The vortex transport properties of the superconducting films

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Abstract

Superconductors, materials that have no resistance to the flow of electricity, are one of the last great frontiers of scientific discovery. Not only have the limits of superconductivity not yet been reached, but the theories that explain superconductor behavior seem to be constantly under review. The explanations of the energy losses in the different practical applications of high temperature superconductors take significant part in modern investigation of transportation phenomena in the superconducting films. This work considers the movement of the vortices in superconductors which produces a pseudo-resistance and the undesirable energy losses.

1 History of superconductivity

In 1911, superconductivity was first observed in mercury by Dutch physicist Heike Kamerlingh Onnes of Leiden University. When he cooled it to the temperature of liquid helium, 4 degrees Kelvin, its resistance suddenly disappeared. The Kelvin scale represents an "absolute" scale of temperature. Thus, it was necessary for Onnes to come within 4 degrees of the coldest temperature that is theoretically attainable to witness the phenomenon of superconductivity. The next great milestone in understanding how matter behaves at extreme cold temperatures occurred in 1933. Walter Meissner and Robert Ochsenfeld discovered that a superconducting material will repel a magnetic field. A magnet moving by a conductor induces currents in the conductor. This is the principle upon which the electric generator operates. But, in a superconductor the induced currents exactly mirror the field that would have otherwise penetrated the superconducting material - causing the magnet to be repulsed. This phenomenon is known as diamagnetism and is today often referred to as the "*Meissner effect*". The Meissner effect is so strong that a magnet can actually be levitated over a superconductive material. In subsequent decades other superconducting metals, alloys and compounds were discovered. In 1941 Niobium-Nitride was found to superconduct at 16 K. In 1953 Vanadium-Silicon displayed superconductive properties at 17.5 K. And, in 1962 scientists at Westinghouse developed the first commercial superconducting wire, an alloy of Niobium and Titanium. The first use of this wire in high-energy, particle-accelerator electromagnets, however, did not come until 1987 when it was employed at the Fermilab Tevatron.

2 Type I and Type II superconductors

The Type I category of superconductors is mainly comprised of pure metals that normally show some conductivity at room temperature. They require incredible cold to slow down molecular vibrations sufficiently to facilitate unimpeded electron flow in accordance with what is known as BCS theory. BCS theory suggests that electrons team up in "Cooper pairs" in order to help each other overcome molecular obstacles - much like race cars on a track drafting each other in order to go faster. Type I superconductors - also known as the "soft" superconductors - were discovered first and require the coldest temperatures to become superconductive. They are characterized by a very sharp transition to a superconducting state and by "perfect" diamagnetism - the ability to repel a magnetic field. Below is a list of known Type I superconductors along with the critical transition temperature - known as T_c - below which each of them aresuperconducts.Ironically,*copper*,*silver* and *gold*, three of the best metallic conductors, do not rank among the superconductive elements (Fig.1). Other elements can be coaxed into a superconductive state by the application of high pressure. For example, the Type I element with the highest known T_c is sulfur at 17 K. But, it requires compression pressures of 930,000 atmospheres to become superconductive and 1.6 million atmospheres to reach a T_c of 17 K. Except for the elements vanadium, technetium and niobium, the Type II category of superconductors is comprised of metallic compounds and alloys. The recently-discovered superconducting "perovskites" (metal-oxide ceramics that normally have a ratio of 2 metal atoms to every 3 oxygen atoms) belong to this Type II group. They achieve higher T_c's than Type I superconductors by a mechanism that is still not completely understood. Conventional wisdom holds that it

Lead (Pb)	7.2K
Lanthanum (La)	4.9 K
Tantalum (Ta)	4.47 K
Mercury (Hg)	4.15 K (First superconductor discovered)
Tin (Sn)	3.72 K
Indium (In)	3.40 K
Gadolinium (Gd)	1.083 K(ferromagnetic above T_c; diamagnetic below Tc)

Figure 1: *Type I superconductors and their critical temperatures.*

relates to the planar layering within the crystalline structure. Although, other recent research suggests the holes of hypocharged oxygen in the charge reservoirs are responsible. The superconducting cuprates (copper-oxides) have achieved astonishingly high T_c's when you consider that by 1985 known T_c's had only reached 23 K. To date, the highest T_c attained at ambient pressure has been 138 K. One theory (V. Kresin, et al) predicts an upper limit of about 200 K for the layered cuprates. Others assert there is no limit. Either way, it is almost certain that other, more-synergistic compounds still await discovery among the high-temperature superconductors. The first superconducting Type II compound, an alloy of lead and bismuth, was fabricated in 1930 by W. de Haas and J. Voogd. But, was not recognized as such until much later, after the Meissner effect had been discovered. Type II superconductors - also known as the "hard" superconductors - differ from Type I in that their transition from a normal to a superconducting state is gradual across a region of "mixed state" behavior. A Type II will also allow some penetration by an external magnetic field into its surface. A Type I will not. While there are far too many to list in totality, some of the more interesting Type II superconductors are listed below by similarity and with descending T_c's.

