Problem set 6, Due Friday, February 17

1 Calculate the specific heat (at constant density) and the linear magnetic suscetibility of a free elecron gas of constant density in the low temperature limit. For simplicity, assume that the electronic density of states is $g(\omega) = a\omega^{1/2}$, and keep only the first nonvanishing term in the low temperature expansion. *Explicitely* account for the temperature dependence of the chemical potential in each case. How good was the approximation, made in class, of ignoring the temperature dependence of μ? (The following (Somerfeld) expansion of the fermi function may be useful $f(\epsilon) \approx \theta(\mu - \epsilon) - \frac{\pi^2}{6}(k_B T)^2 \delta'(\epsilon - \mu)$.)

2 First Sound. Sound can propagate in a Fermi Liquid in two ways. If τ is the scattering lifetime ($1/\tau$ the scattering rate), then clearly when $\tau \approx 1/\omega$ no sound will propagate since the excitations are not sufficiently long lived. However, τ is a function of temperature. Thus, at high temperatures τ is small, and sound will propagate more-or-less normally. As the temperature is lowered, when $\tau \approx 1/\omega$, sound ceases to propagate. When it is lowered further, sound again begins to propagate. However, this type of sound called zero sound, may no longer be though of as a simple compression wave.

Consider sound propagation at high temperatures or low frequencies so that $\omega\tau \ll 1$. Then, the speed of sound is given by

$$c^2 = \frac{\partial P}{\partial \rho} = \frac{\partial P}{\partial \left(m\mathcal{N}/\mathcal{V}\right)}$$

This may be related to things we know better, using the Maxwell's relations. If the chemical potential is $\mathcal{F} = \mathcal{E} - T\mathcal{S}$, then $d\mathcal{F} = -Pd\mathcal{V} - \mathcal{S}dT + \mu d\mathcal{N}$. So, since μ and P are functions of $n = \mathcal{N}/\mathcal{V}$ only,

$$\frac{\partial \mu}{\partial \mathcal{N}} = -\frac{\mathcal{V}}{\mathcal{N}}\frac{\partial \mu}{\partial \mathcal{V}} = -\frac{\mathcal{V}}{\mathcal{N}}\frac{\partial}{\partial \mathcal{V}}\frac{\partial \mathcal{F}}{\partial \mathcal{N}} = \frac{\mathcal{V}}{\mathcal{N}}\frac{\partial P}{\partial \mathcal{N}} = \frac{1}{\mathcal{N}}\frac{\partial P}{\partial n}\,.$$

Thus,

$$c^2 = \frac{\mathcal{N}}{m}\frac{\partial \mu}{\partial \mathcal{N}}\,.$$

Use this (and perhaps the fact that $\delta\mathcal{N} = \frac{\mathcal{V}}{\pi^2 p_F^2}\delta p_F$) to find a general relationship between c^2 and the interaction parameters F_l^s in an isotropic homogenious Fermi liquid.

Perform a similar calculation to calculate the Spin susceptibility of a fermi gas to lowest order using the Landau Fermi Liquid theory.