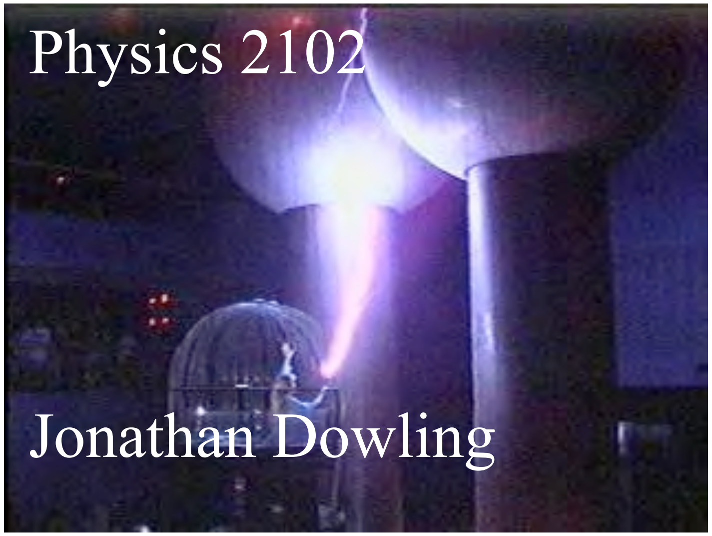
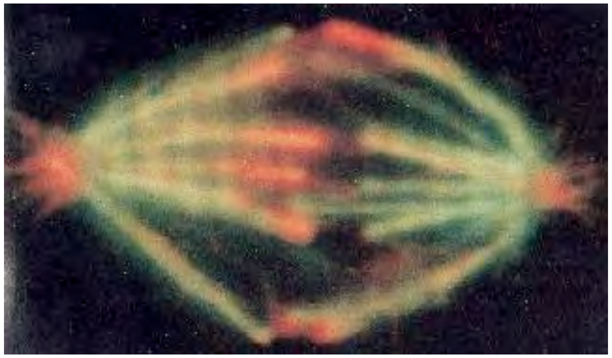


Physics 2102
Lecture 2

Electric Fields



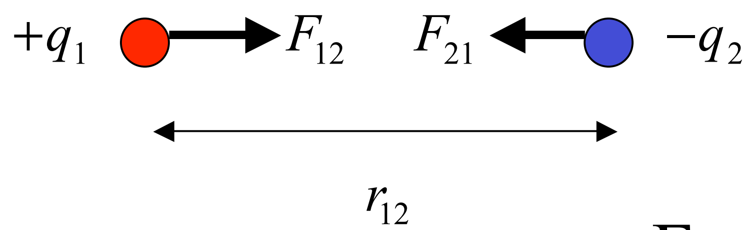
Version: 1/17/07

Charles-Augustin
de Coulomb
(1736-1806)

What are we going to learn?
A road map

- **Electric *charge***
  ➔ Electric *force* on other electric charges
  ➔ Electric *field*, and electric *potential*
- **Moving electric charges : current**
- **Electronic circuit components: batteries, resistors, capacitors**
- **Electric currents ➔ Magnetic field**
  ➔ Magnetic force on moving charges
- **Time-varying magnetic field ➔ Electric Field**
- **More circuit components: inductors.**
- **Electromagnetic waves ➔ light waves**
- **Geometrical Optics (light rays).**
- **Physical optics (light waves)**

Coulomb's law



$$|F_{12}| = \frac{k\ |q_1||q_2|}{r_{12}^2}$$

For charges in a VACUUM

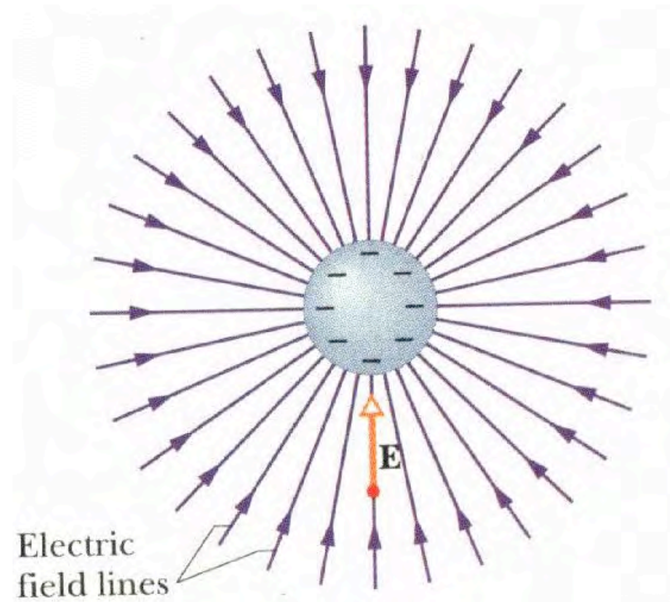
$$k = 8.99\times10^{9}\ \frac{N\,m^2}{C^2}$$

Often, we write *k* as:

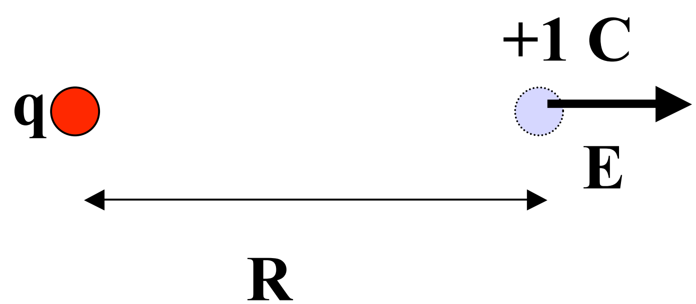
$$k = {}^{1}\!/_{4\pi\varepsilon_0} \text{ with } \varepsilon_0 = 8.85\times10^{-12}\ \frac{C^2}{N\,m^2}$$

Electric Fields

- Electric field E at some point in space is defined as the force experienced by an *imaginary* point charge of +1 C, divided by 1 C.
- Note that E is a VECTOR.
- Since E is the force per unit charge, it is measured in units of N/C.
- We *measure* the electric field using very small "test charges", and dividing the measured force by the magnitude of the charge.



