CONDITIONS FOR STATIC BALANCE FOR THE POST-NEWTONIAN TWO-BODY PROBLEM WITH ELECTRIC CHARGE IN GENERAL RELATIVITY

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The usual condition for static balance, for two bodies with masses and charges m_i and e_i ($i = 1,2$), is $e_i = \pm G^{1/2} m_i$. From a post-Newtonian analysis of the two-body problem, an alternate condition for static balance $e_i = \pm(Gm_1m_2)^{1/2}$ has been found. We do not know if this condition is exact beyond the post-Newtonian approximation.

The two-body post-Newtonian Lagrangian with electric charge in general relativity, [1], $\mathcal{L}$ say, may be written very simply in the center-of-mass system [2]. Let m_i and e_i ($i = 1,2$) denote the masses and charges, G the gravitational coupling constant and set $\boldsymbol{r} = \boldsymbol{r}_1 - \boldsymbol{r}_2$, $\boldsymbol{v} = \boldsymbol{v}_1 - \boldsymbol{v}_2$, $\mu = m_1m_2/(m_1 + m_2)$, and $M = m_1 + m_2$. Then it is found [2] that

$$\mathcal{L} = \tfrac{1}{2}\mu v^2 + \tfrac{1}{8}(1 - 3\mu/M)\mu v^4/c^2 + \frac{G\mu M}{r}\left[1 + \left(\frac{3}{2} + \frac{1}{2}\frac{\mu}{M}\right)\frac{v^2}{c^2} + \frac{1}{2}\frac{\mu}{M}\frac{(\boldsymbol{v}\cdot\boldsymbol{r})^2}{c^2r^2}\right]$$

$$- \frac{e_1e_2}{r}\left[1 + \frac{1}{2}\frac{\mu}{M}\frac{v^2}{c^2} + \frac{1}{2}\frac{\mu}{M}\frac{(\boldsymbol{v}\cdot\boldsymbol{r})^2}{c^2r^2}\right] - \frac{1}{2}\frac{G^2\mu M^2}{c^2r^2} + \frac{Ge_1e_2M}{c^2r^2} - \frac{G(e_1^2m_2 + e_2^2m_1)}{2c^2r^2} \quad . \tag{1}$$

In order to have static balance for all r we require that $v = 0$ and that the static $1/r$ terms and the static $1/r^2$ terms must independently cancel out in the expression for $\mathcal{L}$. We thus must have

$$e_1e_2 = Gm_1m_2 \ , \tag{2}$$

and

$$e_1^2m_2 + e_2^2m_1 = 2e_1e_2(m_1 + m_2) - Gm_1m_2(m_1 + m_2) \ . \tag{3}$$

Using eq. (2) in eq. (3) we obtain

$$m_1e_2(e_2 - e_1) = m_2e_1(e_2 - e_1) \ . \tag{4}$$

It thus follows that there are two solutions for static balance viz. the usual one [3]

$$e_i = \pm G^{1/2}m_i, \quad i = 1,2 \ , \tag{5}$$

and a new condition

$$e_i = \pm(Gm_1m_2)^{1/2}, \qquad i = 1,2 \ . \tag{6}$$

In the special case where $m_1 = m_2$ the two solutions of eq. (5) and (6) become the same. Whereas the former condition is true exactly [3], it is not apparent whether the latter condition will hold for the exact two-body problem.

References

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