CHARMONIUM – THE $^1P_1$ STATE

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Received 27 February 1978

Based on a model which gives agreement with the observed hyperfine and fine splitting of the charmonium states, we predict the mass of $^1P_1$ to be 3372 MeV, substantially lower than any previous predicted values. We have also estimated branching ratios and decay rates for the production of this state via channels which become accessible because of its low mass.

There has been increasing evidence in support of the charmonium spectroscopy [1]. The low-lying charmonium states have been identified as [2,3] $\chi(3554){:}\,^3P_2$, $\chi(3510){:}\,^3P_1$, $\chi(3413){:}\,^3P_0$, $\eta_c(2830){:}\,1^1S_0$, $\eta_c' = \chi(3454){:}\,2^1S_0$, $\psi(3098){:}\,1^3S_1$ and $\psi'(3684){:}\,2^3S_1$.

Recently, a new direct-channel resonance in e^+e^- annihilation has been observed [4] at $M = 3772$ MeV, whose properties fit closely to the long-expected $^3D_1$ ($J^{PC} = 1^{--}$) state, with a small admixture of $^3S_1$, due to the tensor force. Thus, the only missing low-lying state in the charmonium spectroscopy is the $^1P_1$ ($J^{PC} = 1^{+-}$) state. The fact that it has not been observed is not particularly surprising since its quantum numbers are such that it can only be produced via a two-step process from e^+e^- annihilation. Most of the theoretical models give estimates [5,6] of its mass of about 3.5–3.6 GeV, which would almost make it essentially inaccessible for observation in the e^+e^- colliding beam experiment. However, one must recognize that these models underestimate the hyperfine splittings of the S-states by almost an order of magnitude (30–70 MeV compared to the observed values ~250 MeV) and fail to predict the correct spacings of the ^3P states.

One may expect that perhaps the hyperfine splitting of the P states $E(^3P_1) - E(^1P_1)$ should have the same order of magnitude and sign as the hyperfine splitting of the S states, viz. $E(^3S_1) - E(^1S_0)$. Therefore, the true location of the $^1P_1$ level may lie a couple of hundred MeV below 3522 MeV, the center of gravity of the ^3P level. A mass of about 3.3–3.4 GeV for $^1P_1$ would imply a reasonable chance of production of $^1P_1$ by radiative decay from the η_c' and χ_J states.

Our purpose here is, first of all, to calculate the mass and the production and decay rates of the $^1P_1$ state, using a potential model for the $c\bar{c}$ interaction which predicts results in conformity with the existing charmonium spectrum. Secondly, our procedure will make clear the importance of an experimental determination of the properties of this state for the determination of the correct potential model.

Recently one of us (Chan [7] [‡1]) proposed that the hyperfine splitting of the 1S and 2S states and the fine-structure splittings of the 1^3P states can be explained if the quark binding, linearly rising, potential is mainly due to an effective scalar exchange with a small admixture of vector exchange. We shall show in this paper that this model predicts $E(^1P_1) \approx 3372$ MeV and we will also discuss the possible production mechanisms and the dominant decay modes of the $^1P_1$ state [‡2].

In Chan's model [7] the quark binding potential

[1] Work supported in part by the U.S. Department of Energy under contract No. EG-77-S-05-5490.

‡1 Due to misprints in ref. [7] the parameter a appears as $a = 0.149$ GeV^2 and the overall sign of the second term of H_{spin} (eq. (4)) appears as a plus in that paper.

‡2 The production mechanisms and decay modes of a higher mass $^1P_1$ (~3.5 MeV) has been investigated by Renard [8].

$U(r) = ar - \alpha_s/r$ (with the parameters $a = 0.194\ \mathrm{GeV}^2$, $\alpha_s = 0.2$ and $m = 1.6$ GeV from the model of Eichten et al. [3]), is considered to be the sum of $V(r)$ and $S(r)$, i.e. $U(r) = V(r) + S(r)$, where $V(r) = far - \alpha_s/r$ and $S(r) = (1-f)ar$ denote potentials due to vector and scalar exchanges, respectively. The spin-dependent part of the hamiltonian is given by [7]

$$H_{\rm spin} = \left[2(1+\kappa)f\frac{a}{r} - \frac{a}{2r} + \frac{3}{2}\alpha_s\frac{1}{r^3}\right]\frac{1}{m^2}\boldsymbol{L}\cdot\boldsymbol{s}$$

