Problem set 8

1. The simplest many-body model of magnetism is the Ising model described by the Hamiltonian

$$H = J \sum_{<ij>} \sigma_i \sigma_j + h \sum_i \sigma_i \tag{1}$$

where σ_i can take on values of ± 1, and the symbol $< ij >$ denotes that spin i and spin j are nearest neighbors. It is a classical model of easy-axis magnetism. In one and two-dimensions it can be solved exactly.

Read the paper by K. Wilson [Scientific American, 241, 158 (Aug, 1979)], and then use with the xising and xrenor codes which are in the Chap8 directory to study the configurations of the system near the transition. Use xrenor to study the system for a few temperatures just above and just below T_c. Make sure that the system has achieved equilibrium before you draw any conclusions. Perform the experiments discussed in the xising.txt file (Michael Creutz developed the xising code). There is nothing to hand in for problem 1.

2. Mermin Wagner theorem. Derive a general expression for $S- < S_i^z >$ for a spin-S quantum antiferromagnet, in the quadratic spin-wave approximation. Examine this equation in one, two, and three dimensions for simple cubic lattices and small wavevectors $\mathbf{k}$. If $S- < S_i^z >$ diverges, then the spin-wave theory fails. For what dimensions and temperatures is this the case (i.e. consider the form of your equation for $T = 0$ and $T \neq 0$ for one, two and three dimensions. Hint, consider the sum over $\mathbf{k}$ for small $\mathbf{k}$)? Why does the theory fail in these cases (the reason has to do with the validity of the starting point, reconsider the results of problem one)? For the three-dimensional case, you may want to look at T. Oguchi, Phys. Rev. **117** 117, (1960); however, note that this author goes into significantly more detail than is required here.