Homework set for Chapter 8

September 28, 2001

1.Determine the allowed TM and TE modes in a rectangular ($0 < x < a$, $0 < y < b$) wave guide with walls of perfect conductivity.

a In each case find the eigenvalue equation and the lowest allowed frequency. Is the fundamental mode of the guide a TM or TE mode?

b From your equations determine if the TE_{01} mode would be disturbed by narrow slits cut into the waveguide walls at $y = 0, b$ parallel to (1) the x-axis and (2) the z-axis (hint consider the surface current).

c Calculate the time averaged power flow of the TE_{01} mode along the waveguide

2. A lossless dielectric with parameters μ and ϵ fills a rectangular cavity ($0 < x < a$, $0 < y < b$, $0 < z < d$, $a > b > d$).

(A). If the walls are perfect conductors, show that the electric field may be written as

$$E_x(\mathbf{x}, t) = E_x \cos(k_x x) \sin(k_y y) \sin(k_z z) \cos(\omega t)$$

$$E_y(\mathbf{x}, t) = E_y \sin(k_x x) \cos(k_y y) \sin(k_z z) \cos(\omega t)$$

$$E_x(\mathbf{x}, t) = E_z \sin(k_x x) \sin(k_y y) \cos(k_z z) \cos(\omega t)$$

with suitable values for k_x, k_y, and k_z. Are there restrictions on the amplitudes E_x, E_y, and E_z?

(B). Use Maxwell's equations to find the induction $\mathbf{B}$ and the allowed resonant frequencies.

(C). If the cavity oscillates in the fundamental mode, find the instantaneous electric and magnetic energy, and verify that the total energy is constant.

(D) If the walls have a large but finite conductivity, show that the fundamental mode has a Q given by

$$\frac{\mu abd}{\delta \mu_m} \frac{a^2 + b^2}{ab(a^2 + b^2) + 2d(a^3 + b^3)}$$

where μ_m is within the metal.

(E) Discuss the numerical values of ω and Q in the fundamental mode of a cubic cavity where $a = b = d = 1$cm. and $\sigma = 3 \times 10^{17}\text{sec}^{-1}$.

3. An empty spherical cavity of radius R has perfect conducting walls.

(A) Show that Maxwell's equations allows azimuthally symmetric TE modes of the form

$$\mathbf{E}(\mathbf{x}, t) = \hat{\phi} E_\phi(r, \theta) e^{-i\omega t}$$

$$\mathbf{B}(\mathbf{x}, t) = \left(\hat{\mathbf{r}} B_r(r, \theta) + \hat{\theta} B_\theta(r, \theta) \right) e^{-i\omega t}$$

where B_r and B_θ are related to derivatives of E_ϕ. Verify that E_ϕ has solutions of the form $E_\phi(r, \theta) = f(r) \sin\theta$, and obtain the corresponding eigenvalue condition $\tan(kR) = kR$, where $k = \omega/c$. Estimate the lowest frequency for this type of mode. Sketch the associated lines of $\mathbf{E}$ and $\mathbf{B}$

(B) Repeat part (A) for TM modes of the form

$$\mathbf{B}(\mathbf{x}, t) = \hat{\phi} B_\phi(r, \theta) e^{-i\omega t}$$

$$\mathbf{E}(\mathbf{x}, t) = \left(\hat{\mathbf{r}} E_r(r, \theta) + \hat{\theta} E_\theta(r, \theta) \right) e^{-i\omega t}$$

and show that the eigenvalue condition is $\cot(kR) = 1/(kR) - kR$. Which of the two fundamental modes lies lowest?

4. Show that $\gamma^2 > 0$.