Attractive spin-spin contact interactions in the Einstein-Cartan-Sciama-Kibble torsion theory of gravitation

R. F. O'Connell
Department of Physics and Astronomy, Louisiana State University, Baton Rouge, Louisiana 70803
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Kerlick has obtained the unexpected result that the spin-spin contact interactions, which are characteristic of the Einstein-Cartan-Sciama-Kibble theory of gravitation, are attractive for the case of a totally antisymmetric spin angular momentum density τ_{ijk} which, in particular, is appropriate for the Dirac field. Using our previous techniques—where the emphasis is on the use of Lagrangian densities as opposed to energy-momentum densities—we present a simple and explicit verification of this result.

A characteristic feature of the Einstein-Cartan-Sciama-Kibble (ECSK) theory of gravitation[1] is the appearance of spin-spin contact (SSC) interactions. In a recent analysis we compared such interactions with contact interactions which arise in a quantum version of Einstein's theory. Our choice for the spin angular momentum density τ_{ijk} was

$$\tau_{ij}{}^{k} = \epsilon_{ijmn} U^n U^k S^m , \tag{1}$$

which led to a repulsive contribution to the gravitational interaction from the SSC terms in the ECSK theory. However, we also cautioned[2] that different interactions may arise from the use of spin densities different from that given in Eq. (1). In fact, Kerlick[3] has shown that the SSC terms for a Dirac field actually enhance the attractive nature of the gravitational interaction, whereas the opposite result (repulsive contribution) is obtained for a semiclassical spinning fluid. This arises from the fact that τ_{ijk} is totally antisymmetric for the Dirac field. Now Kerlick's analysis was based on the use of energy-momentum tensors, which of course were necessary for his discussion of cosmological models. Here we point out that, if one's attention is confined to an analysis of the basic nature of the interaction, the simplest approach is via the use of a Lagrangian density.

As in Ref. 1, our starting point is the non-Riemannian contribution to the total Lagrangian, viz.,

$$\Delta\mathcal{L} = k(-\tfrac{1}{2}\tau_{ijk}\tau^{ijk} + \tau_{ijk}\tau^{jki} + \tau_{ik}{}^{k}\tau^{il}{}_{l}) , \tag{2}$$

where $k \equiv 8\pi G/c^4$.

In the case where τ_{ijk} is totally antisymmetric, it follows immediately that

$$\Delta\mathcal{L} = \tfrac{1}{2}k\tau_{ijk}\tau^{ijk} . \tag{3}$$

As with Kerlick,[3] we now write

$$\tau_{ijk} = \epsilon_{ijkl}\tau^{l} , \tag{4}$$

where[4]

$$\tau^{l} = \tfrac{1}{4}\bar{\Psi}\gamma_5\gamma^{l}\psi . \tag{5}$$

In the "rest" picture, we can write[5]

$$\tau^{l} = \tfrac{1}{4}(\psi^{+}\sigma^{l}\psi) = \tfrac{1}{2}S^{l} . \tag{6}$$

It immediately follows, from Eqs. (3), (4), and (6) that

$$\Delta\mathcal{L} = \tfrac{3}{4}kS^2 . \tag{7}$$

In other words, the SSC interaction is attractive in the case of a field whose spin angular momentum density is totally antisymmetric. This should be contrasted with the result ($\Delta\mathcal{L} = -kS^2$) obtained with the choice of τ_{ijk} given by Eq. (1).

Comparing $\Delta\mathcal{L}$ with the corresponding term which occurs in a quantum version of Einstein's theory, viz., $\Delta\mathcal{L}_E^{(1)}$ (see Ref. 1) we obtain

$$\Delta\mathcal{L}_E^{(1)} = -\tfrac{2}{9}\Delta\mathcal{L} . \tag{8}$$

Thus, the overall contribution of both spin contact terms is to give an attractive gravitational effect. Of course, repulsive gravitational forces of a different nature[1] are still present.

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[1]R. F. O'Connell, Phys. Rev. Lett. 37, 1653 (1976); 38, 298(E) (1977) contains relevant references. Also, for the most part, we use the notation of this paper.
[2]Reference 11 of Ref. 1.
[3]G. D. Kerlick, Phys. Rev. D 12, 3005 (1975).
[4]F. W. Hehl and B. K. Datta, J. Math. Phys. 12, 1334 (1971).
[5]J. J. Sakurai, *Advanced Quantum Mechanics* (Addison-Wesley, Reading, Massachusetts, 1967), pp. 107 and 108.