$$-\frac{1}{3m^2}\left[(1+\kappa)^2 f\frac{a}{r} + 3\alpha_s\frac{1}{r^3}\right][\boldsymbol{s}_1\cdot\boldsymbol{s}_2 - 3(\boldsymbol{s}_1\cdot\hat{\boldsymbol{r}})(\boldsymbol{s}_2\cdot\hat{\boldsymbol{r}})] \qquad (1)$$

$$+\frac{2}{3m^2}\left[2(1+\kappa)^2 f\frac{a}{r} + 4\pi\,\alpha_s\delta^3(r)\right]\boldsymbol{s}_1\cdot\boldsymbol{s}_2,$$

where $\boldsymbol{s} = \boldsymbol{s}_1 + \boldsymbol{s}_2$ and κ is the effective quark–gluon anomalous magnetic moment which is set equal to zero for the contribution to H from the short-distance part $V(r)$, as expected from the asymptotic freedom of the underlying theory.

The choice of $f = 0.123$ and $\kappa = 5$ yields

$$E(^3\mathrm{P}_2) - E(^3\mathrm{P}_1) = 40\ \mathrm{MeV}\ (44 \pm 8), \qquad (2)$$

$$E(^3\mathrm{P}_1) - E(^3\mathrm{P}_0) = 98\ \mathrm{MeV}\ (95 \pm 8), \qquad (3)$$

$$E(1^3\mathrm{S}_1) - E(1^1\mathrm{S}_0) = 262\ \mathrm{MeV}\ (265 \pm 10), \qquad (4)$$

$$E(2^3\mathrm{S}_1) - E(2^1\mathrm{S}_0) = 225\ \mathrm{MeV}\ (230 \pm 10), \qquad (5)$$

in excellent agreement with the experimental values given in parentheses.

To evaluate $E(^1\mathrm{P}_1)$, it is most convenient to define the center of gravity of the ^{3}P states:

$$E(^3\mathrm{P}) \equiv \tfrac{1}{9}[5E(^3\mathrm{P}_2) + 3E(^3\mathrm{P}_1) + E(^3\mathrm{P}_0)] = 3522\ \mathrm{MeV}. \qquad (6)$$

Then, using eqs. (1) and (6) we calculate that the hyperfine splitting of the P states is

$$E(^3\mathrm{P}) - E(^1\mathrm{P}_1) = (4/3)(1+\kappa)^2 f(a/m^2)\langle 1/r\rangle_{1\mathrm{P}} = 152\ \mathrm{MeV}. \qquad (7)$$

It is given entirely by the spin–spin (last) term of $H_{\rm spin}$ in eq. (1). By comparison the S state hyperfine splitting is

$$E(^3\mathrm{S}_1) - E(^1\mathrm{S}_0) = (4/3)(1+\kappa)^2 f(a/m^2)\langle 1/r\rangle_s + \frac{8\pi}{3}\frac{\alpha_s}{m^2}|\phi_s(0)|^2 = \begin{cases} 262\ \mathrm{MeV} & \text{for 1S,} \\ 225\ \mathrm{MeV} & \text{for 2S.}\end{cases} \qquad (8)$$

In some earlier models, the linearly rising quark binding potential is such that either it does not contribute to the spin dependent hamiltonian ($a = 0$) [5], or it has no vector exchange part ($f = 0$) [9]. Then the only contribution to the S-wave hyperfine splitting is the $(8\pi/3)(\alpha_s/m^2)|\phi_s(0)|^2$ term, which is about 20 MeV. In addition, the splitting of the P-states would vanish, implying that $E(^1\mathrm{P}_1) = E(^3\mathrm{P}) = 3522$ MeV. In Schnitzer's model [10] $f = 1$ so that (correcting Schnitzer's hamiltonian, à la Chan [7], to conform to the correct two-body interaction hamiltonian [11]), $E(^1\mathrm{P}_1) = 3486$ MeV for $\kappa = 0$, and $E(^1\mathrm{P}_1) = 3397$ MeV for $\kappa = 1.13$. For our choice of parameters, viz. $f = 0.123$ and $\kappa = 5$, we obtain $E(^3\mathrm{P}) - E(^1\mathrm{P}_1) = 152$ MeV and $E(^1\mathrm{P}_1) = 3372$ MeV. If we ignore the small contribution from the $|\phi(0)|^2$ term, then

$$E(1^3\mathrm{S}_1) - E(1^1\mathrm{S}_0) : E(2^3\mathrm{S}_1) - E(2^1\mathrm{S}_0) : E(^3\mathrm{P}) - E(^1\mathrm{P}_1) = \langle 1/r\rangle_{1\mathrm{S}} : \langle 1/r\rangle_{2\mathrm{S}} : \langle 1/r\rangle_{1\mathrm{P}}. \qquad (9)$$

