

EFFECT OF THE PROTON MASS ON THE SPECTRUM OF THE HYDROGEN ATOM IN A STRONG MAGNETIC FIELD

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Contrary to a recent claim, we point out that, in the problem of a hydrogen atom in a strong magnetic field, the centre-of-mass motion and the relative motion of the particles can be separated, and that the effect of the finite proton mass on the spectrum is negligible.

Virtamo and Simola have recently claimed [1] that the problem of the hydrogen atom in a strong magnetic field is non-separable. They then carried out a numerical variational calculation, taking into account the proton wave function in the magnetic field, from which they concluded that the spread of the proton wave function leads to a substantial reduction of the binding energy. It is our purpose here to point out that the problem is in fact separable, in agreement with the conclusions of various other authors [2-6], and that the effect of the finite proton mass on the spectrum is negligible.

For the problem at hand we feel that Carter's approach [3], as well as being the most detailed, is the most enlightening, if we make one important addition which we will consider below. The hamiltonian for the problem is (we take $c = \hbar = 1$)

$$H = (2m_1)^{-1} [\mathbf{p}_1 + e\mathbf{A}(r_1)]^2 + (2m_2)^{-1} [\mathbf{p}_2 - e\mathbf{A}(r_2)]^2 - e^2/|\mathbf{r}_1 - \mathbf{r}_2|, \quad (1)$$

where the subscripts 1 and 2 refer to the electron and proton respectively and where $\mathbf{B} = \nabla \times \mathbf{A}$ is the magnetic field. Following Carter [3], we denote the center-of-mass (CM) coordinates by $\mathbf{R} = (x, y, z)$ and the relative coordinates by $\mathbf{r} = (x, y, z)$. We consider that the magnetic field is oriented along the z axis and we choose the Landau gauge

$$A_y(\mathbf{r}) = Bx, \quad A_x = A_z = 0. \quad (2)$$

It can then be shown that $\Pi_Y \equiv -i\partial/\partial Y$ and $\Pi_X \equiv -i\partial/\partial X - eBy$ are simultaneously known constants of the motion, with the result that the most general simultaneous eigenstate of these operators and the hamiltonian H is

$$\Psi(X, Y, \mathbf{r}) = \exp[i(P_X + eBy)X] \exp[iP_Y Y] \psi(\mathbf{r}), \quad (3)$$

where P_X and P_Y are the eigenvalues of Π_X and Π_Y , respectively. In other words, P_X and P_Y are the X and Y components of the CM momentum \mathbf{P} . It then follows that the solution to the Schrödinger equation $H\Psi = E\Psi$ reduces to $h\psi = E\psi$ where, in the CM system of coordinates (i.e. taking $\mathbf{P} = 0$), h is given by (see eq. (31) of ref. [3])

$$h = \frac{1}{2\mu} (\mathbf{p} + e\mathbf{a})^2 - \frac{e^2}{r} + \frac{e^2 B^2}{2Mc^2} (x^2 + y^2), \quad (4)$$

where $\mu = m_1 m_2 / (m_1 + m_2)$, $M = m_1 + m_2$, and where \mathbf{a} is defined to be the vector potential for the effective magnetic field

$$\mathbf{b} \equiv [(m_2 - m_1)/(m_2 + m_1)] \mathbf{B} = \nabla \times \mathbf{a}. \quad (5)$$

Carter [3] then concludes, in his introduction, that "... the effective hamiltonian is formally identical to that for a particle of a different (reduced) mass moving in a different magnetic field and in a harmonic-oscillator potential." However, we wish to point out that an even simpler interpretation is at hand if we make one further important addition to this analysis.

Since the problem is in essence reduced to the con-

sideration of the effective hamiltonian h given by eq. (4), we are now at liberty to re-write this result solely in terms of B by choosing a particular gauge. If we again choose the Landau gauge, similar to eq. (2), no new insight is obtained into the problem. However, on the other hand, if we choose the gauge,

$$a_x = -(1/2)by, \quad a_y = (1/2)bx, \quad a_z = 0, \quad (6)$$

then a remarkable simplification occurs with the result that

$$h = \frac{p^2}{2\mu} - \frac{e^2}{r} + \frac{e}{2\mu} gBL_z + \frac{e^2}{8\mu} B^2(x^2 + y^2), \quad (7)$$

where L_z is the z component of the angular momentum and

$$g \equiv (m_2 - m_1)/(m_2 + m_1). \quad (8)$$

In addition, we note that

$$g_L \equiv (g/\mu) = 1/m_1 - 1/m_2 \quad (9)$$

is Lamb's [2] effective correction to the magnetic moment.

Comparing the hamiltonian given in eq. (7) with the usual hamiltonian for the case of infinite proton mass [7], it is clear that the effect of finite proton mass can be taken into account simply by replacing m_1 by μ everywhere and inserting the additional g factor in the linear B term. Thus, the numerical corrections can only be of relative order (m_2/m_1) , at variance with the results of Virtamo and Simola [1].

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