Electric field of a point charge



$$|E| = \frac{k\,|q|}{R^2}$$

Superposition

- **Question**: How do we figure out the field due to several point charges?
- **Answer**: consider one charge at a time, calculate the field (a vector!) produced by each charge, and then add all the vectors! (“superposition”)
- Useful to look out for SYMMETRY to simplify calculations!

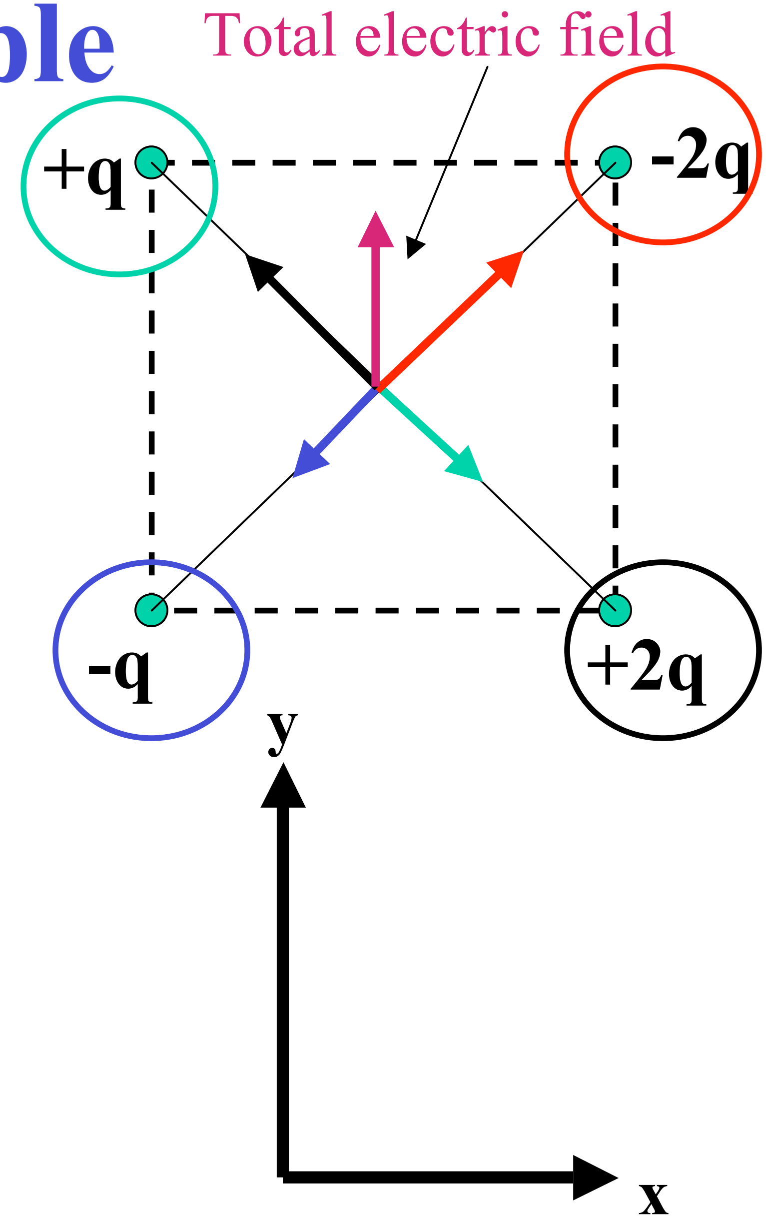
Example

- 4 charges are placed at the corners of a square as shown.
- What is the direction of the electric field at the center of the square?

(a) Field is ZERO!