From these one can deduce an intuitive estimate of the relative size of these hyperfine splittings. It is simple to see that, in general, the hyperfine splitting becomes smaller for higher excited states and higher angular momentum states.

It is useful to express the hyperfine splittings of eq. (2) in terms of the P-state splittings rather than the matrix element. Thus, we find

$$[E(^3\mathrm{P}) - E(^1\mathrm{P}_1)] = \frac{4}{9}\left\{\frac{(1+\kappa)^2 f}{1-(1+\kappa)(3-\kappa)f}\right\} \times \{5[E(^3\mathrm{P}_2) - E(^3\mathrm{P}_1)] - 4[E(^3\mathrm{P}_1) - E(^3\mathrm{P}_0)]\}. \qquad (10)$$

Eq. (10) relates the hyperfine splittings to the P-state fine splitting via the two crucial parameters of this model, viz. f and κ. Therefore, the location of the $^1\mathrm{P}_1$ state would be an important test of the choice of these parameters in this model.

For the case $a = 0$, eq. (10) is identically zero and we recover the results of Eichten et al. [5]. On the other hand if $a \neq 0$ and $f = 1$, $\kappa = 0$ and eq. (10) is equivalent to the sum rule of Gupta and Khare [6] for arbitrary potential. Using the observed $^3\mathrm{P}_J$ mass they obtain $E(^1\mathrm{P}_1) = 3561 \pm 10$ MeV.

If the mass of the $^1P_1$ state is below 3.4 GeV as suggested in our model it opens the possibility for it being observed in the decays $\chi_J \to {}^1P_1 + \gamma$ and $\eta_c' \to {}^1P_1 + \gamma$. It is clear that the combined branching ratio from ψ' to $^1P_1$ is rather small. In the following we shall give a rough estimate of these branching ratios.

$\Psi' \to \chi_J + \gamma$ is an electric dipole (E1) transition. The experimental branching ratio is about 7% for each J channel ‡3. The decay $\chi_J \to {}^1P_1 + \gamma$ is a magnetic dipole (M1) transition. Assuming that the charm quark has mass $m \sim 1.6$ GeV, $e_Q = 2/3$, and a normal magnetic moment, we obtain an upper bound of

$$\Gamma(\chi_J \to {}^1P_1 + \gamma) < \tfrac{4}{3}\alpha e_Q^2 \frac{k^3}{m^2} \sim \begin{cases} 9\ \text{keV} & \text{for } J=2, \\ 3\ \text{keV} & \text{for } J=1, \\ 0.1\ \text{keV} & \text{for } J=0. \end{cases} \tag{11}$$

Since the total widths of χ_J have not been established we can only estimate the branching ratio, B say, from the accompanying E1 decay $\chi_J \to \psi + \gamma$ [12] ‡4:

$$\Gamma(\chi_2 \to \psi\gamma) = 400\ \text{keV}, \quad B(\chi_2 \to \psi\gamma) = 14 \pm 6\%,$$
$$\Gamma(\chi_1 \to \psi\gamma) = 300\ \text{keV}, \quad B(\chi_1 \to \psi\gamma) = 35 \pm 7\%, \tag{12}$$
$$\Gamma(\chi_0 \to \psi\gamma) = 140\ \text{keV}, \quad B(\chi_0 \to \psi\gamma) = 3 \pm 3\%.$$

Then we obtain

$$B(\chi_2 \to {}^1P_1 + \gamma) < 0.3\%, \quad B(\chi_1 \to {}^1P_1 + \gamma) < 0.4\%,$$
$$B(\chi_0 \to {}^1P + \gamma) < 0.002\%. \tag{13}$$

A branching ratio much less than 0.3% would be difficult to detect. Similar calculations of decay rate from the magnetic dipole transition $\psi \to \eta_c + \gamma$ indicates [13] that the three branching ratios can be overestimated by an order-of-magnitude. However, Feinberg and Sucher [14] showed that the M1 transition rates are quite sensitive to the Dirac covariant character of the potential. Factors of 10 or more away from the naive estimate of eq. (11) are possible. These authors suggested that the M1 decay may be a sensitive probe of the quark interaction. Therefore, it would be important to have a relativistic calculation of radiative M1 transitions for Chan's model.

