Homework set 14

November 9, 2001

1. Complete the analysis of the Fizeau experiment begun in class. I.e., derive (to lowest order in $\beta$, and for constant $n$)

$$v_{\text{phase}} = \frac{c}{n} \pm v \left(1 - \frac{1}{n^2}\right) + O(\beta^2)$$

2. In a certain region of space, an observer at rest relative to the local neighborhood of stars sees the distribution of stars to be isotropic. A second observer moves relative to the first in the positive $z$-direction with speed $v = \beta c$.

(a) Show that the second observer sees a distribution of stars which depend upon the polar angle $\theta'$ (in spherical coordinates) according to

$$n(\theta') = \frac{N}{4\pi (1 - \beta \cos \theta')^2}$$

where $N$ is the total number of visible stars. Discuss this result.

Plot $n(\theta')$ versus $\theta'$ for $\beta = 0.1, 0.5, 0.9$. How would the sky appear to the second observer as $\beta \to 1$?

(b) To the first observer, all the stars visible emit light of the same color: for simplicity assume that each star is a monochromatic emitter of light of wavelength $\lambda_0$. For the second observer, find the observed wavelength $\lambda$ as a function of the observed angular position of $\theta'$ of the star. Find the value of $\theta'$ for which (for a given $\beta$) there is neither a red shift nor a blue shift. Plot $\lambda(\theta')$ versus $\theta'$ for $\beta = 0.1, 0.5, 0.9$ and $\lambda_0 = 5000\,\text{rA}$. Describe the appearance of
the sky as a function of $\beta$ for the second observer.

(c) The stars are of course not monochromatic emitters. Over a broad range frequencies they emit a black-body spectrum. For a black body of temperature $T$, the number of phonons present per unit volume within $d^3k$ of $k$ is

$$n(k)d^3k = \frac{1}{4\pi^3} \frac{1}{(e^{\frac{hc}{k_BT}} - 1)} d^3k.$$  

Show that to the second observer, a star at apparent position $\theta'$ has a black-body spectrum too, but for a different temperature than that of the first observer. Find this temperature in the frame of the second observer in terms of the temperature $T$ in the frame of the first observer, $\beta$, and $\theta'$. In light of this result, discuss the appearance of the sky to the second observer.

3. Jackson 11.11.