(b) Along +y

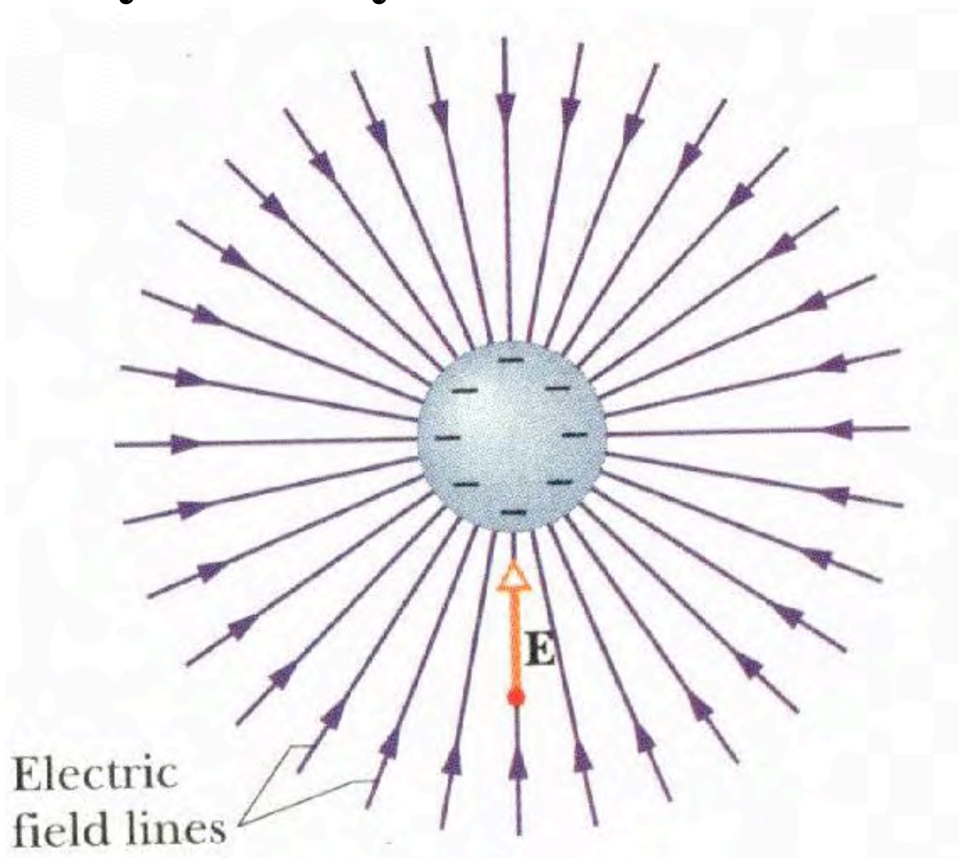
(c) Along +x



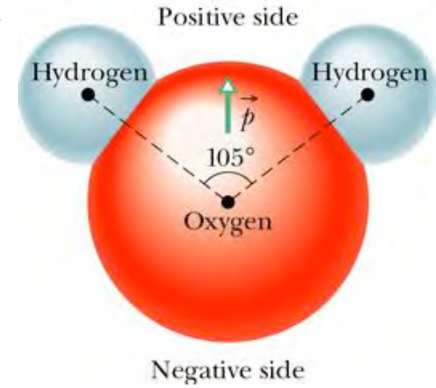
Electric Field Lines

- Field lines: useful way to visualize electric field E
- Field lines start at a positive charge, end at negative charge
- E at any point in space is tangential to field line
- Field lines are closer where E is stronger

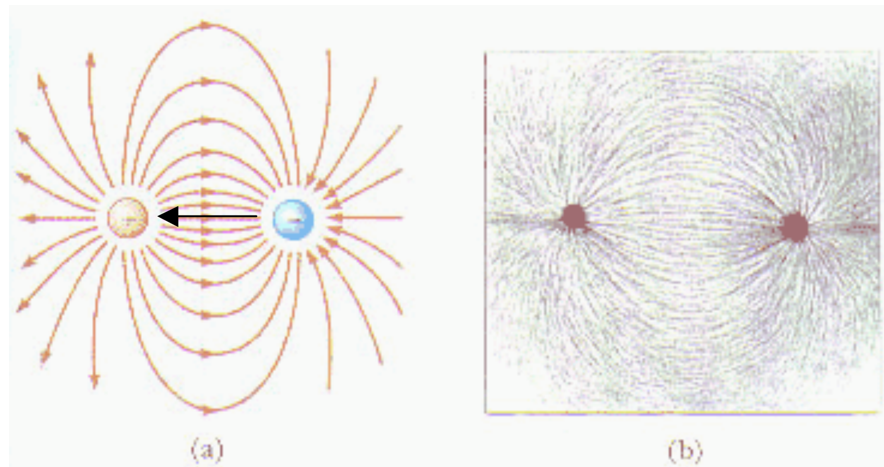
Example: a negative point charge — note spherical symmetry



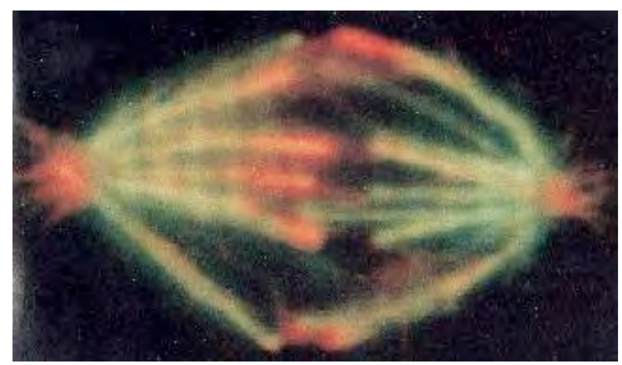
Electric Field of a Dipole



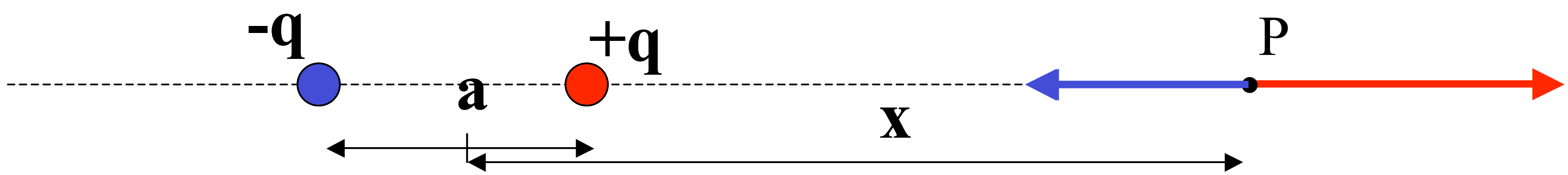
- **Electric dipole: two point charges +q and –q separated by a distance d**
- **Common arrangement in Nature: molecules, antennae, …**
- **Note axial or cylindrical symmetry**
- **Define "dipole moment" vector p: from –q to +q, with magnitude qd**



Cancer, Cisplatin and electric dipoles:
http://chemcases.com/cisplat/cisplat01.htm



