Magneto-acoustic Effects in Stellar Winds

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Abstract

Here we treat the outflowing material from the stars as a highly conductive plasma in order to derive relevant equations governing the wind behaviour. An estimation , based on above approach, has been made to determine the mass loss rate of a red giant star.

1 Introduction

Stars emit not only radiation, but also particles. The emission of particles is called the stellar wind. The two most important parrameters regarding a stellar wind that can be derived from observations are mass loss rate dM/dt, which is the amount of mass lost by the star per unit time, and the terminal velosity v_{term}, which is the velosity of the stellar wind at a large distance from the star. By convention the mass loss rate dM/dt is expressed in units of sollar masses per year.

$$M_{\odot} yr^{-1} = 6.303 \times 10^{25} g s^{-1} \tag{1}$$

A star with $dM/dt = 10^{-6} M_{\odot} yr^{-1}$ which is not unusual value loses an amount equal to the total mass of the earth in three years. The terminal velosity v_{term} of a stellar wind ranges typically from 10 km/s for a cool supergiant star to 3000 km/s for a luminous hot star. If we assume that the winds are stationary and spherically symmetric then the equation of continuity yields:

$$dM/dt = 4\pi r^2 \rho(r) v(r) \tag{2}$$

In the above equation r is the distance from the center of the star, ρ is the density and v is the velocity.

The values of dM/dt and v_{term} are important because:

(1)dM/dt describes how much material is lost by the star per unit time. This is important for the evolution of the stars because stars with high mass loss rates will evolve differently from those with low mass loss rates.

(2)The gas that escapes from the star into space carries kinetic energy into the interstellar medium. The amount of kinetic energy that a stellar wind deposits into the interstellar medium per unit time is

$$E = \frac{1}{2} v_{term}^2 \frac{dM}{dt} \tag{3}$$

In order to study the effect of a stellar wind on its surrounding interstellar material E has to be known.

(3)Different stellar models predict different mass loss rates and different terminal velosities for a star. By comparing the observed values with the predictions we can learn which mechanism is responsible for the mass loss from a star.

Another important thing is that the stellar wind is treated usually as a high conductive plasma which has significant consequences.

2 Alfven Model

In hot stars the stellar winds are driven from the surface by the radiation pressure of the star. However in order to make the picture more realistic we have to concider the effect of the magnetic field. Moreover some stars(red giants) have wind speeds of less than about 30km/s, which are well below the escape speed from the surface of the star:

$$v_{esc} = \sqrt{\frac{2GM_*}{r_*}} \tag{4}$$

In order to give an explanation to this paradox Hartmann and Mac Gregor (1980) proposed that the winds are driven by Alfven waves, and the terminal speed is reduced by wave damping [1].

Now we shall treat the wind as a highly conductive fluid in order to study the propagation of Alfven waves. According to the Ohm's law we have:

$$\vec{j} = \sigma(\vec{E} + \frac{\vec{v}}{c} \times \vec{B}) \tag{5}$$

or

$$\vec{E} = \eta\vec{j} - \frac{1}{c}\vec{v} \times \vec{B} \tag{6}$$

Combining (6) with the Maxwell equations we can find an equation for the temporal behaviour of $\vec{B}$:

$$\nabla \times \vec{E} = -\frac{1}{c}\frac{\partial \vec{B}}{\partial t} \tag{7}$$

$$\frac{\partial \vec{B}}{\partial t} = -\nabla \times c\eta\vec{j} + \nabla \times (\vec{v} \times \vec{B}) \tag{8}$$

$$\frac{\partial \vec{B}}{\partial t} = \nabla \times (\vec{v} \times \vec{B}) - \frac{c}{4\pi}\nabla \times (\eta\nabla \times \vec{B}) \tag{9}$$

$$\frac{\partial \vec{B}}{\partial t} = \nabla \times (\vec{v} \times \vec{B}) - \frac{c}{4\pi}\nabla\eta \times (\nabla \times \vec{B}) + \frac{c\eta}{4\pi}\nabla^2\vec{B} \tag{10}$$

The second term on the RHS represents the effect of the spatial variation of resistivity on the currents generated by magnetic field. For simplicity we will now assume that the resistivity is uniform so that term disappear.

$$\frac{\partial \vec{B}}{\partial t} = \nabla \times (\vec{v} \times \vec{B}) + \frac{c\eta}{4\pi}\nabla^2\vec{B} \tag{11}$$

The first term on the RHS in the last equation is called the convective term and will be seen to represent the tendency of magnetic field to be 'frozen' in the fluid. The second term is diffusive and represents the resistive leakage of magnetic field lines across the conductive fluid.

