This problem deals with the vibrations of the two-dimensional gas-atom surface. A monolayer of gas atoms is deposited on an atomically perfect surface. Consider first a model in which the effect of the surface is simply to constrain the atoms to move in the $z = 0$ plane. The atoms form a square lattice (with $a = 3\text{Å}$), and for small $\mathbf{k} = (k_x, k_y)$, the equations of motion give
\[-M\omega^2 e_i(\mathbf{k}) = -A(k_x^2 + k_y^2)e_i(\mathbf{k}) \quad i = x, y\]
where $M = 6.7 \times 10^{-23}\text{ gm}$, and $A = 6.7 \times 10^{-12}\text{ gnm}^2/\text{sec}^2$.

a. Find the normalized density of states (frequencies) per unit frequency near $\omega = 0$.

b. Give an expression for the low temperature specific heat as a function of temperature. What is the specific heat at 100K?

c. Is the Debye-Waller factor finite or zero and why?

Now account for the potential of the surface, i.e. allow for the corrugation of the surface by adding a potential energy
\[\phi = \frac{K}{2} \sum_{n,i} s_{ni}^2\]
where $s$ is, as usual, the displacement from equilibrium, and $K = 6.7 \times 10^4\text{ gm/sec}^2$. The wave solutions are of the form
\[s_{ni} = e_{ni}e^{(i\mathbf{k} \cdot \mathbf{r}_n - \omega t)}\]

d. What are the new frequencies for $\mathbf{k} = 0$?

e. What is the form of the temperature dependence of the specific heat neat $T = 0$?

f. Is the Debye-Waller factor finite or zero and why?