Electric Field ON axis of dipole



$$\text{Superposition}: \quad \vec{E} = \vec{E}_+ + \vec{E}_-$$

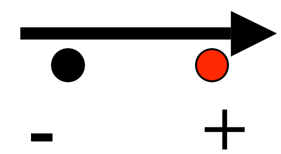
$$\vec{E}_+ = \frac{kq}{\left(x - \frac{a}{2}\right)^2} \qquad \vec{E}_- = -\frac{kq}{\left(x + \frac{a}{2}\right)^2}$$

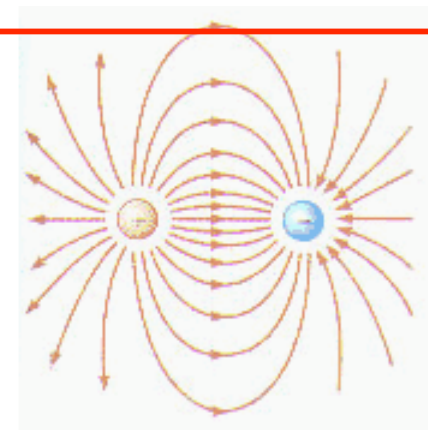
$$\vec{E} = kq\left[\frac{1}{\left(x - \frac{a}{2}\right)^2} - \frac{1}{\left(x + \frac{a}{2}\right)^2}\right] = kq\frac{2xa}{\left(x^2 - \frac{a^2}{4}\right)^2}$$

Electric Field ON axis of dipole

$$E = kq\frac{2xa}{\left(x^2 - \frac{a^2}{4}\right)^2} = \frac{2kpx}{\left(x^2 - \frac{a^2}{4}\right)^2}$$

p = qa
“dipole moment”
-- VECTOR




(a)

What if x>> a? (i.e. very far away)

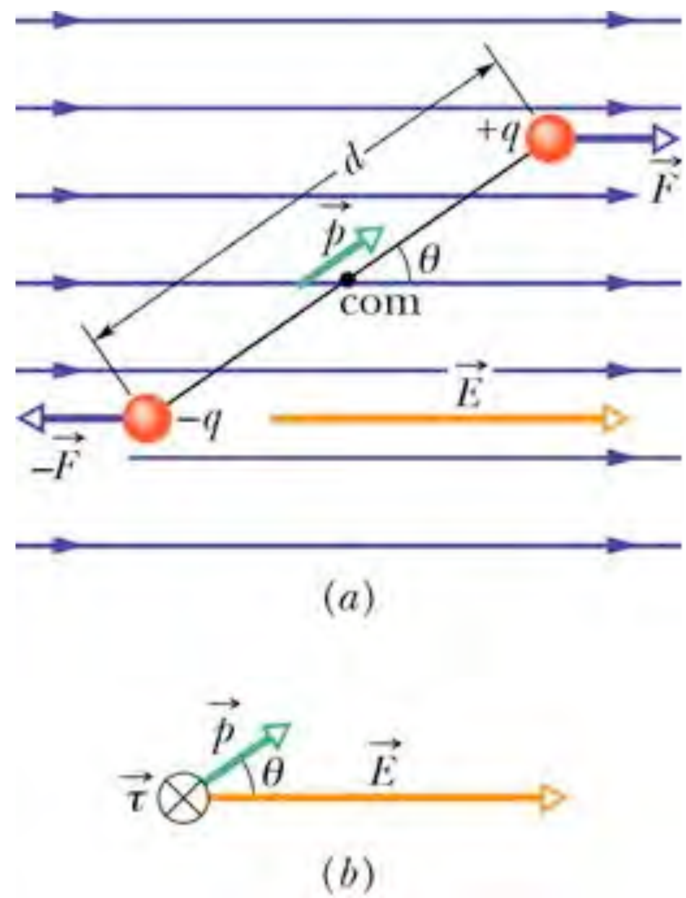
$$E \approx \frac{2kpx}{x^4} = \frac{2kp}{x^3} \Rightarrow \left|\vec{E}\right| \propto \frac{\left|\vec{p}\right|}{r^3}$$

$E \sim p/r^3$ is actually true for ANY point far from a dipole (not just on axis)

Electric Dipole in a Uniform Field

- **Net force on dipole = 0; center of mass stays where it is.**
- **Net TORQUE τ: INTO page. Dipole rotates to line up in direction of E.**
- **| τ | = 2(QE)(d/2)(sin θ)**
  = (Qd)(E)sinθ
  =|p| E sinθ
  = |p x E|
- **The dipole tends to "align" itself with the field lines.**
- **What happens if the field is NOT UNIFORM??**

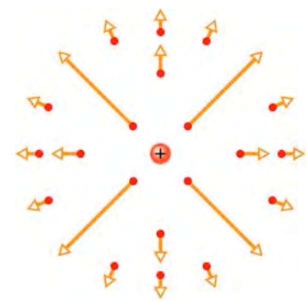
Distance between charges = d

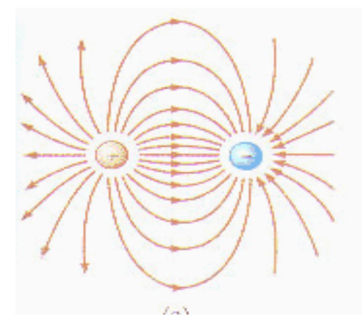


Electric charges and fields

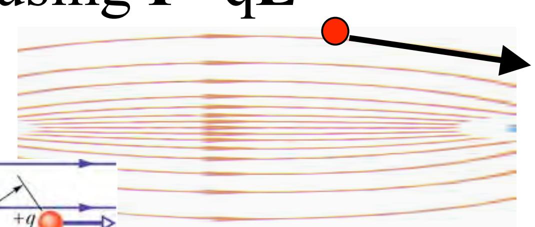
We work with two different kinds of problems, easily confused:

• **Given certain electric charges**, we calculate the electric field produced by those charges
(using $\mathbf{E}=kq\mathbf{r}/r^3$ for each charge)
Example: the electric field produced by a single charge, or by a dipole:

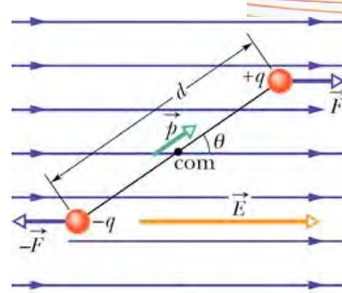




• **Given an electric field**, we calculate the forces applied by this electric field on charges that come into the field, using $\mathbf{F}=q\mathbf{E}$

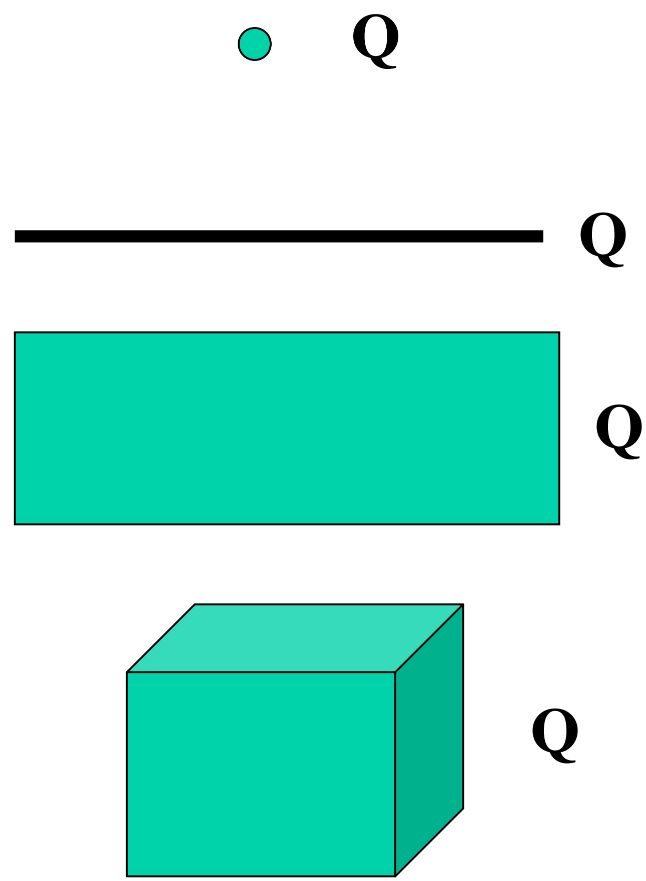


Examples: forces on a single charge when immersed in the field of a dipole, torque on a dipole when immersed in an uniform electric field.



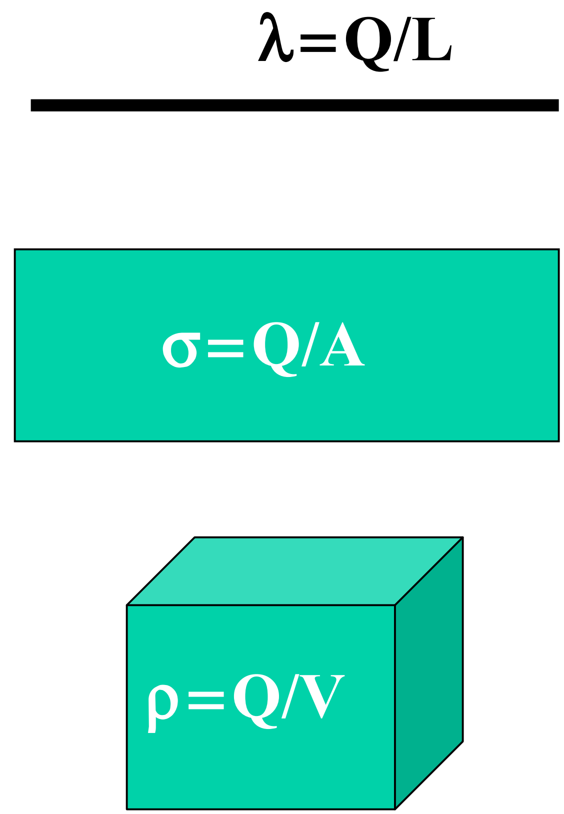
Continuous Charge Distribution

- Thus far, we have only dealt with discrete, point charges.
- Imagine instead that a charge Q is smeared out over a:
  - LINE
  - AREA
  - VOLUME
- How to compute the electric field E??



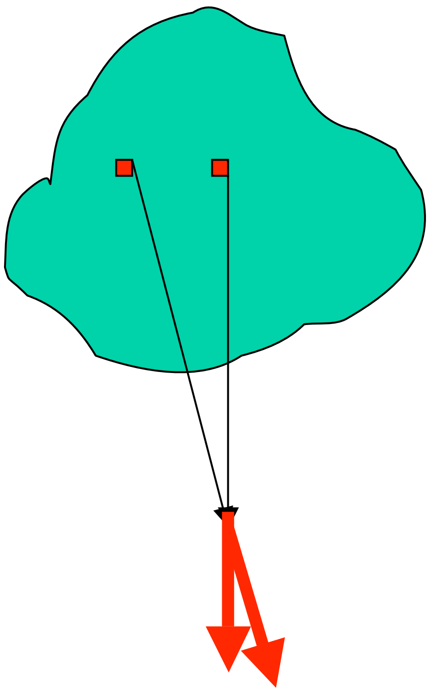
Charge Density

- Useful idea: charge density
- Line of charge:
  charge per unit length $= \lambda$
- Sheet of charge:
  charge per unit area $= \sigma$
- Volume of charge:
  charge per unit volume $= \rho$



Computing electric field of continuous charge distribution

- Approach: divide the continuous charge distribution into infinitesimally small elements
- Treat each element as a POINT charge & compute its electric field
- Sum (integrate) over all elements
- Always look for symmetry to simplify life!