When the conductivity is infinite, η tends to zero and therefore

$$\frac{\partial \vec{B}}{\partial t} = \nabla \times (\vec{v} \times \vec{B}) \tag{12}$$

Using $\nabla.\vec{B} = 0$ we get

$$\frac{d\vec{B}}{dt} = (\vec{B}.\nabla)\vec{v} - (\nabla.\vec{v})\vec{B} \tag{13}$$

From continuity equation we get:

$$\frac{d\vec{B}}{dt} = (\vec{B}.\nabla)\vec{v} + \frac{\vec{B}}{\rho}\frac{d\vec{B}}{d\rho} \tag{14}$$

$$\frac{d}{dt}\frac{\vec{B}}{\rho} = (\frac{\vec{B}}{\rho}.\nabla)\vec{v} \tag{15}$$

Let us now concider the evolution of a small vector $\delta\vec{x}$ connecting two neighbouring points in the fluid, as the fluid moves with the prescribed velocity field. The point initially at position $\vec{x}$ at time t will be displaced to the position $\vec{x}+v(\vec{x})\Delta t$ at time $t+\Delta t$. The point initially at $\vec{x}+\delta\vec{x}$ at time t will be displaced to the position $\vec{x}+\delta\vec{x}+\vec{v}(\vec{x}+\delta\vec{x})\Delta t$ at time $t+\Delta t$.Hence we find that

$$\frac{d}{dt}(\delta\vec{x}) = (\delta\vec{x}.\nabla)\vec{v} \tag{16}$$

We see that $\frac{\vec{B}}{\rho}$ and $\delta\vec{x}$ satisfy the same equations for all time. Hence if initially $\delta\vec{x} = \rho^{-1}\vec{B}$ this same relation will hold for all time. This justifies the concept of the frozen field. Whenever the plasma starts to move, the magnetic field will be 'frozen' and follow the motion of plasma [2,3].

We can conclude that in a perfectly conducting fluid the plasma particles behave as if they were tied to the magnetic field lines. So the field lines bahave as if they were mass-loaded elastic strings under tension. Thus by analogy with the transverse vibrations of elastic strings, we expect that whenever the conducting fluid is slightly disturbed from the equilibrium conditions, the magnetic field lines will perform transverse vibrations. The speed of propagation of these transverse vibrations are expected to be given by $v_A = (\frac{tension}{density})^{1/2}$ which is known to be the Alfven speed.

The apropriate equations needed to describe the wave motion of a conducting fluid in the presence of magnetic field are:

$$\frac{\partial\rho}{\partial t} + \nabla.(\rho\vec{v}) = 0 \tag{17}$$

$$\rho\frac{\partial\vec{v}}{\partial t} + \rho(\vec{v}.\nabla)\vec{v} = -\nabla\rho - \frac{1}{4\pi}\vec{B}\times(\nabla\times\vec{B}) \tag{18}$$

$$\frac{\partial\vec{B}}{\partial t} = \nabla\times(\vec{v}\times\vec{B}) \tag{19}$$

The two new equations are continuity equation (17) and the force equation (18). We shall assume that equilibrium velocity is zero. In order to deduce dispersion relation for small amplitude waves let us consider small amplitude departures from the equilibrium values.

$$\vec{B}(\vec{x},t) = \vec{B}_0 + \vec{B}_1(\vec{x},t) \tag{20}$$

$$\rho(\vec{x},t) = \rho_0 + \rho_1(\vec{x},t) \tag{21}$$

$$\vec{u}(\vec{x},t) = \vec{v_1}(\vec{x},t) \tag{22}$$

$$\frac{\partial^2 \vec{v}}{\partial t^2} - s^2\nabla(\nabla.\vec{v_1}) + \vec{v_A}\times\nabla\times[\nabla\times(\vec{v_1}\times\vec{v_A})] \tag{23}$$

where $s = \sqrt{\frac{\partial p}{\partial \rho}}$is the speed of sound and $v_A = \frac{B_0}{\sqrt{4\pi\rho_0}}$ is the Alfven speed. Assuming $\vec{v_1}(\vec{x},t)$ to be a plane wave

$$\vec{v_1}(\vec{x},t) = \vec{v_1}e^{i(\vec{k}.\vec{x}-\omega t)} \tag{24}$$

equation (23) becomes:

$$-\omega^2\vec{v_1} + (s^2+\vec{v_A}^2)(\vec{k}.\vec{v_1})\vec{k} + \vec{v_A}.\vec{k}[(\vec{v_A}.\vec{k})\vec{v_1} - (\vec{v_A}.\vec{v_1})\vec{k} - (\vec{k}.\vec{v_1})\vec{v_A}] = 0 \tag{25}$$

