de Haas-van Alphen effect

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Abstract

Theory of the de Haas-van Alphen effect is briefly explained as well as its relation to the fermi surface. Two different techniques to measure it are discussed. Also we discussed how the fermi surface can be obtained from the knowledge of the periods of the quantum oscillations.
Introduction

The fermi surface is related to many important physical properties. Information about the transport coefficients can be obtained from a knowledge of the shape of the fermi surface. Also equilibrium and optical properties can be learned from it. It is also important when testing new band structures calculations. Therefore it becomes important to be able to measure experimentally properties of the fermi surface [1].

One of the techniques used the most in obtaining information about the fermi surface is based on the "de Haas-van Alphen" effect. The effect was discovered in 1930 by de Haas-van Alphen when he observed oscillation in M/H where M is the magnetization and H is the applied magnetic field to a sample of bismuth. Later Onsanger related the frequency of the oscillation to the cross sectional area of the fermi surface in a plane normal to the magnetig field. The main point is that the period of oscillation due to a change in 1/H where H is the magnetic field applied will be [1]:

$$\Delta(1/H) = 2\pi e/\hbar c A_e$$

where $A_e$ is any extremal cross-sectional area of normal to H. Therefore from the knowledge of the frequency of the quantum oscillation we can obtain information about the fermi surface and its shape. Many observation have been made of this oscillatory dependence in the susceptibility as well as in other physical quantities such as the conductivity. In fact the quantum oscillation are observed in many physical phenomenon.

These quantum oscillation can only be explained from a pure quantum mechanical point of view, and no classical or semiclassical theory would explain them. Basically the effect has its origin in the quantization of closed electronics orbits in a magnetic field [1] [5].

Observing the de Haas-van Alphen effect.

Torque Method

The de Haas-van Alphen effect has been observed using several tecniques, but there are two that have been widely exploited. One of them is based on the fact that a magnetized material in a magnetic field will experience a torque. This torque will be proportionnal to the magnetic moment itself. Therefore measuring the oscillation in angular position of a sample of a metal as a function of the magnetic field will give information about oscillation in the magnetic field. This method is widely discussed in Shoenberg and the idea is as follow: the torque on the crystal will be related to the cross sectional area of the fermi surface, the parallel component of the magnetization, and the angle specifying the direction of the magnetic field H. The relation is given by [2]:

$$T = (-1/F)dF/d\Theta M_\parallel HV$$

where $M_\parallel$ is the parallel component of the magnetization, $\Theta$ is the angle specifying the direction of the magnetic field and F is a quantitie related to the fermi surface in the
following way $F = (e\hbar/2\pi c)A_e$. This torque can be measured using simple equipment. In figure 1 we have diagram of the apparatus used. The torque acting on the sample is will twist the mirror slightly against the torsional restrain of the wire, therefore the reflected image of an illuminated slit formed by the lens $l$. There is damping to the vibration coming from the vanes $V$ in figure 1. Under favorable condition, typical values for the torque will be around $10$ dyne cm, and it turns out that this is easily measurable with a good precision, usually around $1$ percent and this performance can be considerably improved with more sophisticated equipment, usually novel methods of angle sensing has been used combined with others new techniques. Originally the torque method was essentially a low field since a vertical axis of suspension requires a horizontal magnetic field and this cannot usually exceed $3/10^9 G$, but with the development of superconducting magnets, rather higher fields have become possible. This is an important consideration since limitation to low fields implies that only small extremal areas of a complicated Fermi surface can be observed. It is important to notice that the actual performance depends very much on the electronic noise and drift and how it is reduced. Some data obtained using this method is shown in figure 2.

Modulation Techniques

The field modulation technique is a simple approach for dHvA studies. This technique measures the voltage induced in a pickup coil surrounding the sample when a field is applied. This technique is used for shorter periods where large fields are required, usually
Figure 2: Torque dHvA oscillations in Au at 1.2 K. The torque is plotted versus the inverse of the field

Figure 3: Apparatus used to measure the dHvA oscillation using the modulation technique of the order of 10 T at a temperature of 1 K. With this technique we can observe the effect satisfactorily for those short periods [3]. To obtain the high fields, superconducting solenoids are used creating a high field $H_o$. The samples are typically single-crystal wires in a small pickup coil. A periodic variation in the field is achieved by superimposing a small sinusoidal periodic field $H_1 \cos(\omega t)$ to the field $H_o$. Doing a taylor series about the average field, we can get an expression for the magnetization that will include derivatives of the magnetization with respect to the field evaluated at $H_0$.

$$M(t) = M(H_0) + H_1 \cos(\omega t)(dM/dH)_{H_o} + 1/2[H_1 \cos(\omega t)]^2(d^2 M/dH^2)_{H_o} + ...$$

Now the derivative of the magnetization with respect to time will give the emf induced in the pickup coil.

$$-dM/dt = +\omega H_1 (dM/dH)_{H_o} \sin(\omega t) + 1/2 \omega H_1^2 (d^2 H/dH^2)_{H_o} \sin(2\omega t) + ...$$

The oscillation of the magnetization associated with the de Haas-van Alphen effect imply a non zero value for the second derivative $d^2 H/dH^2$ in the Taylor expansion which
oscillates as a function of the applied field $H_0$ and is also the coefficient of the second harmonic in the Tailor expansion [3]. This is basically has information of the period as a function of the inverse of the applied field. Now if we employ a phase sensitive detection at frequency $2\omega$, (where $\omega$ is the natural frequency of the modulating field) we can observe the effect and reduce the signal to noise ratio. Usually the modulating amplitude is chosen to be compared with the period of the oscillation. Recently field of the order of 28 T, were produced using hybrid magnets [4]. The apparatus is shown in figure 3. An AC field of up to 30 mT is generated in the modulation coil using a high power amplifier. The time derivative of the magnetization is detected from the coils and the induced voltages are measured. The temperature for this experiment was of 4.4 K. In figure 4 we can see some of the results from this experiment.

Measuring the Fermi Surface.

Once the de Haas-van Alphen effect has been detected and the frequency has been measured, we can obtain information about the fermi surface using equation 1. When we change the magnetic field direction we can get different extremal areas, these extremal areas are mapped out. This will be enough to reconstruct the actual shape of the fermi surface. This is a complex task, in many cases it is easier to guess at what the surface is from some theoretical approximation to the band structure, and improving the guess when comparing it with the data. For instance for gold for quite a wide range of field directions Shoenberg observed that the magnetic moment has a period of 2 x10-19 gauss -1, and this correspond to a extremal area of 4.8x10-6 cm -2, and this value is in agreement with the experimental value of the fermi wave vector [5].

Conclusions.

We can see that besides being an interesting phenomenon by itself due to the fact that it is a pure quantum phenomenon, the de-Haas-van Alphen effect provides information in an effective way about the fermi surface and therefore about many important quantities of
the crystal. The techniques to measure it are in general simple and easy to set up and good precision can be obtained.

References