Example: Field on Bisector of Charged Rod

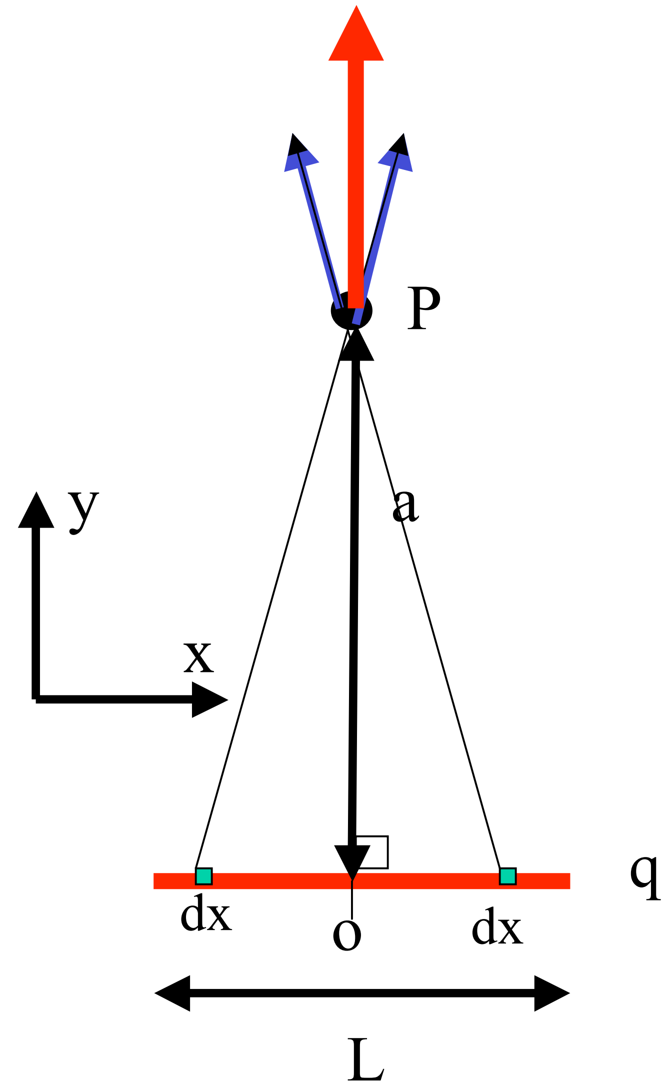
- Uniform line of charge +Q spread over length L
- What is the direction of the electric field at a point P on the perpendicular bisector?

(a) Field is 0.

