$$H_0 = \sum_{ij} \langle i | H_0 | j \rangle c_{i\sigma}^{\dagger} c_{j\sigma}$$

Let the new basis be a set of atomic orbitals

$$c_{j\sigma}^{\dagger} = \int d^3x \, \Phi(x - R_j) \, \psi_\sigma^{\dagger}(x)$$

Then as you learned in elementary solid state physics

$$\langle i | H_0 | j \rangle = \begin{cases} \epsilon & i = j \\ -t & i = j + \delta \quad \text{(ie } i \text{ \& } j \text{ are nn.)} \\ 0 & \text{else} \end{cases}$$

negative since $U(x)$ presumably is

Similarly since $\Phi(x - R_i)$ falls off exponentially as $|x - R_j|$ increases

$$\langle l m | V | p n \rangle = \begin{cases} U & l = m = p = n \\ 0 & \text{else} \end{cases}$$

$$V = \frac{U}{2} \sum_{j\sigma\sigma'} c_{j\sigma}^{\dagger} c_{j\sigma'}^{\dagger} c_{j\sigma'} c_{j\sigma}$$

$$= U \sum c_{j\uparrow}^{\dagger} c_{j\downarrow}^{\dagger} c_{j\downarrow} c_{j\uparrow} = U \sum_j n_{j\uparrow} n_{j\downarrow}$$

so that

$$H = -t \sum_{\langle ij \rangle \sigma} \left(c_{i\sigma}^{\dagger} c_{j\sigma} + h.c. \right) + \epsilon \sum_{j\sigma} c_{j\sigma}^{\dagger} c_{j\sigma} + U \sum_j n_{j\uparrow} n_{j\downarrow}$$