$Hg_{0.8}Tl_{0.2}Ba_2Ca_2Cu_3O_{8.33}$	138 K (record-holder)
$HgBa_2Ca_2Cu_3O_8$	133-135 K
$HgBa_2Ca_{1-x}Sr_xCu_2O_{6+}$	123-124 K
$HgBa_2CuO_{4+}$	94-98 K

Figure 2: *Type II superconductors and their critical temperatures.*

3 Evolution of the theory of superconductivity

The first phenomenological theory of superconductivity was proposed by brothers Fritz and Heintz London in 1935 [2]. It is based on empirical facts and not deduced from any microscopic mechanisms within the material. The equation of state according to that theory are:

$$\vec{J} = -\frac{1}{\mu \cdot \lambda_L^2} \cdot \vec{A} \tag{1}$$

- London equation,

$$\nabla \times \vec{J} = -\frac{1}{\mu \cdot \lambda_L^2} \cdot \vec{B}, \tag{2}$$

$$\nabla \cdot \vec{B} = \mu_0 \cdot \vec{J}, \tag{3}$$

$$\nabla^2 \cdot \vec{B} = -\mu_0 \nabla \times \vec{J}, \tag{4}$$

These equations along with $J = nqv$, $v = m/(p - \frac{q}{cA})$ - velocity of superelectrons;n-concentration of superelectrons; and $\vec{J} = -(nq^2/m) \cdot \vec{A}$, $q = -2e$ for the pair electrons, where $\lambda_L^2 = m/\mu_0 nq^2$ - London penetration depth , give the distribution of the magnetic field at the surface of superconductor:

$$B(x) = B(0)e^{-\frac{x}{\lambda_L}}, \tag{5}$$

which is presented on Fig.3. This theory allows us to calculate a superconducting systems

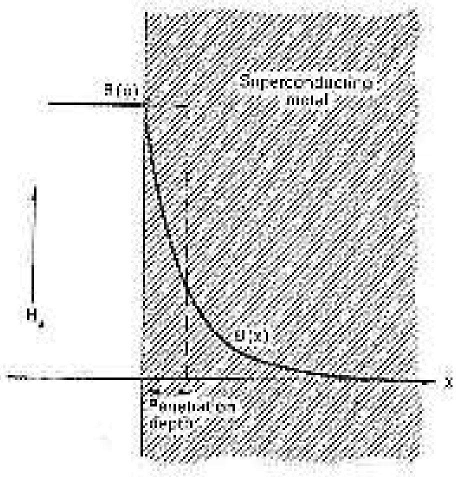

Figure 3: *Distribution of the magnetic field at the surface of superconductor.*

relevant parameters [1]. But this theory fails to account for many other properties associated with superconductivity.
The explanation of the absence of the energy losses in superconductors led to the idea of quantization of the magnetic field flux. Quantization of magnetic field in superconductors and the quantum mechanical nature of superconductivity was realized in 1948. In

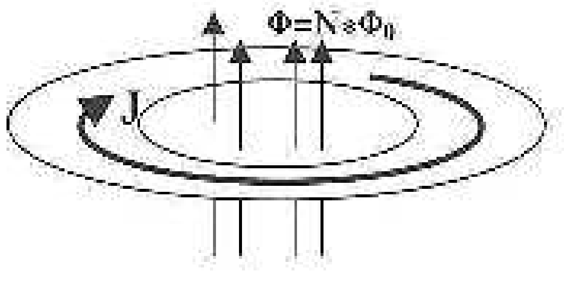

Figure 4: *Magnetic fluxoid* - $\Phi_0 = hc/2e = 2.07 \cdot 10^{-15} Tm^2$.

1950 Vitaly Ginzburg and Lev Landau gave the successful theoretical description of the

phenomena of superconductivity that integrates the electrodynamics, quantum mechanics and thermodynamics. Nevertheless, it doesnt explain how the superconducting state forms from the binding of electrons into *superelectrons* (Cooper pairs).This theory describes the superconducting state by a quantum mechanical wavefunction.
The first widely-accepted theoretical understanding of superconductivity was advanced in 1957 by American physicists John Bardeen, Leon Cooper, and John Schrieffer. Their *Theories of Superconductivity* became know as the BCS theory - derived from the first letter of each man's last name - and won them a Nobel prize in 1972. The mathematically-complex BCS theory explained superconductivity at temperatures close to absolute zero for elements and simple alloys. However, at higher temperatures and with different superconductor systems, the BCS theory has subsequently become inadequate to fully explain how superconductivity is occurring.

