Homework set 1, Due April 13

November 9, 2001

1. Consider a swarm of charged particles for which the force the i'th particle is

$$\mathbf{F}_i = q_i \left(\mathbf{E}(\mathbf{x}_i) + \mathbf{v}_i \times \mathbf{B}(\mathbf{x}_i) \right)$$

Thus the rate at which the fields do work is

$$-\frac{dW}{dt} = \sum_i \mathbf{F}_i \cdot \mathbf{v}_i = \int_V d^3x \, \mathbf{E}(\mathbf{x}) \cdot \mathbf{J}(\mathbf{x})$$

Starting with the fully time dependent Maxwell's equations, show that for a closed system of fields from enclosed particles,

$$W_{EM} = \frac{1}{8\pi} \int_V d^3x \; \left[|\mathbf{B}(\mathbf{x})|^2 + |\mathbf{E}(\mathbf{x})|^2 \right]$$

2. Jackson problem 6.1.

3. What is the flux though the i'th loop in terms of the inductances and the currents for the system of currents considered in 6.1..

4. Consider again the same system of currents. Derive an expression for the *total* δW due to small displacements of the loops and small changes in the field (Hint, for the differential energy of the field you may with to start with $\delta W_{field} = -\int_V d^3x \, \mathbf{E}(\mathbf{x}) \cdot \mathbf{J}(\mathbf{x}) \;\; \delta t$). Your final result should be a natural function of the postitions of the loops and of the fluxes through them. Thus show that when the fluxes are held fixed, the force and current through the i'th loop are given by

$$\mathbf{F}_i = -\left(\frac{\partial W}{\partial \mathbf{x}_i} \right) \Bigg|_F \;\; ; \;\; I_i = c \left(\frac{\partial W}{\partial F_i} \right) \Bigg|_{\mathbf{x}} .$$

Now suppose that we displace one of the loops, but keep the currents (rather than the fluxes) fixed. Find the appropriate theromdynamic potential W': that for which the minimun corresponds to the most stable state of the system. Write W' in terms of W, and show that

$$\mathbf{F}_i = +\left(\frac{\partial W}{\partial \mathbf{x}_i} \right) \Bigg|_I \;\; ; \;\; F_i = c \left(\frac{\partial W}{\partial I_i} \right) \Bigg|_{\mathbf{x}} .$$