If $\vec{k}$ is perpendicular to $\vec{v_A}$ last term vanishes and we have a longitudinal magnetosonic wave with phase velocity $u_{long} = \sqrt{s^2+v_A^2}$.

If $\vec{k}$ is paralel to $\vec{v_A}$

$$(\vec{k}^2.\vec{v_A}^2 - \omega^2)\vec{v_1}^2 + (\frac{s^2}{v_A{}^2} - 1)\vec{k}^2(\vec{v_A}.\vec{v_1})\vec{v_A} = 0 \tag{26}$$

In this case there are two modes: a longitudinal wave with phase velocity equal to the sound velocity s and a transverse wave ($\vec{v_1}.\vec{v_A} = 0$) with phase velocity equal to v_A. The magnetic field for the last mode is $\vec{B_1} = -\frac{k}{\omega}B_0\vec{v_1}$

If the conductivity of the fluid is not infinite we expect to have damping of oscillations. For the case of interest when we have Alfven wave parallel to the field the dispersion relation becomes:

$$k^2 v_A{}^2 = \omega^2(1 + i\frac{c^2k^2}{4\pi\sigma\omega})(1 + i\frac{k^2\chi}{\rho_0\omega}) \tag{27}$$

where χ is the fluid viscosity In order to simplify the result we assume the correction terms to be small. In this limit

$$k = \frac{\omega}{v_A} + \frac{i(\xi_m+\xi_k)\omega^2}{2v_A{}^3} \tag{28}$$

where ξ_m and ξ_k represent the magnetic and kinetic viscosities correspondingly. The positive imaginary part in this expression for $k(\omega)$ implies wave damping. The amplitude of the wave is exponentially decreasing. We see that the attenuation increases rapidlywith frecuencyand decreases with increasing the magnetic field intensity.

An expression allowing numerical estimation of $\frac{dM}{dt}$ based on the above model has been given in [1]. The mass loss rate of a typical Alfven wave driven wind is:

$$\frac{dM}{dt} = 9.1\times10^{-17}\frac{B_0^4 R_*^{7/2}}{M_*^{3/2}10^{13}\rho_0}M_\odot yr^{-1} \tag{29}$$

with ρ_0 in units gcm^{-1}.

For a red giant star $R = 300R_{\odot}$ and $B_0 = 1 \times 10^{-5} Gauss$, $M_* = 1M_{\odot}$. The latter yelds:

$$\frac{dM}{dt} = 8 \times 10^{-7} M_{\odot} yr^{-1} \tag{30}$$

The above result matches perfectly with the observed value of the mass loss rate of a red giant star. Curiously to say, this estimation is of the same order as the rate of mass loss in a wind driven purely by a radiation pressure gradient. But unlike the radiation-driven wind theories, the model that equation (29) is based on explains adequately the low speeds of the red giants winds.

3 Conclusion

The Alfven wave driving wind mechanism discussed here and the radiation driving mechanism are the most commonly occuring classes of wind mechanisms across the Hertzsprung-Russell diagram. Alfven waves are a valuable addition to the list of mass loss mechanisms, because the waves can produce a significant mass loss even in stars which have neither hot coronal zones nor strong radiative fluxes. Consequently, Alfven waves have been considered as a possible cause for the mass loss in every portion of the HR diagram, where the radiative driving mechanism is ineffective [1]. As it was proposed by Hartmann and Macgregor, the winds from the red giants are driven by Alfven waves and the wave damping due to the finite conductivity of the plasma is responsible for the reduced terminal speed of the wind.

References

[1] Henry J.G.L.M. Lamers, Joseph P. Cassimelli,1999, Introduction to Stellar Winds

[2] Peter A. Sturrock,1994,Plasma Physics

[3] J. D. Jackson,1975, Classical electrodynamics