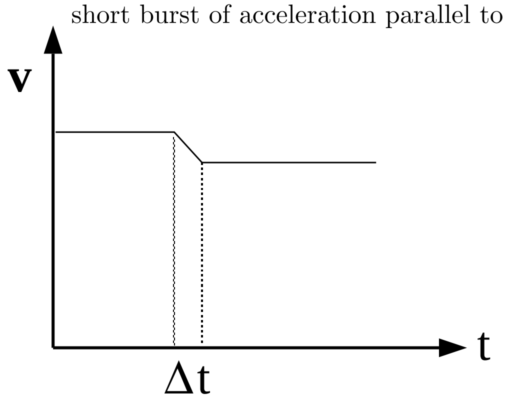
Homework set for Chapter 14

1. In the notes, we derived the relativistic Larmor formula by demanding conservation of energy. In the book, it is derived by rewriting the nonrelativistic result in terms of momentum 4-vectors, and considering how these transform. However, there is another way. The Larmor formula states that the radiated power is proportional to the acceleration of a charged particle squared. By considering how acceleration transforms from frame to frame, derive the relativistic Larmor formula, or the Liénard formula.

2. Apply what you learned in Section IV.A of the notes to the simple case of a particle which feels a short burst of acceleration parallel to its motion.



Assume that $|\Delta \mathbf{v}| \ll |\mathbf{v}|$, and since the region of acceleration is small, make a dipole approximation to calculate the radiation portion of $\mathbf{E}(\mathbf{x}, \omega)$. Then calculate $dI(\omega)/d\Omega$ as a function of θ, ω, Δt and $\Delta \mathbf{v}$. Plot the frequency distribution for fixed θ, note the width of the central peak. Is this what you expected? Make the small angle approximation, and show the the angular distribtion for a fast particle is peaked near $\theta \sim 1/\gamma$.

3. A particle of positive charge q is released at rest at the point x_0 in an electric field $E(x) = E_0 e^{-kx}$ $(E_0, k > 0)$. The particle is accelerated in the positive x direction and at $x = \infty$ has velocity $v = \beta c$.

1. Find the total energy $\mathcal{E}(x)$ of the particle as a function of x by integrating the fourth component of the force equation.

2. Find the total radiation energy dissipated by the particle as it goes from x_0 to ∞. (neglect radiation reaction effects)

The following integral may be useful.

$$\int_a^b du \frac{u(b-u)}{\sqrt{u^2 - a^2}} = \frac{b}{2}\sqrt{b^2 - a^2} - \frac{a^2}{2} \log\left(\frac{b + \sqrt{b^2 - a^2}}{a}\right)$$

4. Jackson 14.5 (second edition).

5. Jackson 14.7(second edition).

6. Two particles of charges q_1 and q_2 are oscillating along the z-axis, their positions are given by $z_1(t) = a\cos(\omega_0 t)$ $z_2(t) = -a\cos(\omega_0 t)$ respectively. Given that the energy per cycle per unit solid angle radiated at frequency $\omega_m = m\omega_0$ of a single particle of charge e, executing harmonic motion of amplitude a along the z-axis is the single-particle problem is

$$\frac{dW_m(1)}{d\Omega} = \frac{e^2 c\beta^2}{2\pi a^2} m^2 \tan^2(\theta) J_m(m\beta\cos\theta)$$

where $\beta = \omega_0 a/c$, consider the following:

1. In the two cases, $q_1 = q_2 = q$ and $q_1 = -q_2 = q$ illustrated below, find the frequency distribution of the radiation $\frac{dW_m}{d\Omega}$ of the mth harmonic of ω_0.

2. In the non-relativistic limit find the ratio of the total power radiated in the two cases illustrated below.

7. Jackson 14.10 (second edition).