Homework set 12

November 9, 2001

1. Jackson problem 7.3. Please show a detailed calculation including all necessary steps.

2. Jackson problem 7.5. Please show a detailed calculation including all necessary steps.

3. (A). Consider two electromagnetic plane waves propagating in a dispersive medium. The first wave has frequency ω and wavenumber k, while the second wave has frequency $\omega + \Delta\omega$ and wavenumber $k+\Delta k$. At time t there are points in space where the two waves are in phase, producing an interference maximum. Show that, for $\Delta k \to 0$, the velocity of these points is

$$V_g = \frac{d\omega}{dk}$$

V_g is the group velocity. Sketch V_g and the pahse velocity V_p as a function of ω for a collisionless plasma. Find V_g in terms of V_p for such a medium.

(B). A rotating neutron star emits radio pulses of duration $\approx$ 5–50 msec at intervals of $\approx$ 1 sec. These pulses propagate through the interstellar medium of ionized hydrogen.

It is observed that if a pulse is detected at frequency ω at time t, a pulse at frequency $\omega + \Delta\omega$ will be detected at time $t + \Delta t$. Find the relation between Δt, $\Delta\omega$, and the dispersive measure

$$DM = \int_0^L n_e dl,$$

where L is the distance to a neutron star, n_e is the electron number density, and dl an element of path length along the line of sight. Discuss the sign of $\Delta t/\Delta\omega$.

(C). Consider the following data for the neutron star PSR 0950 + 08: THe average density fo

electrons in the solar neighborhood is $0.03\ \mathrm{cm}^{-3}$. Find the distance to PSR 0950 + 08.

(D). The observed dispersion in the data for PSR 0950 + 08 could be due in part to the photon having a finite mass. Find an upper limit to the mass of the photon.

4. Jackson problem 7.14.

5. Jackson problem 7.19.