‡3 A good review on the experimental branching ratios and decay rates of the charmonium states is given in ref. [12].

‡4 The partial widths are derived from the calculation of Eichten et al. [5], corrected for the latest transition energies.

Alternatively $^1P_1$ can be produced from $\psi' \to \eta_c' + \gamma$ and $\eta_c' \to {}^1P_1 + \gamma$. The state $\eta_c'(3454)$ has been observed in the processes $\psi' \to \eta_c' + \gamma$ and $\eta_c' \to \psi + \gamma$. The first process is a M1 transition. The second is a forbidden M1 transition. The combined branch ratio [12] of $\psi' \to \psi + \gamma + \gamma$ is $0.6 \pm 0.4\%$. Because of low statistics the existence of η_c' remained doubtful for some time. It has been recently confirmed by PLUTO [15]. The E1 transition rate $\Gamma(\eta_c' \to {}^1P_1 + \gamma) = \frac{4}{3}\alpha e_Q^2 k^3 \times |\langle 1P|r|2S\rangle|$ is estimated to be about 8.5 keV. An upper bound for this rate can be obtained by combining the Thomas–Reiche–Kuhn sum rule ‡5

$$2\mu \sum_j \omega_{ji} |\langle j|r|i\rangle|^2 = 3, \tag{14}$$

and a dipole sum rule [17]

$$2\mu \sum_n \omega_{ns,1P} |\langle ns|r|1P\rangle|^2 = -1. \tag{15}$$

We obtain

$$\Gamma(\eta_c' \to {}^1P_1 + \gamma) < \tfrac{4}{3}\alpha e_Q^2 k^2/\mu = 34\ \text{keV}. \tag{16}$$

A decay rate of a few keV for $\eta_c' \to {}^1P_1 + \gamma$ should compete favorably with the forbidden M1 transition $\eta_c' \to \psi + \gamma$. However, the $^1P_1$ is not as simple to detect as the ψ. The decay $^1P_1 \to \psi + \gamma$ is forbidden by C invariance. The major decay mode is probably $^1P_1 \to \eta_c + \gamma$. An upper bound and a lower bound for the E1 transition rate, obtained by the sum rules eqs. (14) and (15) respectively, are

$$\Gamma({}^1P_1 \to \eta_c + \gamma) < \tfrac{2}{3}(\alpha k^2/\mu) e_Q^2 = 670\ \text{keV},$$
$$\Gamma({}^1P_1 \to \eta_c + \gamma) > \tfrac{2}{9}\alpha e_Q^2 k^2 = 217\ \text{keV}. \tag{17}$$

The major problem is that η_c has only been observed in the $\eta_c \to 2\gamma$ mode. Therefore in this sequence of cascades $\psi' \to \eta_c' + \gamma$, $\eta_c' \to {}^1P_1 + \gamma$, $^1P_1 \to \eta_c + \gamma$, and $\eta_c \to \gamma\gamma$, the combined process is really $\psi' \to 5\gamma$! Compensated by the large phase space the Iizuka–Okubo–Zweig rule violating decay rate $\Gamma({}^1P_1 \to \eta' + \gamma)$ is about 200 keV. Alternatively, one can look for the $^1P_1$ state from hadrons produced in decay modes such as $\bar{P}P$ and 3π. The total hadronic width has been estimated to be about 100 keV [8,18].

‡5 The combination of these two sum rules has been applied to obtain an upper bound for $\Gamma(\psi' \to \chi_J + \gamma)$ by Jackson [16].

In summary, based on a model which gives the correct prediction of the hyperfine and fine splitting of the charmonium states, we have predicted the mass of $^1P_1$ to be 3372 MeV, substantially lower than any previous predicted values. We have also estimated branching ratios and decay rates for the production of the $^1P_1$ state via channels which become accessible because of its low mass. The ultimate discovery of this state would certainly give strong support to the charmonium spectroscopy. The location of the $^1P_1$ state would be a crucial test of various charmonium models.

One of us (LHC) would like to thank A.R.P. Rau for many useful conversations.

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