4 Vortices and theory of superconductivity

The first superconductors discovered belong to a class known as type I superconductors. At applied magnetic field strength less than $H_c(T)$, a type I superconductor will expel magnetic flux from the material, and hence is in the "Meissner state" (Fig.5.). In contrast, a material such as Pb-Bi is the type II superconductor. Its behavior can be characterized by two critical fields for these materials. For applied field strength less than the lower critical field H_{c1}, the superconductor will exibit the usual the "Meissner effect". Applied fields greater than the upper critical H_{c2} destroy the superconductivity altogether. For the

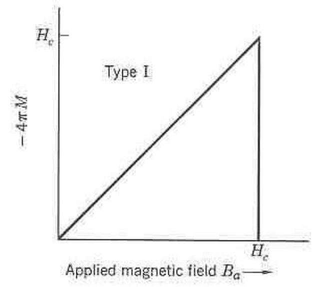

Figure 5: *Magnetization of type I superconductor.*

range $H_{c1} \leq H \leq H_{c2}$ the superconductor is in the mixed (or vortex) state (Fig.6.). Type

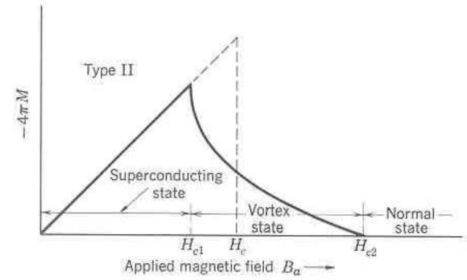

Figure 6: *Magnetization of type II superconductor.*

II superconductors are most often used for applications since they remain super conducting in much higher magnetic fields and at higher temperatures than type I superconductors. Typically, type I superconductors will be superconducting up to critical fields of less than 0.01-0.1 T and they will have critical temperatures less than 9 K. In contrast, the type II superconductors Nb_3Sn remains superconducting to up to critical field $\mu_0 H_{c2}$ of nearly 30 T, and has T_c of about 18 K. Moreover, the high temperature superconductors are type II materials and some are estimated to remain superconducting in flux densities exceeding 100 T. In 1957 Alexei Abrikosov formulated a successful theoretical explanation for these two classes of magnetic materials. The entering magnetic field is observed in the discrete array of entities known as vortices. These vortices are a manifestation of the fluxiod quantization. Each vortex has a single flux quanta Φ_0 associated with it (the magnetic flux quantum is $\Phi_0 = 2.07 \cdot 10^{-15} Tm^2$). Fields and currents associated with vortices will be found later. Except of this, each vortex can be characterized by its radius or the *coherence length* ξ . In fact , two types of superconductors can be distinguished by comparing λ_L and ξ [3].Type I superconductors have $\lambda_L \leq \xi$ while type II superconductors have $\lambda_L \geq \xi$. We shall see that magnetic fields and currents from a vortex are more conveniently described by modifying the second London equation. Now I am going to pay more attention to the structure of the sole vortex . Suppose we have a slab with a type II superconductor of thickness $a \gg \lambda_L$. The applied field is perpendicular to the surface of slab as shown in the figure 7:

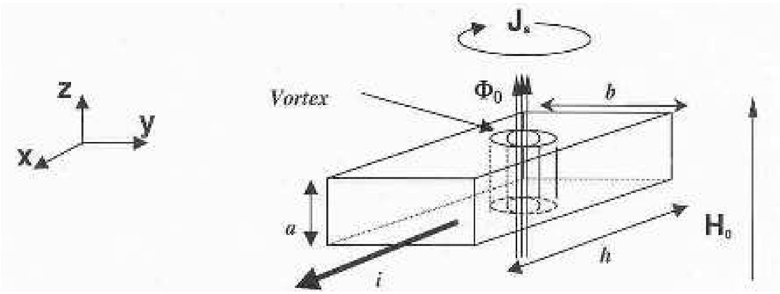

Figure 7: *Vortex in the superconducting slab.*

The current density around the core of vortex $\vec{J_s} = Js \cdot \vec{\varphi}$, where J_s - the current density of the vortex. In the limiting case when r approaches to zero:

$$\lim_{r \to 0} \vec{J_s} = \frac{\Phi_0}{2\pi\mu_0\lambda_L^2 r} \cdot \vec{\varphi}$$

Because the current density has only φ -component and inversely proportional to the radius according to the above formula, it would increase without bounds as the center of the vortex is approached. Such a large current is certainly not physical because it demands an unbounded amount of energy. In order to overcome this problem the model of vortex with the normal core of thickness (radius) ξ should be chosen. The supercurent thus flows in around the core in the superconducting region and not in the normal region of the core. The maximum the current density is :

$$\vec{J_s} = \frac{\Phi_0}{2\pi\mu_0\lambda_L^2 \xi} \cdot \vec{\varphi}. \tag{6}$$

