Faraday Rotation in the Interstellar Medium

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Abstract

Although wavelength and intensity of radiation are the most frequently used measures in astrophysics, some distant sources of light also have polarization properties that can be used to learn about their constitution. This polarized light is also affected by the interstellar medium, which can be treated as a rarefied electron gas, and the effect of the medium is dependent on the presence and intensity of magnetic fields, as well as upon the density of the medium itself. The presence of magnetic fields is indicated by Faraday rotation, a change in the polarization plane of a signal as it passes through a medium in the presence of a magnetic field. These measurements can easily be applied to determine the gross magnetic field of our galaxy.

The light from a majority of stellar sources is not polarized to any measurable degree. There are a few exceptions: light from "reddened stars," which has been scattered by grains of dust, often shows signs of polarization due to such scattering, and pulsar light, while not constant in its polarization, can be averaged over a sample of perhaps 100 pulses to give a stable mean polarization [1]. Pulsar light has the added advantage that each pulse contains light of many different frequencies which all originated at the same instant, so the spread of the pulse is due to the frequency-dependent conductivity of the interstellar medium, and thus can be used to determine its path length while in the plasma. For a plasma made up of ions and electrons, we can assume that only the electrons contribute to the conductivity, the ions being so massive as to be practically immobile. The result for an isotropic plasma is well-known,

$$\epsilon = 1 - (\frac{\omega^2}{\omega_p^2})$$

where ω_p is the plasma frequency, defined by

$$\omega_p^2 \equiv \frac{4\pi n e^2}{m}$$

Clearly if $\omega < \omega_p$ then ϵ is negative and the amplitude of the wave falls off exponentially, so ω_p is also called the *plasma cutoff frequency*, below which there is no propagation of the wave through the plasma.[2] Now consider propagation through such a plasma in the presence of an external magnetic field (B_0). The system is no longer isotropic, but has azimuthal symmetry around the direction of the magnetic field lines. Any plane-polarized wave can be decomposed into a component parallel to the magnetic field and a component perpendicular to the field. Assuming (B_0) is large compared to the magnetic field of the wave, the equation of motion for an electron in the plasma will be

$$m(d\mathbf{v}/dt) = -e\mathbf{E} - (e/c)\mathbf{v} \times \mathbf{B_0}$$

(1)

For a circularly polarized incoming wave, we know

$$\mathbf{E}(t) = Ee^{-i\omega t}(\epsilon_1 \epsilon_2)$$

Defining the cyclotron frequency $\omega_B \equiv \frac{eB_{0,parallel}}{mc}$ and solving for the steady-state velocity of the electron, we find

$$\mathbf{v}(t) = \frac{-ie}{m(\omega \pm \omega_B)}\mathbf{E}(t) \tag{2}$$

which gives

$$\epsilon_R = 1 - \frac{\omega_p^2}{\omega(\omega + \omega_B)} \tag{3}$$

for a right-handed polarization, and

$$\epsilon_L = 1 - \frac{\omega_p^2}{\omega(\omega - \omega_B)} \tag{4}$$

for a left-handed polarization.

A plane-polarized wave can be treated as the superposition of a left-hand and a right-hand circularly polarized wave of equal weight. As a result of the differential in speed between the different left- and right-hand modes, the plane of polarization will rotate as it passes through the plasma. This is called FARADAY ROTATION. In general, the angle through which the electric vector of a circularly polarized wave proceeds is given by

$$\phi = \int_0^d k ds \tag{5}$$

where k is the wave number,

$$k = \frac{\omega}{c}$$

Assuming that $\omega \gg \omega_p$ and $\omega \gg \omega_B$, we can expand k as

$$k \approx \frac{\omega}{c}[1 - \frac{\omega_p^2}{2\omega^2}(1 + \frac{\omega_B}{\omega})] \tag{6}$$

Substituting the known expressions for ω_p^2 and ω_b, we obtain for the Faraday rotation

$$\Delta\theta = \frac{2\pi e^3}{m^2 c^2 \omega^2} \int_0^d n B_{parallel} ds \tag{7}$$

Since $\Delta\theta$ varies with frequency in a known manner, measurements made at different frequencies can be combined to give a value for the integral.[2]

One useful astrophysical application of Faraday rotation is the determination of the magnetic field of the galaxy using extragalactic pulsars. The dispersion relation given in equation (1) leads to an expression for the travel time for a pulse of a given frequency through a path length L of plasma

$$t(\nu) = \frac{L}{c} + \frac{e^2}{2\pi mc} \times \frac{D_m}{\nu^2}$$

where D_m is the "dispersion measure",

$$D_m = \int_0^L n ds$$

D_m can be determined for pulsars based on the frequency spread of what was originally a simultaneous pulse. Then $B_{parallel}$ is directly proportional to the ratio of $\Delta\theta$ to D_m, and the constant of proportionality is known.

[1]

The following plot was constructed from a study of 26 pulsars; the solid line is the least squares fit, which yields a net value of $(2.2 \pm 0.4) \times 10^{-6}$ gauss for the galactic magnetic field.

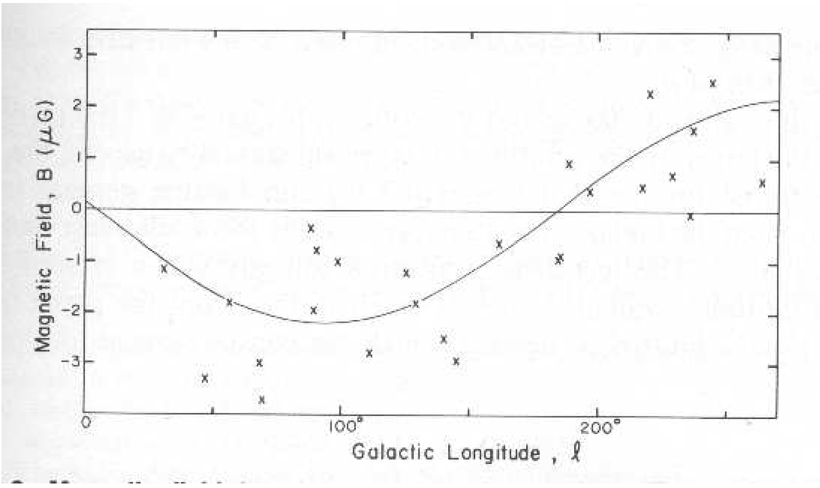

Figure 1: *Galactic Magnetic field as a function of Galactic Longitude*

References

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