Homework set 14

November 9, 2001

1. Complete the analysis of the Fizeau experiment begun in class. I.e., derive (to lowest order in β, and for constant n)

$$v_{phase} = \frac{c}{n} \pm v\left(1 - \frac{1}{n^2}\right) + O(\beta^2)$$

2. In a certain region of space, an observer at rest relative to the local neighborhood of stars sees the distribution of stars to be isotropic. A second observer moves relative to the first in the positive z-direction with speed $v = \beta c$.

(a) Show that the second observer sees a distribution of stars which depend upon the polar angle θ' (in spherical corrdinates) according to

$$n(\theta') = \frac{N}{4\pi} \frac{(1-\beta^2)}{(1-\beta\cos\theta')^2}$$

where N is the total number of visible stars. Discuss this result. Plot $n(\theta')$ versus θ' for $\beta = 0.1\ , 0.5\ , 0.9$. How would the sky appear to the second observer as $\beta \to 1$?

(b) To the first observer, all the stars visible emit light of the same color: for simplicity assume that each star is a monochromatic emitter of light of wavelengh λ_0. For the second observer, find the observed wavelength λ as a function of the observed angular position of θ' of the star. Find the value of θ' for which (for a given β) there is neither a red shift nor a blue shift. Plot $\lambda(\theta')$ versus θ' for $\beta = 0.1\ , 0.5\ , 0.9$ and $\lambda_0 = 5000\mathbf{r}A$. Describe the appearance of the sky as a function of β for the second observer.

(c) The stars are of course not monochromatic emitters. Over a broad range frequencies they emit a black-body spectrum. For a black body of temperature T, the number of phonons present per unit volume within d^3k of $\mathbf{k}$ is

$$n(\mathbf{k})d^3k = \frac{1}{4\pi^3}\frac{1}{\left(e^{\hbar ck/k_B T} - 1\right)}d^3k\,.$$

Show that to the second observer, a star at apparent position θ' has a black-body spectrum too, but for a different temperature than that of the first observer. Find this temperature in the frame of the second observer in terms of the temperature T in the frame of the first observer, β, and θ'. In light of this result, discuss the appearance of the sky to the second observer

3. Jackson 11.11.

3. Jackson 11.12.

3. Jackson 11.19.