(b) Along +y

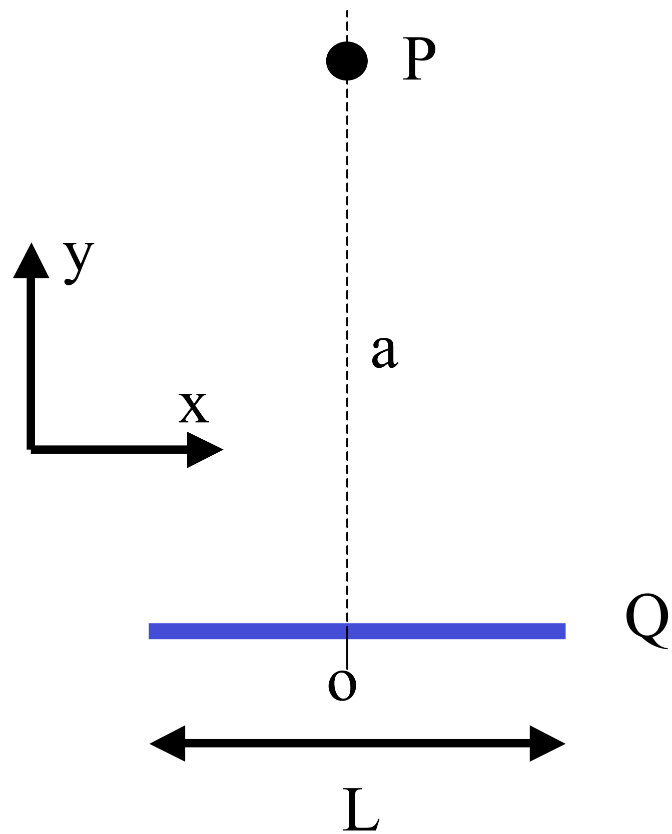
(c) Along +x

- Choose symmetrically located elements of length dx
- x components of E cancel

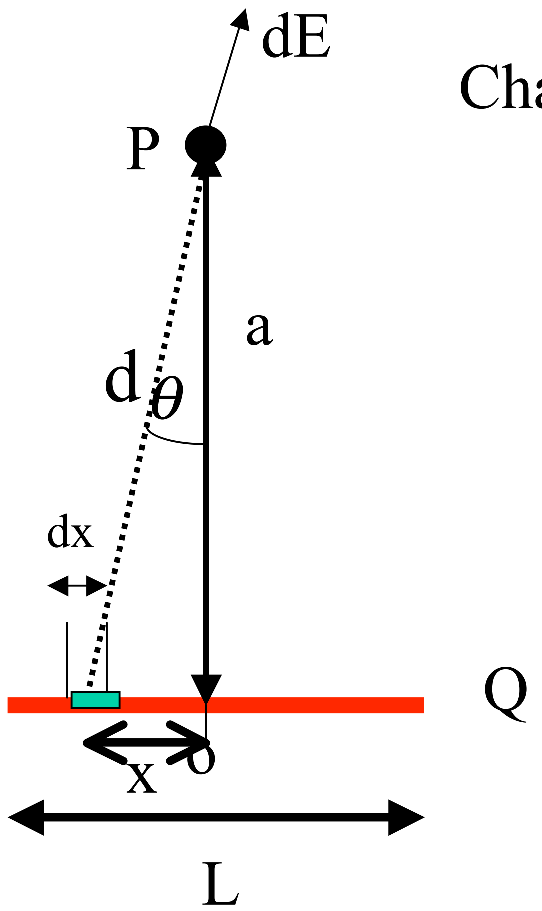


Example --Line of Charge: Quantitative

- Uniform line of charge, length L, total charge Q
- Compute explicitly the magnitude of E at point P on perpendicular bisector
- Showed earlier that the net field at P is in the y direction -- let's now compute this!



Line Of Charge: Field on bisector



Distance $d = \sqrt{a^2 + x^2}$

Charge per unit length $\lambda = \dfrac{q}{L}$

$$dE = \frac{k(dq)}{d^2}$$

$$dE_y = dE\cos\theta = \frac{k(\lambda\, dx)a}{(a^2 + x^2)^{3/2}}$$

$$\cos\theta = \frac{a}{(a^2 + x^2)^{1/2}}$$

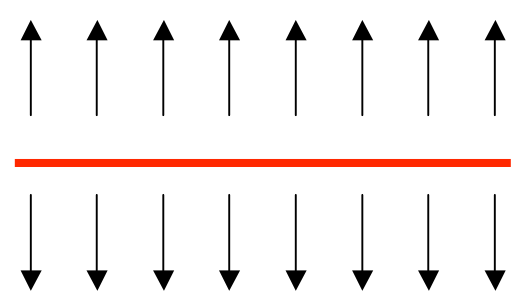
Line Of Charge: Field on bisector

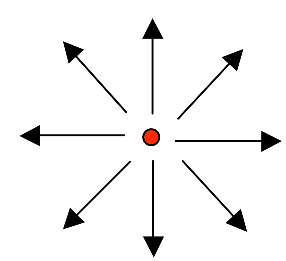
$$E_y = k\lambda\, a \int_{-L/2}^{L/2} \frac{dx}{(a^2+x^2)^{3/2}} = k\lambda\, a \left[\frac{x}{a^2\sqrt{x^2+a^2}} \right]_{-L/2}^{L/2}$$

$$= \frac{2k\lambda L}{a\sqrt{4a^2+L^2}}$$

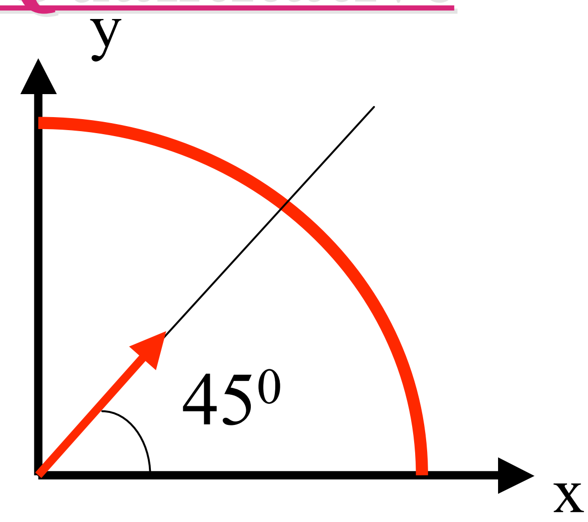
What is E very far away from the line (L<<a)?
What is E if the line is infinitely long (L >> a)?

$$E_y = \frac{2k\lambda L}{a\sqrt{L^2}} = \frac{2k\lambda}{a}$$



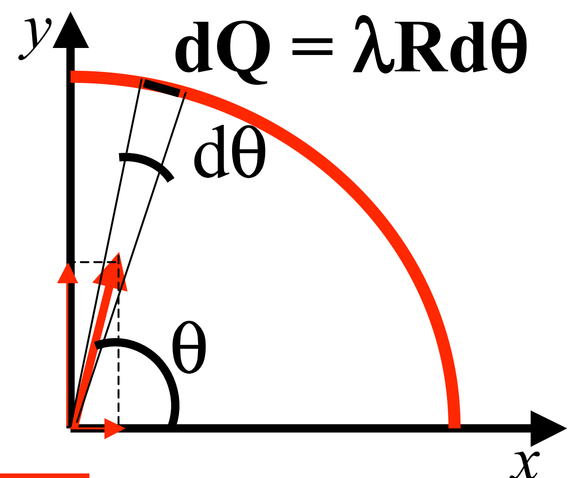


Example -- Arc of Charge: Quantitative



- Figure shows a uniformly charged rod of charge **–Q** bent into a circular arc of radius R, centered at (0,0).
- Compute the direction & magnitude of E at the origin.

$$dE_x = dE\,\cos\theta = \frac{kdQ}{R^2}\cos\theta$$



dQ = λRdθ

$$E_x = \int_0^{\pi/2} \frac{k(\lambda R d\theta)\cos\theta}{R^2} = \frac{k\lambda}{R}\int_0^{\pi/2}\cos\theta d\theta$$

$$\boxed{E_x = \frac{k\lambda}{R}} \quad \boxed{E_y = \frac{k\lambda}{R}} \quad \boxed{E = \sqrt{2}\frac{k\lambda}{R}}$$