The vortex current encircling a normal core interacts with the magnetic field produced by the vortex current encircling any other core and as a result, any two cores repel each other. Because of this mutual interaction the cores threding a superconductor in the mixed state

do not lie at random but arrange themseves into a regular periodic hexagonal array as shown on the figure 8.There are special techniques for the direct observation of this surfase hexagonal structure of vortices and their demensions [6,7].
Lets now consider a superconducting specimen of the length h , cross-sectional area $A = a \times b$ which carries a current i in the applaied magnetic field B which is at the angle θ to the direction of the current. Due to the metal is in the mixed state, it is threaded by the magnetic flux associated with the normal cores (Fig.7,8). There will be an electromagnetic

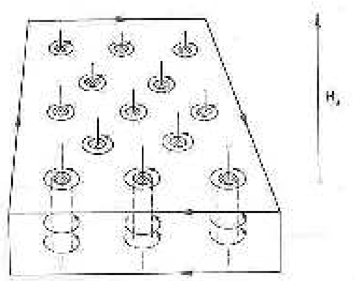

Figure 8: *Supercurrents encircle the normal cores.*

force (Lorentz) between this flux and the current. If the flux density is B electromagnetic force (Lorentz) on the specimen is:

$$F_L = h \cdot i \cdot B \cdot sin\theta. \tag{7}$$

However, since each vortex encloses an amount of flux Φ_0 , the mean flux density is:

$$B = n_\nu \cdot \Phi_0 \tag{8}$$

where n_ν - the number of vortices per unit area perpendicular to B.So the resulting force acting on a single vortex from the magnetic field f_L is :

$$f_L = J \times \Phi_0, \tag{9}$$

where J - the average current density equal i/A , A - the cross section area of the specimen. The vortices tend to be pinned at the imperfection in the material. Consequently,if the Lorentz force f_L is not too great,the vortex remains stationary and doesn't move under its action. If the transport current density J is enough to exceed the average pinning force f_P the core lattice will move.

5 Dynamics of the vortices and energy losses

We now consider what motion the vortices take up when the transport current is raised above the critical value j_p, so that the vortices become detached from the pinning centers. In this case, as follows from the experiments, the electrical resistivity doesnt depend on the purity of the superconductor, so to each material we can ascribe a "viscosity constant" η such that, if the net force $\vec{f} = \vec{f_L} - \vec{f}_{drag}$ acting on a core, the core will acquire a velocity:

$$\vec{u} = \vec{f}/\eta \tag{10}$$

The sum of Lorentzlike force due to current and drag force will cause the vortex to move with some velocity u_y in y direction (Fig.7). Experiments reveal that for a constant driving current $i = j \cdot A$ the velocity is constant. According to *Faraday*'s law , some electromotive force is expected across the sample:

$$V = \frac{d\Phi}{dt} = \Phi_0 n_v h u_y, \tag{11}$$

where the n_v concentration of vortices per unit area. There must be power dissipated P_{dis} in superconductor due to the work done against the drag force:

$$P_{dis} = i\Phi_0 n_v h u_y. \tag{12}$$

The dissipation of energy can be calculated as the dissipation in each normal core of vortex [4] that is given by:

$$P_{dis}^{core} = \int_{core} E_{in} \times J_{in}(r) dv. \tag{13}$$

Both the electric field and current density are independent of position inside the core so the direct integration gives:

$$P_{dis}^{core} = \frac{u_y^2 \Phi_0^2 \sigma_0}{4\pi\xi^2} a. \tag{14}$$

$4\pi\xi_2 a$ the volume of the core,σ_0 - conductivity of the normal core. The total power dissipation in the slab is the number of vortices in the slab $(n_v bh)$ times the power dissipation in a single vortex:

$$P_{dis} = P_{dis}^{core} \cdot n_v bh = \frac{u_y^2 \Phi_0^2 \sigma_0}{4\pi\xi^2} a. \tag{15}$$

This power dissipation must be the same that was obtained from Faradays law. The equivalence of both expressions gives that the vortices move with velocity:

$$u_y = \frac{2\pi\xi^2}{\Phi_0 \sigma_0} \frac{1}{2ab} i. \tag{16}$$

So the velocity, and, in turn, dissipation in type II superconductors depends on the uniformly flowing current i. Practically, a type II superconductor in an applied field with moving vortices is useless (even comparing with the ordinary copper at low temperatures) as practical conductor. Its resistivity is 1000 times bigger then that of copper. If the vortices could be prevented from moving, it could eliminate dissipation significantly.

6 Conclusions

The movement of vortices can produce a pseudo-resistance and, as such, is undesirable. To be of practical value for conductors in magnets, type II superconductors must be made in such a manner that the vortices are prevented from moving. This prevention of the vortex motion is known as pinning the vortices.

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