Homework set 12

November 9, 2001

1. Jackson problem 7.3. Please show a detailed calculation including all necessary steps.

2. Jackson problem 7.5. Please show a detailed calculation including all necessary steps.

3. (A). Consider two electromagnetic plane waves propagating in a dispersive medium. The first wave has frequency $\omega$ and wavenumber $k$, while the second wave has frequency $\omega + \Delta \omega$ and wavenumber $k + \Delta k$. At time $t$ there are points in space where the two waves are in phase, producing an interference maximum. Show that, for $\Delta k \rightarrow 0$, the velocity of these points is

$$V_g = \frac{d\omega}{dk}$$

$V_g$ is the group velocity. Sketch $V_g$ and the phase velocity $V_p$ as a function of $\omega$ for a collisionless plasma. Find $V_g$ in terms of $V_p$ for such a medium.

(B). A rotating neutron star emits radio pulses of duration $\approx 5-50$ msec at intervals of $\approx 1$ sec. These pulses propagate through the interstellar medium of ionized hydrogen.

It is observed that if a pulse is detected at frequency $\omega$ at time $t$, a pulse at frequency $\omega + \Delta \omega$ will be detected at time $t + \Delta t$. Find the relation between $\Delta t$, $\Delta \omega$, and the dispersive measure

$$DM = \int_0^L n_e dl,$$

where $L$ is the distance to a neutron star, $n_e$ is the electron number density, and $dl$ an element of path length along the line of sight. Discuss the sign of $\Delta t/\Delta \omega$.

(C). Consider the following data for the neutron star PSR 0950 + 08: The average density for
electrons in the solar neighborhood is 0.03 cm$^{-3}$. Find the distance to PSR 0950 + 08.

(D). The observed dispersion in the data for PSR 0950 + 08 could be due in part to the photon having a finite mass. Find an upper limit to the mass of the photon.
