Homework set 3

1. Jackson problem 3.1 Please do **not** completely solve this problem. Rather it is easier to split this problem into a sum of two problems by superposition: one which is completely symmetric and one which is antisymmetric.

- What are these two problems?
- What series of special functions would you use to solve for the potential?
- What property of these polynomials are you exploiting by splitting this problem into two problems?

2. For the example of a Sphere With a Specified Potential presented in class (Sec. IA.2) a point charge is now introduced along the polar axis a distance d from the center of the sphere, but the charge is also added and redistributed on the sphere so as to maintain the above boundary condition. Now find the potential for all $a < r < d$.

3. The surface of a nearly spherical conductor is specified by

$$r(\theta) = a\left[1 + \sum_{l=0}^{\infty} \delta_l \mathbf{P}_l(\cos\theta)\right] ,$$

where $\delta_l \ll 1$.

- Explain clearly why δ_0 and δ_1 may be neglected.
- If the conductor is given a charge q, find the potential throughout the exterior region correct to first order in δ_l.
- Show that the surface charge density, to first order in δ_l, is

$$\sigma \approx \frac{q}{4\pi a^2}\left[1 + \sum_{m=2}^{\infty} \delta_m (m-1)\mathbf{P}_m(\cos\theta)\right]$$

  and find the capacity of the conductor.

4. Reconsider example IA.2 in the notes; Hemispheres of opposite charge. How fast do you think the series Eq. 52 will converge? Are there places in the enclosed volume where you would expect the convergence to be faster (slower)? Now use Mathematica to test your assumptions: Use Plot3D to plot the function $\Phi(r,\theta)$, $0 < r < 1$ and $0 < \theta < 2\pi$ when $a = 1$, $V = 1$ using 3 terms in the sum. Repeat using 6 terms in the sum. Be sure to use a sufficient number of PlotPoints so that you can see the difference between 3 and 6 terms in the series. Now use Plot to plot the angular dependence of the potential at fixed r. Do this for $r = a$ with 3,6 and 9 terms in the sum. Repeat this procedure for $r = 0.9a$. Please comment on the convergence of the series.

5. An empty cylindrical region is bounded by the surfaces $z = 0$, $z = c$, and $\rho = a$; where a and c are positive constants. The potential on the ends ($z = 0$ and $z = c$) of the cylinders is zero. The potential on the surface $\rho = a$ is $V(z, \phi) = V_0 \sin(\phi)$ where V_0 is a constant. Devise an expansion for the potential within the cylinder, and evaluate the coefficients in the expansion.

6. Jackson problem 3.9 (note that problem 3.8 was solved in class).

7. The potential on a spherical surface of radius a is given by $V(\theta, \phi) = V_o \sin\phi$, where V_o is a constant. The space within the sphere is empty. Devise an expansion for the potential in this region and obtain an integral expression for the coefficients in this expansion. Which coefficients must be non-zero? Estimate how many terms must be kept in the expansion for it to be accurate to within $10^{-3} V_o$ for $r < a/2$.