λ=2Q/(πR)

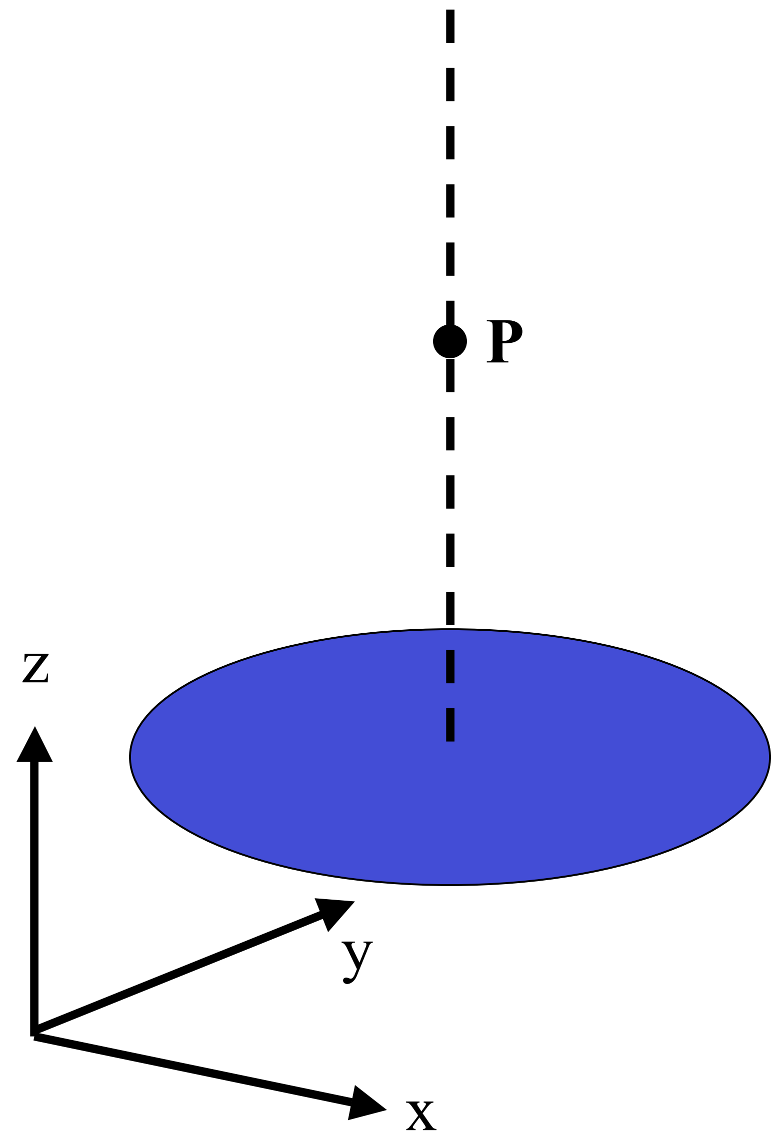
Example : Field on Axis of Charged Disk

- A uniformly charged circular disk (with positive charge)
- What is the direction of E at point P on the axis?

(a) Field is 0

(b) Along +z

(c) Somewhere in the x-y plane



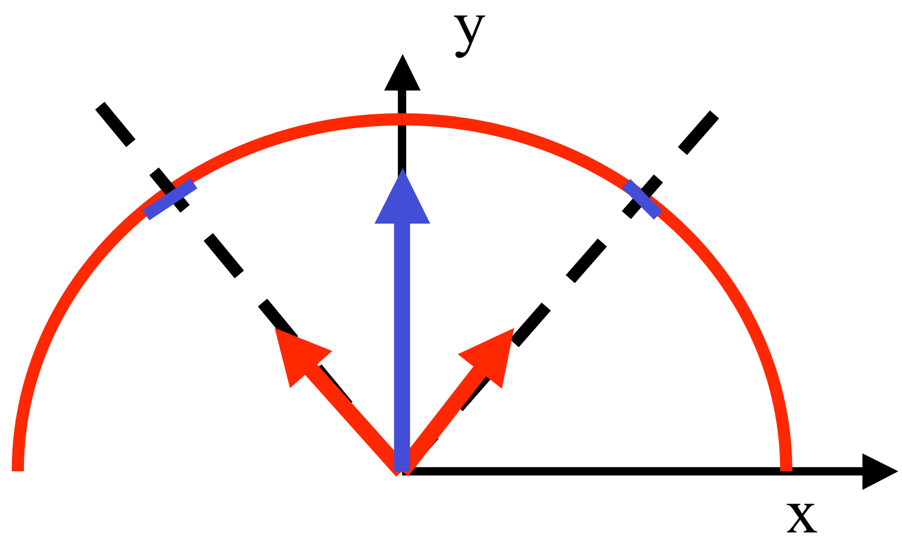
Example : Arc of Charge

- Figure shows a uniformly charged rod of charge **-Q** bent into a circular arc of radius R, centered at (0,0).
- What is the direction of the electric field at the origin?

(a) Field is 0.

(b) Along +y

(c) Along -y



- Choose symmetric elements
- x components cancel

Summary

- The electric field produced by a system of charges at any point in space is the force per unit charge they produce at that point.
- We can draw field lines to visualize the electric field produced by electric charges.
- Electric field of a point charge: $E=kq/r^2$
- Electric field of a dipole: $E \sim kp/r^3$
- An **electric dipole** in an electric field rotates to align itself with the field.
- Use CALCULUS to find E-field from a continuous charge distribution.